A COMPARATIVE STUDY OF SOME TIME-FREQUENCY DISTRIBUTIONS USING RENYI CRITERION

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ABSTRACT

The paper presents a quantitative comparison study of some time-frequency distributions i.e. (TFDs). The comparison is in terms of a criterion known as the Renyi measure. The assessment of the TFDs is accomplished by evaluating the Rényi measure which yields the best time-frequency resolution. We show, using synthetic as well as real-life data, that a recently proposed TFD outperforms existing TFDs. In particular, we show that this proposed TFD presents a high time-frequency resolution while suppressing the undesirable cross-terms.

1. INTRODUCTION

In this paper, we present a quantitative comparative study between some existing time-frequency distributions (TFDs) and a recently proposed one [1] in the analysis of multi-component signals. In the comparison, we consider the cross-terms suppression and the high energy concentration of the signal around its instantaneous frequency (IF).The widely used spectrogram (SP), which is in general a cross-terms free TFD, suffers from the undesirable trade-off between time resolution and frequency resolution [2], [3], [4], and [5]. On the other hand, the Wigner-Ville distribution (WVD) has a high time-frequency resolution but is known to suffer from the presence of cross-terms [2]. To address the problem of cross-terms suppression, while keeping a high time-frequency resolution, other TFDs have been proposed. Among these, one can cite the Zhao-Atlas-Marks distribution (ZAMD) [6], the B-distribution (BD) [7] and the Modified B-distribution (MBD) [8], just to name a few. In the sequel, we will compare these reduced cross-terms TFDs to a newly proposed TFD, inspired from the Butterworth kernel quadratic TFD [9]. The comparison is performed by evaluating the Rényi criterion for each of the considered TFDs. Since these TFDs are function of some parameters, we first obtain the optimal parameters in terms of the minimal values of the Rényi information for each individual one. Then, we compare them to each other. Synthetic as well as real-life data are used in this comparative study. The paper is organized as follows: In Section 2, we give a brief theoretical background of the various TFDs used in the comparison and present some properties of the comparison tool (i.e., the Rényi information criterion). In Section 3, we present some simulations results as well as a discussion. Section 4 concludes the paper.

2. QUADRATIC TIME-FREQUENCY DISTRIBUTION

Quadratic, a.k.a. bilinear or Cohen's, TFDs constitute a powerful tool in the analysis of non-stationary signals, i.e., signals whose frequency contents vary with time. Many of these representations are invariant to time and frequency translations and can be considered as energy distribution in the time-frequency plane. The quadratic class can be expressed as [2, 4].

$$C_x(t,f)=\iiint e^{j2\pi(\xi t-\xi f-f\tau)}\Phi_x(\xi,\tau)x\left(u+\tfrac{\tau}{2}\right)x^*\left(u-\tfrac{\tau}{2}\right)du\,d\tau\,d\xi \quad (1)$$

where x (t) represents the analytical form of signal under consideration and $\Phi_x(\xi,\tau)$ is called the kernel of the distribution. All the integrals are from $-\infty$ to $+\infty$, unless otherwise stated. A choice of a particular kernel function yields a particular quadratic TFD with its own specificities [2], [3]. All the reduced cross-terms TFDs mentioned earlier are members of the quadratic class. In particular, the kernel of the Butterworth kernel is given by [9]

$$\Phi x(\xi,\tau)=\frac{1}{1+\left(\frac{\xi}{\sigma_\xi}\right)^{2N}\left(\frac{\tau}{\sigma_\tau}\right)^{2M}} \quad (2)$$

This kernel, which is a general form of the exponential kernel, is considered as a low-pass filter in the ambiguity domain. A suitable choice of the parameters N, M, σ_ξ and

σ_τ helps remove the cross-terms from the TFD, in the analysis of a multicomponent signal.

A recently proposed TFD, inspired from the Butterworth kernel, was shown to possess a good trade-off between cross-terms suppression and high time-frequency resolution [1]. This TFD kernel is expressed as

$$\Phi_x(\xi,\tau) = \frac{\pi\sigma_\xi}{1+\left(\frac{\xi}{\sigma_\xi}\right)^{2N}} \cdot \frac{1}{1+\left(\frac{\tau}{\sigma_\tau}\right)^{2M}} \quad (3)$$

Using the inverse Fourier Transform and fixing N equal to unity, we obtain the time-lag kernel expression given by

$$\Phi_x(t,\tau) = \frac{1}{1+\left(\frac{\tau}{\sigma_\tau}\right)^{2M}} \exp\left(-\pi\sigma_\xi |t|\right) \quad (4)$$

Now, by substituting expression (3) in Equation (1), we obtain the proposed TFD expression as

$$C_x(t,f) = \iint \frac{1}{1+\left(\frac{\tau}{\sigma_\tau^2}\right)^{2M}} e^{-\pi\sigma_\xi|s-t|} x\left(s+\frac{\tau}{2}\right) x^*\left(s-\frac{\tau}{2}\right) e^{-j2\pi f\tau} ds\, d\tau \quad (5)$$

In this comparison study, we have used the Rényi information of order α defined as:

$$R_\alpha = \frac{1}{1-\alpha} \log_2 \iint C_x^\alpha(t,f)\, dt\, df \quad (6)$$

α: Rank of the Rényi measure, $\alpha \geq 2$.

