



RADIATION IMPEDANCE OF CIRCULAR cMUT MEMBRANES

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ABSTRACT

A method to estimate the radiation impedance of immersed Capacitive Micromachined Ultrasonic Transducer membranes is presented in this paper. The method employs the lumped element equivalent circuit of the membrane and FEM simulation results of an immersed membrane. FEM simulations are carried out by driving the membrane by a uniform force distribution and the mechanical impedance of an immersed membrane is obtained as the ratio of the average velocity phasor to the total force phasor. This “loaded” membrane impedance is inverted to find the radiation impedance of the membrane, using the lumped element parametric model of the membrane. It is shown that the radiation impedance of thin membranes is a series combination of the radiation impedance of an equal sized piston and a mass of water, within the useful bandwidth. This mass is approximately equal to the mass of water contained in the half of the hemi-spherical volume defined by the membrane aperture. The sensitivity of this mass to different membrane materials and membrane dimensions is also discussed.

INTRODUCTION

Capacitive micromachined ultrasonic transducers (cMUT) received attention due to their compatibility with the current silicon fabrication processes and due to their wide bandwidth when immersed in water. This frequency band starts from very low frequencies compared to the series resonance frequency and extends almost up to the first parallel resonance of the clamped membrane. There have been many observations on the impedance of an immersed cMUT, both through measurements and by simulation [1], [2]. When a cMUT is immersed in water, all resonance frequencies shift to lower values and there is a remarkable change in the input impedance of the cMUT. The interaction of the radiation impedance of the membrane and the membrane dynamics produces this input impedance.

In this paper, we study the mechanical input impedance of a circular cMUT membrane by means of finite element simulations (FEM). We show that although the radiation impedance of a membrane differs from that of a rigid piston in general, it can be well approximated for thin membranes within the useful bandwidth by the piston’s radiation impedance in series with an extra mass of water.

RADIATION IMPEDANCE: CLAMPED CIRCULAR MEMBRANE AND PISTON

If the radius of a rigid piston is less than $\lambda/4$ in the frequency range of interest, where λ is the wavelength in the immersion fluid, the pressure components generated in the medium everywhere on the surface are all constructive and do not interfere with each other [3], [4]. First, we consider the case when the membrane radius, a , is also less than $\lambda/4$ at all frequencies up to the first parallel resonance frequency, ω_p .

The particle velocity distribution across a clamped membrane surface is well known, both when it is unloaded and it is loaded with water. This distribution has a profile similar to the one in Figure 1 (a) as long as the frequency of excitation is less than the first parallel resonance frequency and is not significantly larger than the series resonance frequency, ω_s . The particle velocity is always in the same sense everywhere across the surface. The contribution to the average velocity is constructive at all regions of the membrane. Furthermore, since the

amplitude of particle displacement is very small compared to the radius of the membrane, the membrane surface is basically planar. In this frequency range, the average velocity describes the motion of a rigid disc whose radius is equal to that of the membrane. This is similar to defining a piston moving with average velocity, which is less than the peak velocity, and with an effective aperture of equal size with the membrane. The radiation resistance of the membrane at these frequencies must be similar to that of a piston.

The radiation reactance of the membrane must be different than the piston's reactance, because there is a residual particle velocity distribution across the membrane surface other than the average velocity. This residual velocity moves a mass of water and takes energy from the mechanical input, hence must contribute to the radiation reactance.

