OPTIMUM SPECTRAL SHAPING FOR DISCRETE MULTITONE DEMODULATION

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ABSTRACT

In this paper the problem of optimum shape (frequency weighting) of signal and noise before discrete-multitone demodulation (DMT) is analysed. Such a weighting is usually possible to some extent with the time domain equalizer (TEQ). The optimality criterion applied is the total capacity over all frequency subchannels. Optimum weighting is shown to be the maximum of a nonlinear function of weights. The feasible approach to solving the problem is a nonlinear Newton-type search supported by analytic derivative of the criterion. However, a suboptimal easy to evaluate solution is proven to be noise whitening weighting which achieves optimality with negligible bias. This weighting is shown to compensate FFT noise spreading in various noise and SNR conditions. These results form a step towards fast design technique of TEQ equalizer with objectives of impulse response shortening and spectral shaping.

1. INTRODUCTION

Nowadays, data transmission uses broadly digital signal processing algorithms instead of analog circuits. That is also the case with modulation and demodulation in wireline and wireless multi-frequency transmission systems [1]. The pair of Inverse Fast Fourier Transform (IFFT) and Fast Fourier Transform (FFT) algorithms is applied in discrete multitone (DMT) wireline setup of ADSL and VDSL modems and in orthogonal frequency division multiplexing (OFDM) setup of wireless WiFi 802.11a and WiMAX 802.16 standards [2], [3]. Numerical frequency-time IFFT transform assures stable independency (orthogonality) of frequency subchannels. The role of these transforms in DMT transmission is explained in figure 1.

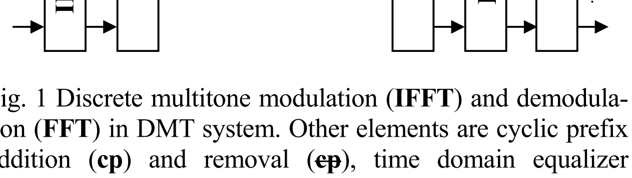


Fig. 1 Discrete multitone modulation (**IFFT**) and demodulation (**FFT**) in DMT system. Other elements are cyclic prefix addition (**cp**) and removal (~~**cp**~~), time domain equalizer (**TEQ**), frequency domain equalizer (**FEQ**). Signals are distorted by fading channel (**h**) and noise (**s**).

The focus of the paper is on influence of demodulation filters realized via FFT on transmission characteristics. The IFFT-FFT modulator-demodulator pair creates orthogonal frequency subchannels. However, with orthogonality principle held, the subchannels are overlapping besides tone-grid. That may cause degradation of quality of a good channel with relatively low noise and high SNR by high noise from a neighbouring channel (e.g. interfered by amateur radio). This degradation can be lowered in the receiver by spectrally shaping equally signal and noise before demodulation to balance mutual influence of the channels. Shaping signal power at the transmitter, with change of incoming SNR, is the well known water-filling algorithm. In contrast, optimum shaping cooperation of the usual TEQ with FFT demodulator, has not been reported in literature. The solution to this problem is developed in the following sections.

2. FFT NOISE SPREADING

Frequency subchannels of DMT/OFDM transmission are mutually independent as far as signal is concerned, what is the principle of orthogonal frequency division multiplexing. Even rather poor sidelobe attenuation of FFT filters at -13dB does not matter in this case, because zeros of demodulation filters lie at the centres of other filters, i.e. at other tones.
However, if we consider noise with a continuous powers spectral density (PSD), the leakage effect appears (also called noise smearing or FFT spreading) [1]. In this case sidelobe attenuation does matter as well as the width of main lobe (frequency selectivity of the demodulator). Both transmission principles are presented in figure 2.

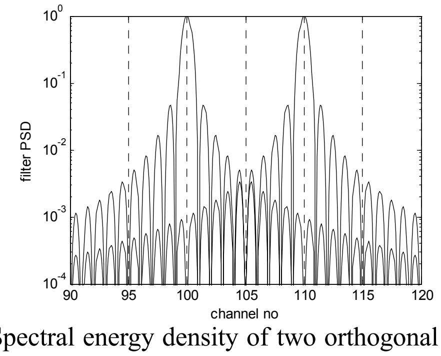


Fig. 2 Spectral energy density of two orthogonal demodulator filters of FFT for channel 100 and 110

To state the problem in a simplified form feasible for general parametric approach we introduce spectral noise spreading matrix D, with element $D_{i,j}$ representing mean energy of the i-th FFT filter in the j-th subchannel. That is clearly discrete approximation of the characteristic presented in figure 2 with the following properties:

- dimension of D is (n,n), where n is the number of subchannels (tones),
- elements of D are greater than 0 and lower then 1,
- diagonal elements are lower then 1,
- rows sum up to 1 ($\sum_{j=1\ldots n} D_{i,j} = 1$, i=1..n).