This criterion was used in [10], [11], and [12] to evaluate the complexity of a signal in the time frequency plane. Four schemes have been studied in [12] but both of them using the normalized form versus volume have been proved to have useful properties. The normalization operation assures that the TFD behave like a probability density function (pdf). So minimising the Rényi entropy for a given TFD is equivalent to maximizing its concentration, peakiness and resolution [13]. Then the best parameters of kernel for TFD with respect to the minimal value of the Rényi will give a good localization of the energy. Recently in [14] the performance of minimum entropy kernels for best TFDs and component counting is also demonstrated. In our study we have used the second and the third order of Rényi entropy with volume normalized. Moreover the discrete-time formulation of the Rényi entropy for TFD [12] with normalization volume is given by:

$$RV_\alpha = \frac{1}{1-\alpha} \log_2 \sum_{k=-K}^{K} \sum_{n=-N}^{N} \left(\frac{C(n,k)}{\sum_{k=-K}^{K} \sum_{n=-N}^{N} |C(n,k)|} \right)^\alpha \quad (7)$$

where n and k are variables for discrete-time and discrete frequency respectively , N and K are number of samples in time and frequency respectively.

3. EXAMPLES AND DISCUSSIONS

All analysis is done using third order Rényi entropy RV_3 since it has been proved [10-11] that is the well choice for measuring time-frequency uncertainty. We have particularly used the normalized scheme versus the volume which has been used for adaptive kernel design [11, 12]. The goal is to

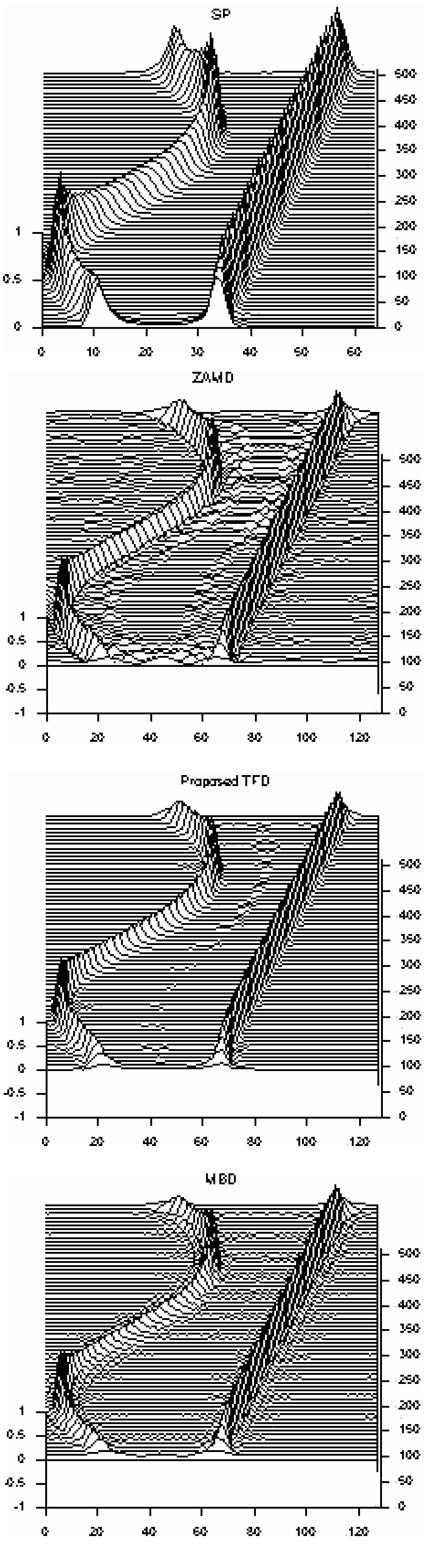

Fig. 1. TFDs results of signal given in example 1.