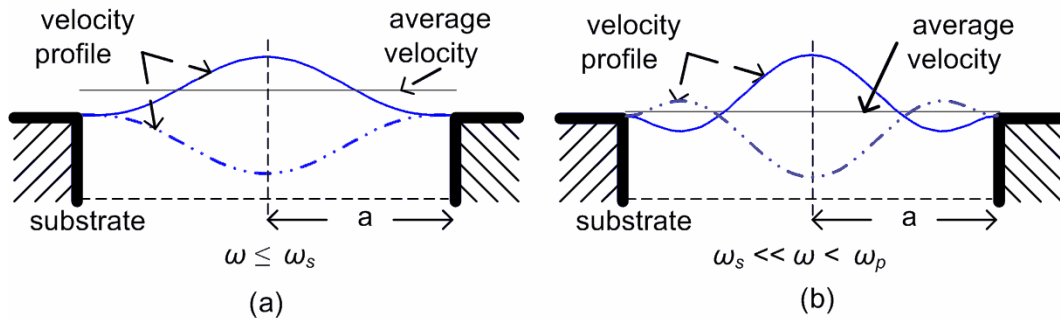


Figure 1 - Velocity profile of a cMUT membrane

When we increase the frequency to the vicinity of the parallel resonance of a small aperture membrane ($a < \lambda/4$), the particle velocity is no longer in the same sense across the membrane. Local phase distribution of the particle velocity at the outer region becomes opposite of that of the center, as depicted in Figure 1(b). The average velocity can become very small, yielding a high mechanical input resistance. However the radiation resistance must essentially be equal to that of a rigid piston, since $a < \lambda/4$ condition is maintained. The difference between the input impedance of the membrane and its radiation impedance is because of the membrane mechanics.

If the radius of a rigid piston is large compared to $\lambda/4$ at the operation frequency, contribution of the mutual effects of the forces generated in the medium by the central part and the periphery of the piston becomes destructive, while the particle velocity is still uniformly distributed across the piston [4]. This effect eventually sets a limit on the radiation impedance as the frequency is increased.

Large membranes are affected differently by this phenomenon, because the distributions of particle velocity amplitude and its phase are not uniform. The peripheral region contributes to radiation to a very limited extent in clamped membranes. The central region is the main driver. The force generated in water by the peripheral region is minimal and hence its destructive addition effect in the central region is ineffective. We can expect larger radiation resistance compared to an equal size piston. For example, it has been shown in [4] that we can have a significantly larger radiation resistance in a 5X5 array of piston elements, each with $\lambda/4$ radius and separated by $5\lambda/4$, compared to a large piston of 25 times the area of individual array elements.

MECHANICAL INPUT IMPEDANCE OF CLAMPED MEMBRANE

FEM simulations provide very accurate estimates of the behavior of immersed membranes. We simulated immersed clamped 20 micrometer radius silicon nitride membranes with different thickness, l_m , and calculated their mechanical input impedance. The thickness of the membrane is chosen such that ratio of the membrane radius to its thickness, a/l_m , are 5, 10, 20, 40 and 80. The membrane is driven by a uniform force distribution in these simulations and the mechanical input impedance, $Z_{in}(\omega)$, is calculated as the ratio of total force phasor to the average particle

velocity phasor on the surface. The real part of $Z_{in}(ka)$ of each membrane and an equal sized piston, normalized to the product of surface area and the specific acoustic impedance of water, is given in Figure 2(a). k is wave number in water, $2\pi/\lambda$. The stiffening effect of increasing membrane thickness shifts the resonances to higher frequencies. The relative positions of parallel resonance frequencies are clearly marked by an impedance peak for each membrane. For example, the parallel resonance of a $4\mu\text{m}$ thick membrane, $a/l_t = 5$, occurs at a frequency which is more than five times the resonance frequency of a $1\mu\text{m}$ thick membrane, $a/l_t = 20$. The input resistances of thin membranes follow the input resistance of a thick one at lower frequencies and depart from this impedance as the frequency approaches to the parallel resonance of a particular membrane.

