Spring 2002
EEE 532 Homework Set 1
Due: 22 February 2002

1. **Hadamard transform (HT):** The HT of a single qubit is defined in the measurement basis $|0\rangle$, $|1\rangle$ by the matrix

$$H = \frac{1}{\sqrt{2}} \begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix}$$

   (a) Sketch a Bloch sphere to indicate the action of H on the basis states $|0\rangle$ and $|1\rangle$.

   (b) Let $|x\rangle = |x_1, \ldots, x_n\rangle$ be an n-qubit state where each x_i is either 0 or 1, i.e., $|x\rangle$ is an element of the measurement basis for an n-qubit system. The HT of $|x\rangle$ is defined by

$$H^{\otimes n}|x\rangle = H|x_1\rangle \otimes \cdots \otimes H|x_n\rangle,$$

   which is the tensor product of the HT of individual qubits. Show that this is given by the expression

$$H^{\otimes n}|x\rangle = \frac{1}{\sqrt{2^n}} \sum_{z=0}^{2^n-1} (-1)^{x\cdot z}|z\rangle$$

   where the sum is over all measurement basis states $|z\rangle = |z_1, \ldots, z_n\rangle$ and $x \cdot z = \sum_{i=1}^{n} x_i z_i$.

2. **Deutsch's problem:** Recall that Deutsch's problem is about Alice finding out whether Bob's function $f : \{0,1\}^n \to \{0,1\}$ is constant or even.

   (a) How many even functions are there from n bits to 1 bit?

   (b) For $n = 2$ simple enumeration shows that there are 6 even functions. What are the measurement results and the corresponding probabilities for each of these 6 even functions and the two constant functions in the Deutsch-Josza algorithm for this case? If the measurement outcome at the end of the algorithm is $|x\rangle = |11\rangle$ which of the 8 possible functions f are consistent with this observation?