Spring 2002
PHYS/EEE 532 Homework Set 4
Due: 19 April March 2002

1. Consider a POVM defined by four positive operators

$$P_1 = \frac{1}{2}|0\rangle\langle 0|, \quad P_2 = \frac{1}{2}|1\rangle\langle 1|, \quad P_3 = \frac{1}{2}|a\rangle\langle a|, \quad P_4 = \frac{1}{2}|b\rangle\langle b|$$

   where

$$|a\rangle = \frac{1}{\sqrt{2}}\left(|0\rangle + |1\rangle\right), \quad |b\rangle = \frac{1}{\sqrt{2}}\left(|0\rangle - |1\rangle\right).$$

   Show how this POVM can be realized as an projective measurement if one ancilla qubit is introduced.

2. For the two-qubit state

$$|\psi\rangle = \frac{1}{\sqrt{2}}|a_1\rangle\left(\frac{1}{2}|b_1\rangle + \frac{\sqrt{3}}{2}|b_2\rangle\right) + \frac{1}{\sqrt{2}}|a_2\rangle\left(\frac{\sqrt{3}}{2}|b_1\rangle + \frac{1}{2}|b_2\rangle\right),$$

   - Compute $\rho^A = \mathrm{tr}_B(|\psi\rangle\langle\psi|)$ and $\rho^B = \mathrm{tr}_A(|\psi\rangle\langle\psi|)$.
   - Find the Schmidt decomposition of $|\psi\rangle$.

3. Exercise 2.72