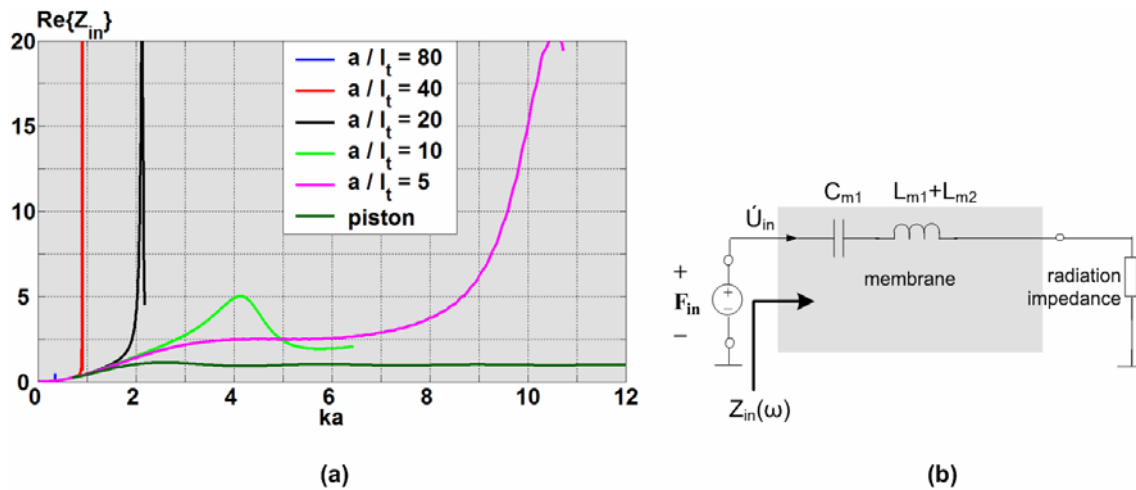


Figure 2 - (a) Real part of the normalized input impedance for a piston and for membranes with radius to thickness ratio, a/l_t , of 5, 10, 20, 40 and 80, as a function of ka ; (b) Low frequency model relating Z_{in} and radiation impedance through membrane mechanics.

Estimating the radiation impedance of a clamped membrane from $Z_{in}(\omega)$

$Z_{in}(\omega)$ includes the radiation impedance combined with the membrane mechanics. Extracting the radiation resistance at all frequencies from $Z_{in}(\omega)$ is possible only if an accurate analytical model of the membrane is available. On the other hand, a series L-C model describes the membrane dynamics adequately at frequencies significantly lower than the parallel resonance of a particular membrane. For example, at the series resonance frequency, which is usually about 4 times less than the frequency of the parallel resonance in an immersed membrane, the real part of radiation impedance can be estimated as the real part of the input impedance. The approximate connection model between $Z_{in}(\omega)$ and the radiation impedance for a clamped membrane at low frequencies is depicted in Figure 2(b). A good estimate of radiation impedance is possible by subtracting the reactance of series model from the $Z_{in}(\omega)$ for each membrane.

A method to determine membrane model parameters accurately for thick membranes is given in [5]. We used these model parameters for each membrane (each thickness) and obtained an estimate of the radiation impedance, $Z_R(\omega)$, for each membrane. The real and imaginary parts of $Z_R(ka)$, $R_R(ka)$ and $X_R(ka)$, again normalized to the product of surface area and the specific acoustic impedance of water, is depicted in Figure 3. The radiation impedance estimate for each membrane is plotted up to a frequency close to but less than its parallel resonance in order to differentiate between the membranes. Accuracy of the estimated respective radiation impedances deteriorates after the about twice the frequency of series resonance of each membrane.

The thickest membrane, $a/l_t = 5$, has a series resonance and a parallel resonance approximately at $ka = 2.5$ and $ka = 10.5$, respectively. Its estimated radiation impedance is very accurate up to about $ka = 2.5$. Since the membrane is thick, and hence much stiffer, its resonances occur at very high frequencies compared to thinner ones of same radius. This allows us to observe the variation of impedance at relatively high values of ka , but still at low

frequencies compared to its parallel resonance. $R_R(ka)$ of this membrane is similar to piston radiation resistance for very low values of ka , but a positive difference becomes very significant as ka is increased. The morphology of variation in $R_R(ka)$ is in agreement with the analysis in [4], as discussed above. The reactive part, $X_R(ka)$, however, displays a positive difference even at very low ka range.