Sample of the content of D matrix is presented in figure 3.

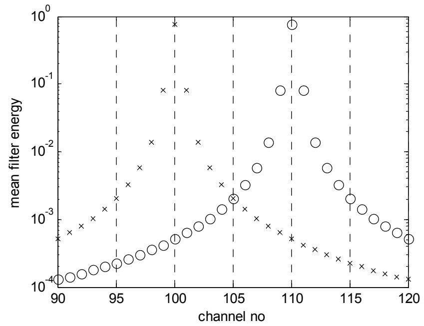


Fig. 3 Rows of noise spreading matrix D corresponding to two orthogonal FFT demodulator filters at channel 100 (crosses) and 110 (circles) (compare with figure 2)

Using this matrix we can approximately describe noise after demodulation at the output of i-th subchannel as $N_i^{out} = \sum_{j=1..n} D_{i,j} N_j^{in}$, where N_j^{in} is the mean PSD of noise in j-th subchannel before FFT demodulation.

3. OPTIMALITY CRITERION

The criterion of efficiency of DMT transmission commonly adopted [4], [5], [6], [7] is the achievable bitrate at the assumed bit error rate (BER). The integer valued bit rate is directly related to the real-valued channel information capacity, which is much easier to optimize. Channel capacity is dependent on signal power P and noise power N at the output of the channel. The noise N comprises thermal and quantization noise, crosstalk and radio interferences, and intersymbol/intrasymbol interferences. Signal P is severely attenuated by transmission line at higher frequencies. DMT transmission divides wideband channel into a set of nearly AWGN narrowband frequency subchannels. Every i-th subchannel is characterized by signal (tone) power density P_i and mean level of noise power density N_i. These quantities observed at the output of the transmission line in the i-th subchannel form signal to noise ratio $SNR_i^{line} = P_i / N_i$.

The usual equalization of the line impulse response using TEQ selectively attenuates/amplifies both signal and noise and at the same time changes the frequency content of interferences. Thus, operation of equalizer completely eliminating interferences can be seen as shaping (weighting) of signal and noise power spectral densities without change of the resulting SNR. We note it for i-th subchannel as (1), where W_i is the resulting weight, $G(e^{j\Omega_i})$ is the TEQ complex value at i-th tone (frequency Ω_i), j is the imaginary unit.

$$W_i = | G(e^{j\Omega_i}) |^2 \qquad (1)$$

As was shown in section 2, imperfect multichannel FFT demodulator changes the SNR in subchannels due to noise leakage from neighbouring subchannels. Using previously introduced matrix D and weighting W we express SNR after TEQ and demodulation in i-th subchannel as

$$SNR_i = \frac{W_i P_i}{\sum_{j=1..n} D_{i,j} W_j N_j} .$$

Summing Shanon's capacities [1] of individual frequency subchannels we get total capacity expressed in bits per symbol (QAM code is two-dimensional) expressed as:

$$c = \sum_{i=1\ldots n} \log_2 \left(1 + SNR_i\right) \qquad (2)$$

To well pose the problem we constrain weights W_i :

- to be nonnegative ($W_i \geq 0$, i=1 … n),
- to sum up to n ($\sum_{i=1..n} W_i = n$).

Let us note that the weights are not related with any specific TEQ design, as we assume freedom of their values apart from practical realization. Now, we proceed to maximization of capacity c with respect to weights W_i.

4. OPTIMUM SPECTRAL SHAPING

The maximization problem at first seems to be highly nonlinear. As a practical approach we suggest nonlinear search by gradient or quasi-Newton algorithm supported by analytical derivative, which for the criterion (2) is:

$$\frac{\partial c}{\partial W_i} = \frac{1}{\log 2} \left(\frac{SNR_i}{1 + SNR_i} \frac{1}{W_i} - \sum_{j=1\ldots n} \frac{SNR_j}{1 + SNR_j} \frac{D_{j,i} N_i}{\sum_{k=1\ldots n} D_{j,k} W_k N_k} \right)$$

That approach, while applied for different transmission lines and noises, has been fast-converging to the same solution from different starting points. However, no analytical proof of unimodality of the function is provided. The results of this approach are presented further on examples.

