CONFLICTUAL PEACETIME INTERNATIONAL POLITICS

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Abstract: Affirmative answers are given to two questions of international politics: Do the parsimonious postulates of structure theory imply any mode of behavior for states, as claimed by the theory of neorealism? Does a principle of the harmony of interests exist for states, as asserted by idealist theories? Answers emanating from an n-person game theoretic model, which portrays states as security maximizing agents that allocate their conflictual resources against each other at peacetime, can be summarized in a "principle of the harmony of security." In a world without a hegemon, if all states are cautious and ambitiously maximize their individual securities, then the world will be in a mode of bilateral-equilibrium in which all states are equally insecure. If two or more strong states vigorously pursue their individual securities, then a bilateral-equilibrium is most plausible, irrespective of whether the rest of the states are irrational, modest, or ambitious.

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1 Introduction

It is now more than three decades since Waltz outlined a structure theory in the book *Theory of International Politics* (1979). The structure of an international system, he postulated, consists of an ordering principle (anarchy or self-help), a function of units of the system (like-units called states), and the distribution of capabilities across the units (relative capabilities of states). He further asserted that "states seek to ensure their survival"(1979, p. 91) or that "the dominant goal of states is security" (1997, p. 915). This motive together with the structural constraints, he claimed, leads to the behavior of balancing: "balances of power tend to form whether some or all states consciously aim to establish and maintain a balance, or whether some or all states aim for universal domination" (1979, p. 119). This is Waltz's balancing imperative.

Since then, the neorealist theory, as the structure theory has come to be known, was subjected to heavy criticisms, not only by opponents of realism, but also by holders of realist viewpoint. Each and every postulate of Waltz, the motive he attributed to states, his balancing imperative, and the derivation of it have all been challenged.

Constructivists maintain that the ordering principle among states is not a primitive of the international system but is a social construct, Wendt (1992), so that "self-help" is in principle interchangeable with "collective security." They also draw attention to "ideational factors" that influence states and to states being embedded in broad transnational activities, Ruggie (1998, 2004). Institutionalists, likewise, question the validity of the assumption of anarchy as well as the definition of units in Waltz's international system as institutions, interdependence, and globalization seem to seriously weaken the autonomy of states, Keohane and Nye (1977) (also see Ohmae (1990), Friedmann (1999)). Liberal democratic view puts forward that states are not like-units at all since democratic states are fundamentally different and are more peaceful than non-democracies, Doyle (1983), Fukuyama (1992), Russett (1993). The postulate that the distribution of capabilities across units is the third pillar of structure is challenged by Wendt (1995) as he argues that it is made not only of material resources but also by shared knowledge and practices. A constant criticism is that international politics and unit-level internal characteristics, like leadership or foreign policy strategies, should be combined in order to incorporate explanatory power, Schweller (1997), Zakaria (1998).

Different motives, such as straining for ever more power (classical or offensive realist motive), responding to a threat (Walt, 1987), expectation of easy gains (Christensen and Snyder, 1990) may lead to quite different modes of behavior than balancing, such as bandwagoning, buck-passing, chain-ganging, and balancing of interests (Schweller, 1994). Each such alternative motive has found support within the realist school and has been argued to be either more basic than or complementary to the motive of security.

The questions one can ask are: Can so minimal assumptions as those of structure theory imply any mode of behavior for states? Does structure indeed shape and shove? Does motive of security imply any kind of balancing behavior? These are questions of consistency for the theory of neorealism. The answers, whether affirmative or negative, would not validate or invalidate the theory but would erase some doubts of inner contradictions.

Classical realism has been shaped by ideas put forward by Machiavelli, Hobbes, Meinecke, Carr, and Morgenthau and made frequent references to "human nature." According to Waltz (1990, 2004), neorealist theory is distinct from realist thought in its strong emphasis on system-level causes as opposed to the unit-level causes. In spite of this difference, the long line of the development of realism, neorealism included, is a gradual refuting of spiritual ethics, utopianism, internationalism, and idealism, lurking behind all of which is the "doctrine of harmony of interests." In its various derivatives, this is a belief that "In pursuing his own interest, the individual pursues that of the community, and in promoting the interest of the community he promotes his own" Carr (1946). Pursuit of interest in realist international politics can range from pursuing ever more power (offensive) to pursuing survival (defensive). A relevant question is then: Does the pursuit of interest in some form by each state lead to a unanimous achievement of that same interest without any outside intervention? Is a harmony of interests doctrine for international politics justifiable?