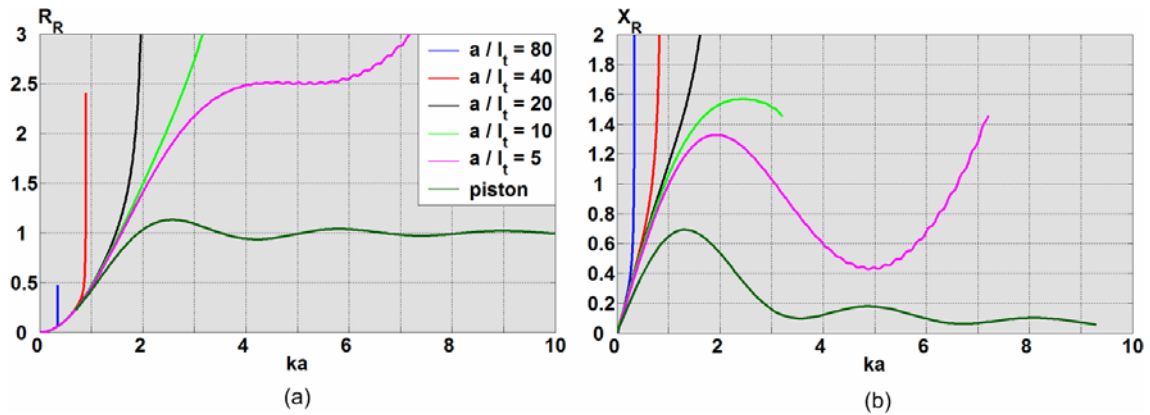


Figure 3 – Estimated radiation impedance of clamped circular membranes (a) normalized radiation resistance; (b) normalized radiation reactance.

RADIATION IMPEDANCE OF A CLAMPED MEMBRANE

Both $R_R(ka)$ and $X_R(ka)$ of each of the thinner membranes follows the $Z_R(ka)$ of the thickest membrane, at lower frequencies and deviates to higher values as the frequency approaches to respective parallel resonance. The series resonances of these membranes occur at ka values lower than 2.5. $Z_R(ka)$ of thickest membrane models the radiation impedance of all other thinner membranes accurately within their respective validity range.

cMUT designs usually have thin membranes. The useful operational frequencies of such membranes fall into a frequency range where $ka < 1$. R_R of thick membrane is similar to a rigid piston radiation resistance in this range, whereas X_R shows an increasing deviation from piston reactance. We fitted a straight line to this difference for $ka < 0.5$ and found that the X_R differs from piston reactance by an inductance corresponding to 20.2% of the mass of water contained in a spherical volume with radius equal to the radius of the membrane, i.e. $(0.20)(4/3)\pi a^3 \rho_w$, where ρ_w is the density of water. Although a linear difference is accurate within this range, the difference gets increasingly larger as ka is increased, reaches to a peak at about $ka = 2.5$ and then decreases. The radiation impedance of thin membranes is basically equal to the radiation impedance of an equal sized rigid piston plus a series connected extra water mass loading.

Extra mass loading in thin membranes

The combination of extra inductance and the radiation reactance of the piston cause a shift in the series resonance frequency when the membrane is loaded by water. This extra inductance represents the mass of water put into motion by the membrane, apart from the piston action, and its value depends on the membrane surface profile.

We studied the acoustic loading effects at different membrane stiffness levels for low ka range. The stiffness of a membrane increases by the third power of its thickness, and its mass increases linearly with the thickness. While increasing the radius makes the membrane more compliant, increasing the thickness yields a stiffer membrane. We limited our discussion to the stiffness levels of the materials used in present cMUT designs and radius to thickness ratios, a/l_t , larger than 20.

We simulated unbiased silicon nitride membranes with a/l_t ratios of 20, 30, 40, 60 and 80, in immersion and obtained the membrane input impedance, Z_{in} . Respective estimates of radiation reactance are obtained by removing the effect of series L-C section. We calculated the extra water mass loading, using a rather low frequency range, from zero to the series resonance of

each immersed membrane. We applied a regression analysis on these masses and found that the extra water mass involved is 25.3 % of the spherical volume of water of radius a . The mean square error in the regression is 4%. This mass is approximately equal to one half of the hemispherical water volume defined by membrane radius and is independent of the thickness of the membrane.

We also examined this proportionality between the extra mass involved when membranes are made of different materials and when they are biased.