TABLE I
OPTIMAL VALUES OF THE KERNEL PARAMETER

TFD	nh	RV_2	RV_3
WVD		5.8912	5.9936
SP	89	5.0359	4.8583
ZAMD a=2	111	5.4923	5.2780
Proposed TFD σ_ξ=0.05	111	4.2892	4.0895
MBD β=0.05	85	4.5840	4.2538

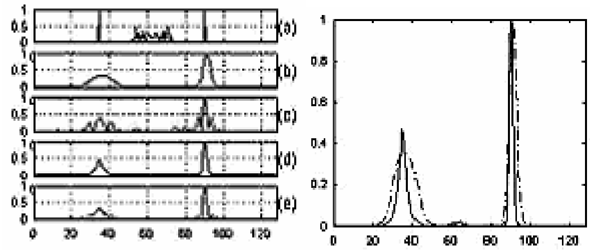

Fig. 2. Left: Slices taken at time instant n=256 for (a) WVD, (b) SP (c) ZAMD (d) Proposed TFD (e) MBD. Right: Performance comparison between SP (dashed line) and the proposed TFD (solid line). Horizontal axis shows frequency in Hz.

find based on Rényi measure the best parameters that can give better TFDs results. Then achieved comparison and interpretation between each TFDs. The data window length which controls the size of the kernel used for analysis is noted *nh*. Some results of TFDs have been realized by TFSA [15] except for the proposed TFD.

3.1. Example 1: Sinusoidal and linear FM components

In this example, the synthetic signals consist of two components the first is sinusoidal FM component and the second is linear FM given by:

$$x(t) = \cos(-50\cos(\pi t) + 10\pi t^2 + 70\pi t) + \cos(25\pi t^2 + 180\pi t)$$

A sampling frequency is equal to fs=256 Hz with a signal length equal to 512. Table I shows the optimal parameters corresponding to the minimal values of the Rényi measure for all TFDs. Each result of TFDs has been obtained after several variations of optional parameters. All optimized TFDs are represented in figure 1, where horizontal axis shows the frequency and vertical axis is the time. The Left part of figure 2 shows slices taken at the same time instant n=256 for different TFDs with optimal parameters when the right part shows slices taken at the same time instant n=256 for the SP and the proposed TFD.

3.2. Example 2: Bat signal.

In this example, the real-life signal consisting of Bat chirp signal[1] digitized at 2.5 microsecond echolocation pulse emitted by the Large Brown Bat, Eptesicus Fuscus. There are approximate 400 samples; the sampling period was 7 microseconds, the signal length used is equal to 512 and the sampling frequency is fixed equal to fs =143.72 KHz. Also, the same TFDs used in the first example are considered here. To find the optimal TFD for resolving the components of the signal we first find the optimal values of the

[1] The authors wish to thank Curtis Condon, Ken White, and Al Feng of the Beckman Institute of the University of Illinois for the bat data and for permission to use it in this paper.

TABLE II
OPTIMAL VALUES OF THE KERNEL PARAMETER FOR BAT SIGNAL ANALYSIS

TFD	WVD	SP nh=65	ZAMD nh=85 a=2	MBD nh=65 β=0.05	Proposed TFD nh=45 σ_ξ=0.05
RV_3	4.6253	2.1148	4.1835	3.3090	2.9961

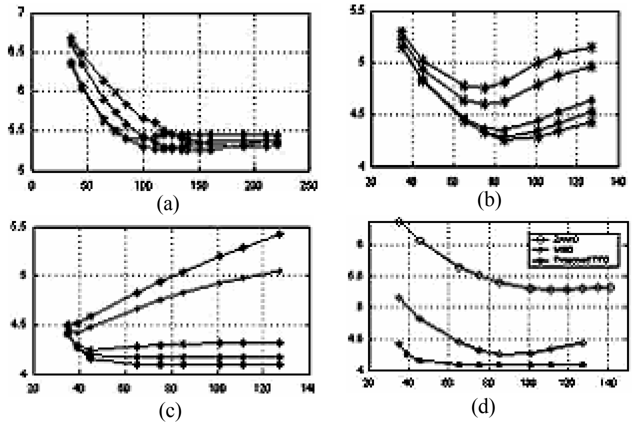

Fig. 3. Evolution of RV3 versus nh for (a) ZAMD (b) MBD (c) Proposed TFD and (d) Optimal values of RV3 for all TFDs.

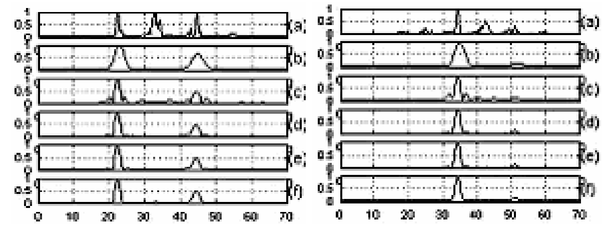

Fig. 5. Slices taken at the same time n=150 (left) and n=250 (right) for (a) WVD, (b) SP, (c) ZAMD, (d) BD, (e) MBD, (f) Proposed TFD. Horizontal axis shows frequency in KHz.