Instead of solving this complicated optimization problem, we observe that optimal solution for W together with FFT demodulator represented by D cannot do much better than demodulation with D being identity matrix (perfect rectangular filter). In fact the slight advantage of optimum shaping appears in peculiar circumstances as will be shown on examples. Thus we are able to predict suboptimal solution as:

$$W_i = w / N_i , \qquad i=1\ldots n \qquad (3)$$

where w is the normalizing constant resulting from assumed constraint. That is the simple noise whitening solution. In fact, with this weighting the joint operation of TEQ and FFT is transparent for SNR ratio (note properties of D):

$$SNR_i = \frac{w P_i / N_i}{w \sum_{j=1\ldots n} D_{i,j} N_j / N_j} = P_i / N_i \equiv SNR_i^{line} .$$

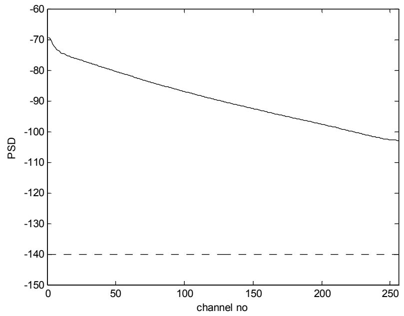
As is shown further, the difference between optimal and suboptimal solution is negligible for realistic spectra, as it is below 1 bit/symbol at capacity above 3 kbit/symbol.

5. EXAMPLES

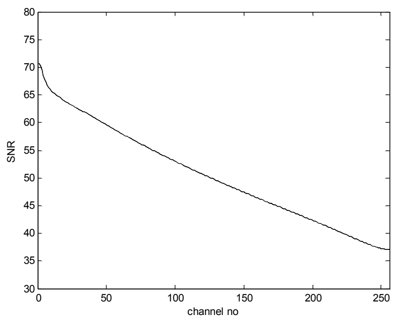
To show the properties of the optimum and suboptimum solutions, typical ADSL environment has been simulated, with 9kft, 26AWG transmission line, uniform transmit signal power spectral density at –40dBm/Hz, and different noise spectra. We assume prefect line equalization (no ISI/ICI interferences).

Results in figures 4, 5, 6 confirm convergence of optimal and suboptimal solution as well as possibility of balancing noise before demodulation to make the pair weighting-demodulation transparent for *SNR* ratio.

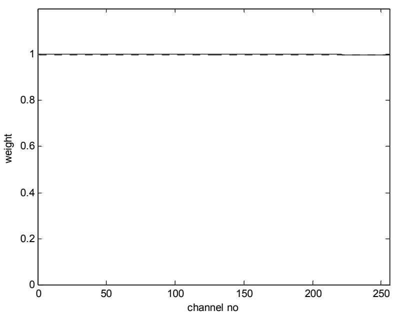
Figures 7 a, b present the problem with the MSSNR [8] approach to equalizer design. More TEQ taps result in worse weighting and high SNR loss in FFT demodulation. Shorter TEQ results (by chance) in weighting closer to optimal and the loss of *SNR* and capacity is much lower.



a) signal (solid line) and noise (dotted line) at the output of the line

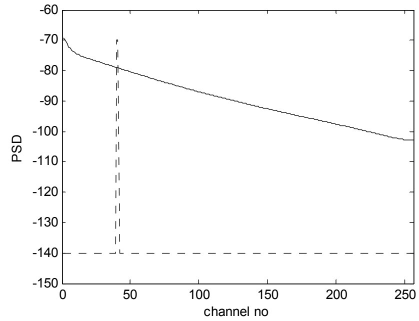


b) SNR at the output of the line, after FFT with unit and optimal weighting (lines are overlapping).

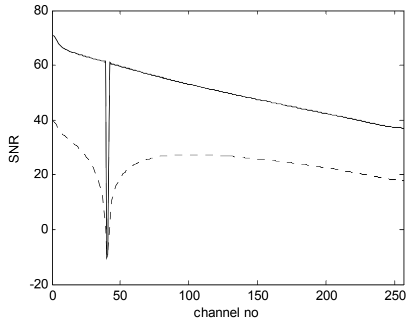


c) optimum and suboptimum weighting (lines are overlapping)

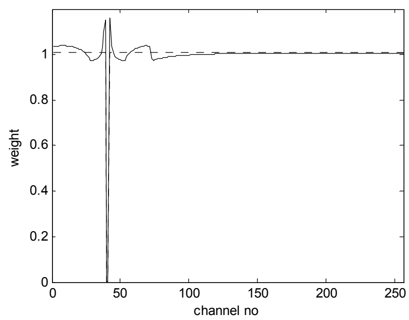
Fig. 4 a, b, c. Results for AWGN noise at -140dBm/Hz.



a) signal (solid line) and noise (dashed line) at the output of the line

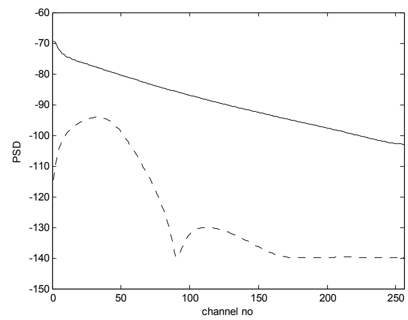


b) SNR at the output of the line (solid line), after FFT with unit (dotted) and optimal (dashed, overlapping with solid) weighting.