For an unbiased membrane, the Young's modulus of the membrane material is varied between $0.5Y_0$ and $2Y_0$, where $Y_0 = 320$ GPascal is the Young's modulus of silicon nitride. The density of the material is varied again between half the density of silicon nitride and twice its density. The proportionality constant between the extra mass loading and the mass of water contained in the spherical volume, α_c , is 0.247 for the stiffest and heaviest combination and 0.266 for another combination. The difference between these figures is 7%.

The biasing has more pronounced effect on α_c at low ka range. We simulated four immersed cMUTs with silicon nitride membranes to observe the effect. All membranes had the same radius of 20 microns, but two of them were 0.25 micron thick and the other two were 1 micron thick. We employed two different gap heights, d_o , 0.25 μ m and 0.75 μ m for each one of two membranes. These dimensions correspond to a/t ratios of 80 and 20 and a/d_o ratios of 80 and 26.7, respectively. The applied dc voltage is taken as 90% of collapse voltage in each case.

When the gap height and thus the bias voltage are low, α_c remains at 0.247 for thinner membrane and 0.267 for thicker membrane. When the gap height is higher at 0.75 μ m, however, the coefficient for thicker membrane increased to 0.297 and the one for thinner membrane decreased to 0.231.

An exhaustive examination of this extra series water mass loading is needed to fully characterize the radiation loading on membrane. However, taking $\alpha_c \approx 0.25$ approximates this water mass well enough for thin membranes, for all possible membrane materials and for all possible operating scenarios as long as the gap height is not very large.

Radiation impedance of a thin membrane can be approximated by a series combination of the radiation impedance of a piston of same diameter with the membrane and an extra inductance equal to the half of the hemispherical volume of water above the membrane, within its useful bandwidth.

CONCLUSIONS

A radiation impedance of circular cMUT membranes can be deduced from the mechanical input impedance of very stiff membranes. We calculated the mechanical impedance of an immersed membrane, removed the effect of membrane mechanics, to obtain an estimate of the impedance observed at the acoustic port. We showed and discussed that this estimate of a thick and hence a stiff membrane models the radiation impedance of thinner membranes, which have lower parallel resonance frequencies. cMUT membrane radiation resistance and reactance are significantly larger than that of a rigid piston, except for very low values of ka . For $ka > 3.5$, radiation resistance of a cMUT membrane is approximately 2.5 times the radiation resistance of an equal sized piston.

Most of current cMUT designs incorporate thin membranes and operate at low ka range. We presented an accurate approximate model for the radiation impedance of such membranes, comprising the radiation impedance of a rigid piston and a series extra mass of water, $(\pi a^3 \rho_w)/3$.

Membrane thickness scales the frequency such that series and parallel resonances shift to higher frequencies as the membrane gets thicker. It is shown in [5] that membranes with different radius-to-thickness ratios but made of same material have different *normalized* model parameters, although the differences are generally small. This is because the velocity profile of each membrane is somewhat different at a comparable frequency, such as, e.g. 70% of the

respective parallel resonance frequency. This difference in velocity profile must affect the radiation impedance to some extent. However it is shown in this paper that the effect is negligible for low values of ka .

A complete description of the radiation impedance can be obtained, if the effect of increasing membrane material stiffness is studied over a large ka range and compared to the large thickness effect discussed in this paper. While the resonance frequencies are again scaled by the material stiffness, it does not have any effect on the *normalized* model parameters of a membrane with a given thickness when operated in vacuum [5]. The membrane velocity distribution profile is the same at the respective frequencies in vacuum, unlike the effect of thickness. It is reasonable to expect that velocity profile of an immersed membrane is affected to a lesser extent when the material stiffness is increased instead of its thickness. This can be assessed by considering extremely stiff materials. When such extremely stiff materials are used in FEM analysis, the parallel resonance occurs at very high ka values. It is then possible to observe the ka range where the radiation resistance and reactance becomes flat, isolated from the effect of membrane mechanics. Using such materials and an approach similar to the one presented in this paper, the radiation impedance of a membrane with any thickness can be found at large ka values. The effect of thickness on the radiation impedance for large ka can be determined. The effect of biasing can similarly be assessed.

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