We attempt to answer these questions by developing a formal model based on the postulates of the structure theory. There are n agents, called states, whose conflictual capabilities are summarized in n numbers called resources. The states live and interact in an anarchic environment. The interaction consists only of each state allocating its resource against some or all of the other states. The environment is strategic as each state tries to maximize its individual security by trying to respond smartly to the allocations of other states against it. An individual security is some composite of a state's $n-1$ bilateral securities that are differences in mutual allocations. The Nash equilibrium of the strategic n-person games are determined under many different definitions of individual security ranging from very modest to rather ambitious objectives. The model thus respects the scanty and economical assumptions of the structure theory, attributes the very moderate motive of ensuring security, and does not portray states in crisis situations in which they may dispute, bargain, or menace. The conflictual model then applies more to peacetime than to dangerous times of emergency. It deviates significantly from the modeling paradigm of international politics.

Wagner (1986) and Niou and Ordeshook (1986, 1989, 1990) are early examples of n-person games that focus on the balancing imperative. They investigate the relation between the distribution of capabilities and alignment decisions, and their conclusions usually diverge from Waltz's assertions. An alternative approach to the study of alignment for $n = 3$ is found in Powell (1999), who concludes that there is no general tendency to balance. Fearon (1995) examines rationalist explanations for war and finds, for instance, that relative power is not a strong factor in reaching or preventing a negotiated settlement. Many rigorous game-theoretic studies focus on crisis bargaining and examine the effects of relative power and relative interests, Nalebuff (1986), Morrow (1989), Fearon (1994), Wagner(1991), Kilgour (1991), Bueno de Mesquita and Lalman (1992).

One part of the question of the harmony of interests, in the context of game theory, may be posed as: Can self-optimizing agents coordinate at (learn, realize, implement) a Pareto efficient equilibrium, which is preferably a social optimum? As we know from one-shot Prisoner's Dilemma, there are many games in which Pareto efficient outcome is not a Nash equilibrium. In its iterative version, however, the cooperative outcome does become a non-cooperative equilibrium, Axelrod (1984), Ross (2010), and experiments show that it can indeed be realized. The question is interesting since there are many games in which Pareto efficient outcomes either do not exist or they exist but are non-unique so that coordination is needed. Game theoretic confirmation of Adam Smith's principle of free market economy, the first welfare theorem of Arrow and Debreu (1954), is a well known affirmative answer to the question. The problem of cooperation emerging in self-help environments has attracted wide attention in the literature of international politics, see e.g., Jervis (1978), Keohane (1984), Krasner (1991), Powell (1991), Snidal (1991), Niou and Ordeshook (1990, 1994). Interest in the problem seems to have declined in recent years mainly because postulate of self-help or anarchy has gradually become unpopular in academic circles.

Our findings indicate that the structure theory is consistent in its claims, at least in the jurisdiction of our model. As far as the peacetime conflictual resource allocation prevails, the motive of security does imply a tendency, in all states, towards a mode of equilibrium. This mode, called bilateral-equilibrium[1], in which all states are barely secure (or, barely insecure) is precarious. Its realization as well as its maintenance is a difficult and risky task in all structures. The degree of difficulty depends on structure. It gets, for instance, higher in a multipolar and lower in a bipolar world. So, structure *is* a strong determinant. There may be differences in observed behavior depending on whether a state, or a group of states, adopt a modest notion of individual security or an ambitious one. Put differently, behavior may vary depending on the vigor with which states pursue the achievement of security. For example, if all states are rather relaxed and get only worried when they have a strictly negative security, then even strong states may find themselves in an equilibrium mode in which they have negative securities against some weaker states.

If a group of states are irrational (not strategic players) among states that hold cautious notions of individual security, or, if there are modest states among ambitious, then many more possibilities for an equilibrium mode emerge. But, these various modes are not too far away from a bilateral-equilibrium. It is shown for instance that one modest state among ambitious security seekers is tolerated and the world is still in bilateral-equilibrium; even with one irrational state among ambitious, a bilateral equilibrium is still most likely unless the irrational is one of the two strongest states. The effect of mixed notions of individual securities among states is that, while other modes of behavior than bilateral-equilibrium emerge as possibilities, they are plausible only under exceptional circumstances. For example, two irrational states among ambitious states may effect a world in disequilibrium if they are so strong that together they constitute a hegemonic-alliance.

A noteworthy result that may serve as a *principle of the harmony of security* for states, some or all of which are security-conscious at peacetime, is: In a world without a hegemon, if all states are cautious and ambitiously maximize their individual securities, then a bilateral-equilibrium will prevail. If two or more *strong* states vigorously pursue their individual securities, then a bilateral-equilibrium is the most plausible Nash profile, irrespective of whether the rest of the states are irrational, modest, or ambitious.