TFDs kernel parameters using Rényi criterion. The values of RV_3 have been measured for all TFDs versus each proper parameter. Different values of window lengths and optional parameters have been used in the evaluation of the Rényi measure. All the minimal values of Rényi that represent the best time-frequency concentration and elimination cross-terms have been summarized on table II. We can see also the optimal value of the kernel parameter for each TFDs. The variations of RV_3 versus the window length for each TFDs can be seen in figure 3. The results of TFDs using the optimal parameters are represented in Figure 4. We take slices of the TFDs at the time instants n= 150 and n=250 (recall that n= 1, 2…512) and we plot the normalized amplitudes of these slices in figure 5. We can see in left part of this figure that the first and the second components have appeared without cross-terms. In the right part we can see the second component and the third component without cross-terms. Also we can remark that the results of the proposed TFD show a better performance, in terms of frequency resolution. However the highest performance is achieved by the MBD and the proposed TFD for signal in consideration. Also we can remark that the proposed TFD not only can successfully appear the third components (the weakest) but it has the best resolution i.e (narrower main-lobe and smaller side-lobes) compared to all the other considered distributions.

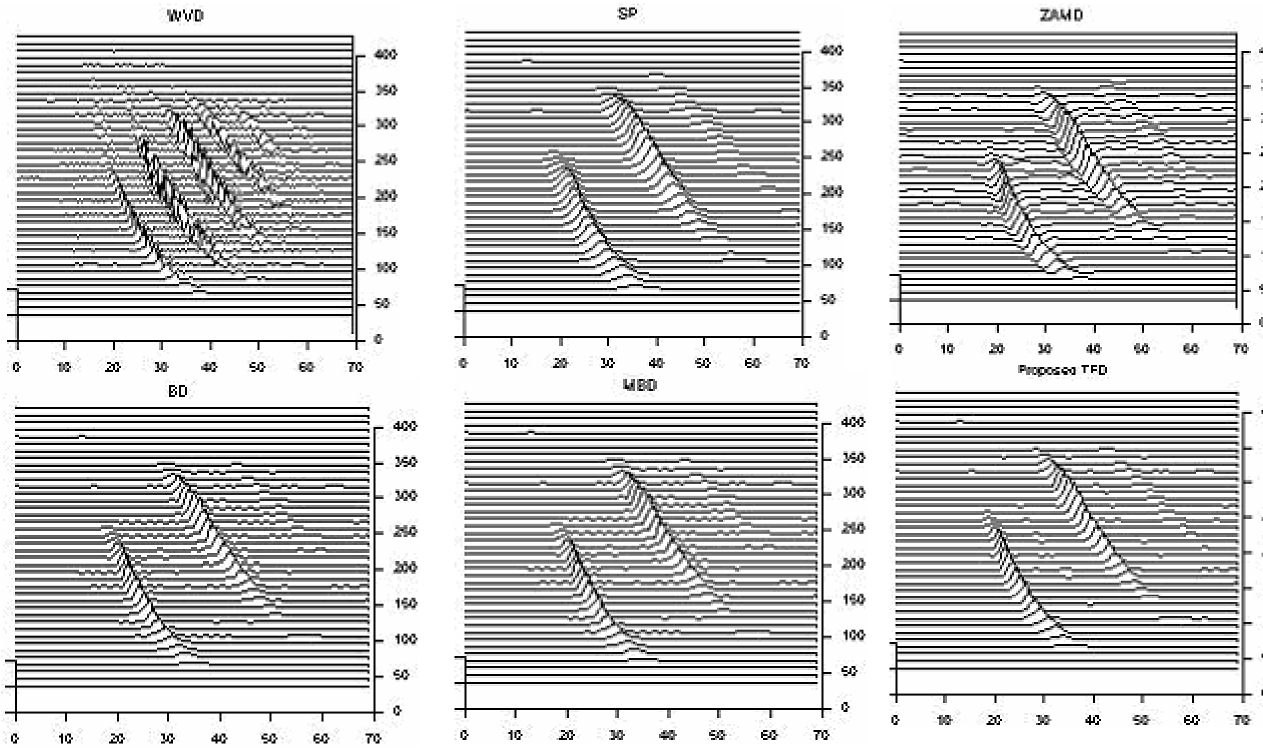

Fig. 4. Bat signal analysis by (a) WVD,(b) SP, (c) ZAMD , (d) BD σ=0.05 nh =45, (e) MBD β=0.05 nh =65 and (f) Proposed TFD σ_{ξ}=0.05 nh =45. Horizontal axis show frequency in KHz, vertical axis show time versus number of samples.

4. CONCLUSION

In this paper, we presented a quantitative comparative study of some quadratic TFDs using synthetic and real-life bat signal. We have used some distributions known for their high cross-terms suppression property in terms of trade-off between cross-terms suppression and high energy concentration in the time-frequency domain. The optimal parameters of all TFDs have been selected based on the Rényi criterion. Our study have show that the proposed TFD exhibits high resolution and very little interference terms between the signal components both on simulated or real signal.

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