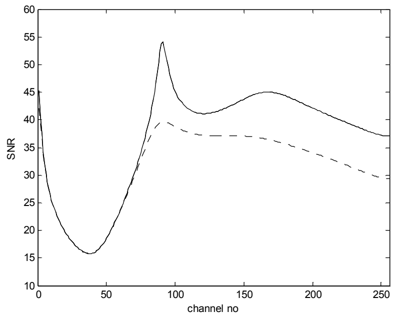


c) optimum after 6 iterations of quasi-Newton algorithm (solid) and suboptimum (dotted) weighting

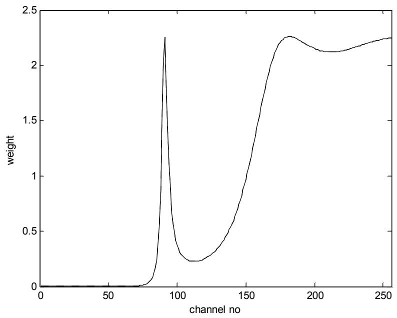
Fig. 5 a, b, c. Results for AWGN noise and radio interference at -70dBm in channels 42-43. Optimal solution slightly outperforms suboptimal one but the difference is negligible.



a) signal (solid line) and noise (dashed line) at the output of the line



b) SNR at the output of the line (solid), after FFT with unit (dashed) and optimal (wide-dashed, overlapping with solid) weighting.



c) optimum after 6 iterations (solid) and suboptimum (dashed) weighting (lines are overlapping)

Fig. 6 a, b, c. Results for AWGN noise and 20 HDSL crosstalkers

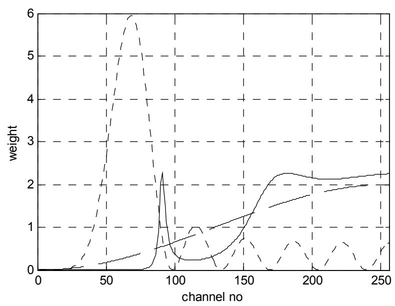


Fig. 7.a Spectral shaping of optimal weighting (solid line), MSSNR 2-tap equalizer (wide-dashed), 16-tap MSSNR equalizer (dashed), for signal and noise as in figure 6.

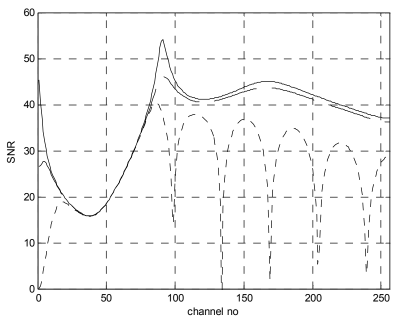


Fig. 7.b SNR for corresponding spectral shaping from figure 7.a. Capacities are 3.11, 3.01, 2.27 kbit/symbol for respective shaping (no ISI/ICI interferences included)

6. CONCLUSIONS

The capacity of the multitone set of frequency subchannels strongly depends on the properties of the FFT demodulator. The exact solution to the problem of maximizing capacity by optimum weighting of signal and noise before FFT demodulation is mathematically complex but numerically tractable. However, simple form noise whitening solution achieves in practice the maximum capacity, as is derived and shown on examples. This simple weighting solution is a potential way to fast equalizer design joining response shortening and optimum shaping, what is developed now by the author.

The criterion adopted in this paper is not directly integer valued bit rate but continuous real valued capacity. The results evaluated for bitrate may differ slightly from the presented ones. Also, the equalization is assumed to perfectly shorten line impulse response. In practice some level of ISI/ICI interferences always remains and is sometimes significant portion of the noise. Thus, the TEQ influence is not only weighting without changing SNR (as assumed in the paper) but also adding ISI/ICI noise to subchannels [9].

Finally, let us observe that defining matched filter bound for DMT transmission, as the quantity determined by noise only without ISI/ICI interferences does not require taking into account negative influence of imperfect FFT demodulator. As is shown in the paper, suboptimal noise whitening weighting makes the pair weigthing-demodulation transparent for subchannel *SNR* ratio.

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