The sections ahead are organized as follows. Section 2 is concerned with the structural aspects of the model and introduces bilateral-equilibrium, the existence and uniqueness conditions of which are given in Theorem 1. Section 3 introduces the n-person strategic games and Theorems 2-5 characterize the Nash equilibrium profiles that result from motives of uniformly maximizing four different individual security functions by all agents. In Section 4, mixed motives are examined. Theorems 6-8 of this section identify Nash equilibrium profiles that result if some states, among vigorous security maximizers, are irrational and adopt no utility or are relaxed and adopt modest utilities. Section 5 examines possible refinements of Nash equilibrium under different scenarios. Theorem 9 identifies the strongly Pareto efficient and socially optimum profiles among the Nash profiles of Theorems 2-5. The effects of multiplicity of Nash equilibria and similarity of views among states are also examined in this section. Section 6 lists the implications of these

[1] In the sense that mutual allocations are pairwise equal, *not* a “bipolar” equilibrium.

results under various assumptions on capability distribution among states. The last section is on conclusions.[2]

2 A Peacetime Model

A peacetime model of an international system that should be useful to critically examine the structure theory and its claims need be as parsimonious as possible. The system under consideration comprises $n \geq 2$ states having resources $r_1, ..., r_n$. Resource may be understood as a source of energy, energy as the capability to do work. At peacetime it is potential energy, as opposed to kinetic (although one may prefer to think of it as *potential power*, Mearsheimar (2001)). Each state may apportion its resource and target it against a number of, or to all, other states. The resource of a state, hence, is a sum total of its conflictual capability only, because resource directed to its internal consumption is left out of consideration. We thus assume that there is no self-allocation and that all other states are adversaries and potential rivals. These are major assumptions. The first says in effect that the resources consumed inside a state remain more or less constant over the time span the model is applied. The second means that a state perceives all other states to be equally threatening to its security and ignores parameters like other states' geographical distance, emotional stability of their leaders, and the like. While it is common to represent the capability of a state by a quantity called resource, the novelty here may be the allocation of portions of it against others in contrast to directing the total resource against one state at a time. Already at this point, there are two important questions to be answered: How is the resource of a state determined? Do states actually allocate their resources against each other?

The capability of a state-i is represented by a positive real number r_i so that it is a measurable quantity and is infinitely divisible, see e.g., Niou, Ordeshook, and Rose (1989). Coming up with such a real number that closely represents capability is difficult and precision impossible. A calculation of a country's conflictual capability needs to take into account as diverse parameters as the size, location, and capabilities of country's military forces; its efforts to develop, acquire, or gain access to advanced technologies that could enhance its military capabilities; its space and cyber capabilities; country's foreign military engagement; and its resources for force modernization, OSD (2010). These parameters obviously transcend the harder quantities such as head-count of soldiers or arms. While it seems possible e.g., to define a "military force potential index" (Hildebrandt, 1980), develop a measure for "military power from a formal model of military capability" (Biddle, 2004), or rank the states according to a Global Firepower formula (GlobalFirepower.com) in arriving at an estimate of conflictual capability, it is also clear that one needs to deal with many subtleties. For instance, the conclusions reached by the guns-versus-butter model of Powell (1993) differ depending on whether the level of military technology is incorporated into the definition of resources or not. How should the level of military technology contribute to resources? Relocating the ground forces of an army takes considerably more time than relocating its naval force and its airforce. A Turkish destroyer at the Russian maritime border in Black Sea is in one day's travel defending the Greek border in Aegean. A fighter plane in an air base at central Anatolia can be deployed to fly over any neighboring country's border in less than two hours. So, how should a country's forces of great mobility be incorporated into

[2]Theorem 1 is from Özgüler, Güner, and Alemdar (1998) and Theorem 5 is from Özgüler, Güner, and Alemdar (2000). The proofs given here in the Appendix are shorter. The conflictual peacetime model and the associated strategic games have their origin in the (2000) paper.

its total resource? Waltz (1993) elaborates on how nuclear weapons limit force at the strategic level to a deterrent role and how they make alliances obsolete. How should one incorporate nuclear capacity of a state to its resource level? All these indicate difficulties associated with representing "capability" by a single number.[3] Nonetheless, in order to apply the model here to a real situation, only the ordering relation among the resources of n-states need be known. Their relative values have a significance, but again there may not be much of a difference between the (normalized) resource values of 1 and 0.9. Resources are quantities that will enable us to attain qualitative conclusions only. Therefore, one needs only have a rough idea about the relative standings of $r_1, ..., r_n$ to each other.[4]

Turning to the second question, let us examine if states in peacetime actually allocate their resources against each other and, if so, how? Resource allocation in international political models has scarcely been considered before. Burns (1957) examined a balance of power under targeting of resources among three or more states which are "roughly of a size." Deutsch and Singer (1964) based their notion of stability in a multipower system on diminishing "share of attention" as the number of actors increases. Blainey's "waterbird's dilemma" that a third party may take advantage of the fight between two parties calls for resource allocation for its resolution, (1988). Powell (1993) examined how a state should allocate its capital stock between producing consumption goods and military goods thereby facing a trade-off between internal and external allocation. The resource apportionment considered here is one of conflictual or strategic mismatch allocation and is, hence, rather distinct from Powell's as well as those in resource allocation models of economics and quality of service management. The allocation in question is very much like the one considered in Colonel Blotto game (Roberson, 2006), where two players place troops across several fronts simultaneously with the objective of dominating each other in as many fronts as possible. It is then worthwhile to look closely into countries' conflictual resources and how they are apportioned.

A brief survey of official websites will indicate that the foreign ministry of any major country is organized in departments or bureaus that specialize in different parts of the outside world, which point out to the existence of *at least* an "allocation of attention" in a country's international political affairs. The structure of army in any major country reflects an allocation scheme based on that country's conception of responsibilities such as meeting threats, setting up defenses, or even planning offenses. US Army's organizational structure begins with a Unified Command Plan showing six geographical regions of responsibilities. In OSD (2010), portions of Chinese military resource available at the Taiwan front are listed item-by-item, which, in principle, can be done for any other front. Naturally, the concentration of forces in certain regions depends on the perceived level of threat to security in that region. But, it also depends on historical animosities with the neighboring country or the presence of natural geographical barriers at near borders, see e.g., OSD (2010, p. 61). This indicates that resource allocation is done according to some underlying rationale. The distribution of resources changes at times of crisis, and the change is drastic at wartime. When a country is at war, all resources are shifted from friendly states to be placed against the hostile ones. Thus, it can safely be assumed that, the amount of allocated resource varies from state to state and that the resource allocation is more evenly distributed

[3]There *are* ways out of these difficulties. For instance, mobility problem can be resolved by calculating all factors that contribute to resource in man-hours. The work done in one hour by a military aircraft is equivalent to, say, 1000 man-hours. If an aircraft can spend 2 hours each at two fronts in the span of a day, then its contribution to two allocations (in the span of a year) would be 2000×365 man each and its contribution to the total resource, 4000×365 man.

[4]A consequence is that the utility functions we examine in Sections 3 and 4 below are ordinal, Ross (2010).

and stable at peacetime than at times of crises. It is still possible that some countries, such as those with not so many neighbors, do not have to physically apportion their resources. They may well be confident that, if need arises, all conflictual resources they command can be used against any threatening rival. But (according to the model used here) such a country must be at least mentally prepared, and do its allocation calculations accordingly, for accommodating simultaneous challenges were they to come from more states than only one.

A *peacetime conflictual resource allocation model of an international system*, then, consists of n states $\mathcal{N} := \{1, ..., n\}$, and n endowed resources $r_1, ..., r_n$ with the following properties. State-i apportions r_i among some or all of the other states so that its $n-1$ allocations a_{ij}, $j = 1, ..., i-1, i+1, ..., n$, add up to r_i. The *set of allocation profiles of state-i* is the set of all such $(n-1)$-tuples, i.e.,

$$A_i = \{a_i \in \mathbf{R}^{n-1} : a_i = (a_{i1}, ..., a_{i(i-1)}, a_{i(i+1)}, ..., a_{in}),\ a_{ij} \geq 0,\ \sum_{j=1, j\neq i}^{n} a_{ij} = r_i\}. \tag{1}$$

The set of allocation profiles of all states combined is $A = A_1 \times ... \times A_n$. It is easier to think of an allocation profile as an $n \times n$ resource allocation matrix R. Its i-th row (omitting $a_{ii} = 0$) corresponds to an allocation profile a_i of state-i so that the i-th row-sum of R is r_i. Given four states with resources $r_1 = 1, r_2 = 0.9, r_3 = 0.8, r_4 = 0.7$, the matrices below give three different allocation profiles:

$$R_1 = \begin{bmatrix} 0 & 0.54 & 0.05 & 0.41 \\ 0.52 & 0 & 0.38 & 0.00 \\ 0.05 & 0.40 & 0 & 0.35 \\ 0.38 & 0.00 & 0.32 & 0 \end{bmatrix}, R_2 = \begin{bmatrix} 0 & 0.2 & 0.1 & 0.7 \\ 0.2 & 0 & 0.7 & 0 \\ 0.1 & 0.7 & 0 & 0 \\ 0.7 & 0 & 0 & 0 \end{bmatrix}, R_3 = \frac{1}{6}\begin{bmatrix} 0 & 2.3 & 2.0 & 1.7 \\ 2.3 & 0 & 1.7 & 1.4 \\ 2.0 & 1.7 & 0 & 1.1 \\ 1.7 & 1.4 & 1.1 & 0 \end{bmatrix}. \tag{2}$$

In R_1, for example, state-1 allocates 0.54 units of its resource against state-2 and state-2 reciprocates by 0.52.

2.1 Bilateral Security

In order to formalize motive of security of a state, one first needs to postulate how two given states assess their securities against each other; and second, how a state assesses its individual security against the rest of the world. The first assessment of security may be called "bilateral" security and, the second, "individual" security. In the allocation model under consideration, a very natural definition of *bilateral security* of a state-i against state-j is

$$s_{ij} = a_{ij} - a_{ji}.$$

Thus, $s_{ij} > 0$ would mean that state-i is secure against state-j, equivalently as $s_{ji} = -s_{ij} < 0$, that state-j is insecure against state-i. If $s_{ij} = 0$, then i and j are both barely secure (or barely insecure) against each other. Ensuring security in this context may roughly mean "keeping all bilateral securities as large as possible at all times". A precise meaning requires a definition of individual security. But first, let us single out a particular mode in the multipolar system under study.

2.2 Bilateral-Equilibrium

A noticeable allocation profile is one in which all bilateral securities are equal to zero.

Definition 1. A *bilateral-equilibrium* is an allocation profile in which every bilateral security is zero, i.e., $a \in A$ such that $a_{ij} = a_{ji}$, $\forall \{i, j\} \subset \mathcal{N}$.

Figure 1 is a picturesque account of attempts at bilateral-equilibrium. The vectors emanating from a state denote its respective allocations and the sum of their magnitudes equals the total resource of that state. As two emanating vectors of the same length meet, this is interpreted as the equality of bilateral allocations. One can visualize the situation in $n = 2$ as two men pushing each other with their right arms and with all strength. In $n = 3$, imagine three men, each pushing the other two with his two arms. In the five-state world, each state is a four-armed creature pushing the other four with all its arms. A bilateral-equilibrium would correspond to a figure standing stationary.

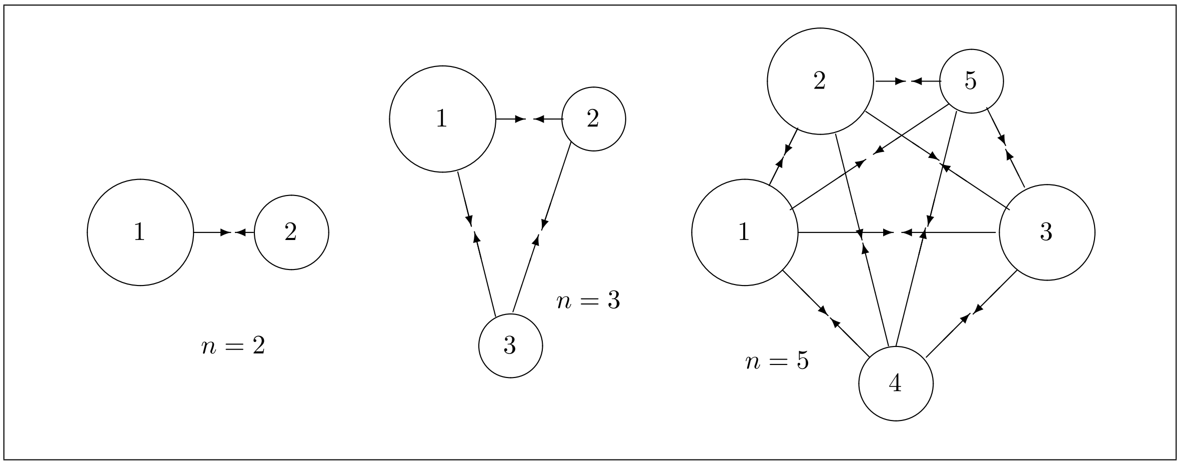

Figure 1: Attempts at bilateral-equilibrium in 2-, 3-, 5-state worlds

A bilateral-equilibrium allocation is represented by and is equivalent to a symmetric resource allocation matrix. The matrix R_1 in (2) represents an allocation profile which is not a bilateral-equilibrium, whereas R_2, R_3 give two different bilateral-equilibria.

If a state's resource endowment strictly exceeds half of the total resource in the system, or equivalently, if it is strictly greater than the total resource of the rest, then that state is called a *hegemon*. Thus, there is no hegemon if and only if for all $i \in \mathcal{N}$

$$r_i \leq \frac{1}{2} \sum_{j \in \mathcal{N}} r_j \quad \text{or, equivalently,} \quad r_i \leq \sum_{j \in \mathcal{N}_{-i}} r_j, \tag{3}$$

where $\mathcal{N}_{-i} := \{1, ..., i-1, i+1, ..., n\}$. A *near hegemon* is defined as a state which owns exactly half of the total resources in the system; state-i is a near hegemon if and only if equalities hold in (3).

Theorem 1. (Özgüler, Güner, and Alemdar (1998)) *In an n-state-system with a given resource distribution, $r_1, ..., r_n > 0$, a bilateral-equilibrium exists if and only if there is no hegemon.*

In the absence of a hegemon, there is a unique bilateral-equilibrium for $n = 2$ and $n = 3$; for $n > 3$, bilateral-equilibrium is unique if there is a near hegemon, and infinitely many if there is none.

If n=2, then there is no hegemon in the system if and only if $r_1 = r_2$ in which case the only possible bilateral-equilibrium is $a_{12} = a_{21} = r_1$. In a three-state-system, there is no hegemon if and only if $r_i \leq r_j + r_k$ for every permutation (i, j, k) of $(1, 2, 3)$. Under this condition, the only possible bilateral-equilibrium is obtained by

$$a_{12} = a_{21} = \frac{r_1 + r_2 - r_3}{2}, a_{13} = a_{31} = \frac{r_1 + r_3 - r_2}{2}, a_{23} = a_{32} = \frac{r_2 + r_3 - r_2}{2}. \tag{4}$$

The smallest size system which, in general, admits an infinity of different bilateral-equilibria is the system of four states. In order to describe such profiles, let us number the states such that $r_1 \geq r_2 \geq r_3 \geq r_4 > 0$. Thus, the system is nonhegemonic if and only if $r_1 \leq r_2 + r_3 + r_4$. Under this condition, the set of all possible bilateral-equilibria are as follows:

$$\begin{array}{l} a_{12} = a_{21} = \frac{1}{2}(r_1 + r_2 - r_3 - r_4) + y, \\ a_{13} = a_{31} = \frac{1}{2}(r_1 + r_3 - r_2 - r_4) + x, \\ a_{23} = a_{32} = \frac{1}{2}(r_2 + r_3 + r_4 - r_1) - (x + y), \\ a_{14} = a_{41} = r_4 - (x + y), \\ a_{24} = a_{42} = x, \\ a_{34} = a_{43} = y, \end{array} \tag{5}$$

for any nonnegative x, y satisfying

$$0 \leq x + y \leq min\{r_4, \frac{1}{2}(r_2 + r_3 + r_4 - r_1)\}. \tag{6}$$

The set of (x, y) described by (6) is a triangle in the xy-plane (The large triangle in Figure 2 below). Every point inside this triangle yields a different bilateral-equilibrium via (5). In (2), the bilateral-equilibrium represented by R_2 is obtained by the point $(x, y) = (0, 0)$ on the triangle and R_3 by $(x, y) = (\frac{1.4}{6}, \frac{1.1}{6})$.

If the first state is a near hegemon, then $r_1 = r_2 + r_3 + r_4$ and the constraint (6) indicates that the triangle degenerates to a point $(x, y) = (0, 0)$. Using this fact in (5), the only possible bilateral-equilibrium is obtained as: $a_{12} = a_{21} = r_2, a_{13} = a_{31} = r_3, a_{14} = a_{41} = r_4, a_{ij} = 0 \; \forall \; i \neq 1, j \neq 1$. All weak states allocate their total resources against the near hegemon and none against each other. The triangle also vanishes as the resource level r_4 of the weakest state approaches zero. This is expected since, then, the world is in effect approaching a world of three states where a bilateral-equilibrium allocation is unique. In fact, as r_4 goes to zero, all allocations in (5) except those of states 1 to 3 approach zero and the nonzero allocations approach (4).

We should emphasize that a world in bilateral-equilibrium is a fictitious world. Even though at some instant, against the laws of probability, all states of the world may find themselves in a bilateral-equilibrium, this situation is bound to change at the slightest disturbance. Nevertheless, like all notions of equilibrium elsewhere, the construct bilateral-equilibrium will be useful.

3 Security Games

A state-i would try to maximize a utility that takes into account some or all of its bilateral securities $\{s_{i1}, ..., s_{i(i-1)}, s_{i(i+1)}, ..., s_{in}\}$. Let us call a utility that is formed in some such manner, the *individual security of state-i*.

In this section, we examine a number of alternative definitions of individual securities that will serve as utilities of states and the equilibria that result from the strategic game defined by each utility under the assumption that *all n states uniformly adopt* that utility. In order to arrive at definitions that may make sense, we may ask "When would a state, being aware of and willing to improve its $n-1$ bilateral securities $\{s_{ij},\ j \in \mathcal{N}_{-i}\}$, feel safe?" When (some type of) total insecurity is zero? When all bilateral securities are nonnegative? When that state is not too far from being equally secure against all others? Or, when each and every one of its bilateral securities is as large as possible? Affirmative reply to each question yields a different definition of utility.

The motive of seeking security and security maximization are sometimes depicted as "do nothing." The argument is that if a state is not a major player in the international arena and is not pursuing "prestige, status, political influence, leadership, political leverage, a positive trade balance, or market shares", then there is no challenge to its security anyway (Schweller, 1997). However, as the arms race security dilemma implies, resource allocation for security requires active involvement in competition and is hence not a passive strategy. Although none of the games we consider below are zero-sum or constant-sum games, in the way we have defined bilateral security, security seeking is a conscious competitive endeavor because a security advantage confers strategic leverage in all sorts of negotiations in peacetime.

3.1 Minimizing Total Insecurity

We first assume that the states are primarily concerned with their nonpositive bilateral securities, i.e., with $\{s_{ij} :\ j \in J_i\}$, where

$$J_i = \{j \in \mathcal{N}_{-i} :\ s_{ij} = a_{ij} - a_{ji} \leq 0\} \tag{7}$$

for each $i \in \mathcal{N}$. The set J_i is, hence, the subset of states that i is not secure against. An appropriate utility for state-i would be

$$v_i(a) = \sum_{j \in J_i} (a_{ij} - a_{ji}),\ a \in A, \tag{8}$$

where $v_i(a) = 0$ if $J_i = \emptyset$. The quantity (8) is negative or zero for each i. Maximizing (8) is equivalent to minimizing $-v_i(a)$, which is in turn equivalent to minimizing the *total insecurity* (the sum of the magnitudes of all negative securities) of state-i.

State-i thus assesses its security against the rest of the world by the negative of its total insecurity. Driven by the motive of security, given any $a_{-i} \in A_1 \times ... \times A_{i-1} \times A_{i+1} \times ... \times A_n$, which includes the allocations against it, a state-i chooses $a_i \in A_i$ such that its individual security $v_i(a)$ is maximum. If each state maximizes its individual security, then a strategic game in which A_i is the strategy space of a state-i results.[5]

A state trying to minimize its total insecurity measured by (8) may be described as very modest or relaxed for two reasons. First, because it disregards its positive bilateral securities, it will be content as soon as it achieves a nonnegative utility function. For example, a state with a utility function of value zero will not try to improve it to some positive value. Second, because it is concerned only with a "total", it underrates the individuality of bilateral securities. A state may stand highly insecure against one state k and reasonably secure against the rest of the states, resulting in a "summed" utility of moderate value, and this situation will be preferable in spite of serious insecurity against k. The main point is that, improvement of its total insecurity alone may not necessarily make a state "feel" secure so that states may be obliged to consider additional aspects of security. Keeping this in mind, we now examine the kind of equilibria that may result under the utility (8).

A subset $\mathcal{L}$ of $\mathcal{N}$ that is not a singleton (i.e., $|\mathcal{L}| > 1$ with $|\mathcal{L}|$ denoting the number of elements of $\mathcal{L}$) will be called an *alliance*. Every allocation profile $a = \{a_{ij} : \{i,j\} \subset \mathcal{N}\}$ in $\mathcal{N}$ induces an internal allocation profile in $\mathcal{L}$ given by $\{a_{ij} \in a : \{i,j\} \subset \mathcal{L}\}$. If $a_{ij} = a_{ji}$ for each $\{i,j\} \subset \mathcal{L}$, then we say that the alliance $\mathcal{L}$ is *internally at bilateral-equilibrium*. Note that internal bilateral-equilibrium disregards allocations against the nonmembers. If the total resource of the alliance is more than the total resource of the rest of the states, i.e., if $r(\mathcal{L}) = \sum_{j\in\mathcal{L}} r_j > \sum_{j\notin\mathcal{L}} r_j$, then $\mathcal{L}$ is called a *hegemonic-alliance*. Note that if a state is a hegemon, then any alliance of which it is a member is a hegemonic-alliance. Of course, in all system structures for $n > 2$, a hegemonic-alliance that excludes some states always exists. An alliance with an internal allocation profile that consist of all zero allocations is called a *coalition*, i.e., a coalition is an alliance with no internal allocations. The coalition members, however, in general, will have nonzero allocations against the nonmembers.

Definition 2. An allocation profile is called a *partitioned-equilibrium* if there exists a disjoint partition $\mathcal{N} = \mathcal{N}_1 \cup \mathcal{N}_2$ into subsets $\mathcal{N}_1, \mathcal{N}_2$, at least one of which is an alliance, such that alliance(s) are internally at bilateral-equilibrium and $a_{il} \geq a_{li}$ for all $i \in \mathcal{N}_1$ and $l \in \mathcal{N}_2$ with at least one strict inequality.

In R_4 below, alliances are $\mathcal{N}_1 = \{1,4\}$, $\mathcal{N}_2 = \{2,3\}$, in R_5 they are $\mathcal{N}_1 = \{1,3\}$, $\mathcal{N}_2 = \{2,4\}$, and in R_6, $\mathcal{N}_1 = \{3,4\}$, $\mathcal{N}_2 = \{1,2\}$; and all are instances of partitioned-equilibria:

$$R_4 = \begin{bmatrix} 0 & 0.5 & 0.4 & 0.1 \\ 0.4 & 0 & 0.2 & 0.3 \\ 0.4 & 0.2 & 0 & 0.2 \\ 0.1 & 0.3 & 0.3 & 0 \end{bmatrix}, R_5 = \begin{bmatrix} 0 & 0.54 & 0.05 & 0.41 \\ 0.50 & 0 & 0.40 & 0.00 \\ 0.05 & 0.40 & 0 & 0.35 \\ 0.40 & 0.00 & 0.30 & 0 \end{bmatrix}, R_6 = \begin{bmatrix} 0 & 0.4 & 0.3 & 0.3 \\ 0.4 & 0 & 0.3 & 0.2 \\ 0.4 & 0.4 & 0 & 0.0 \\ 0.4 & 0.3 & 0.0 & 0 \end{bmatrix}.$$

If there is a hegemon and a partitioned-equilibrium obtains, then $\mathcal{N}_1$ must contain the hegemon so that it is a hegemonic-alliance. In a partitioned-equilibrium of a nonhegemonic world, however, it is possible that $\mathcal{N}_1$, $\mathcal{N}_2$, and neither one is a hegemonic-alliance; this is the case in R_5, R_6, and R_4, respectively.

[5]This game is analogous to a "General Blotto game" (Golman and Page, 2009) somehow extended to n players. However, this viewpoint is not fruitful since here allocated fronts and players coincide. Also, we will see below that, unlike (the zero-sum Colonel Blotto and) many General Blotto games, our game has a plethora of pure strategic equilibria.

Theorem 2. *An allocation profile is a Nash equilibrium of the n-person strategic game with utilities (8) if and only if it is either a bilateral-equilibrium or a partitioned-equilibrium.*

Hence, a Nash equilibrium always exists and can be a bilateral-equilibrium in which all states are equally secure or it can be a partitioned-equilibrium in which one state or an alliance security-wise dominates a state or another alliance. Note that, since partitioned-equilibria is among them, Theorem 2 gives infinitely many possible Nash equilibria even in case $n = 3$. It is easy to see that every bilateral-equilibrium must be a Nash solution of the game with (8). Having zero bilateral securities across the board, as in a bilateral-equilibrium, would leave no incentive for any state to deviate from the allocation profile that achieves it. It may take a little longer to convince oneself that a partitioned equilibrium should also be among the Nash equilibria since, after all, a partitioned equilibrium leaves at least one state with a strictly negative bilateral security. To see why, let us first note (based on the proof of Theorem 2 in the Appendix) that the formation of the partitions in a partitioned-equilibrium is dictated by the following distinction.

Definition 3. Given a subset $\mathcal{A}$ of $\mathcal{N}$ and an allocation profile $a \in A$, let

$$\mathcal{A}_1 = \{i \in \mathcal{A} : r_i \geq \sum_{j \in \mathcal{N}_{-i}} a_{ji}\}, \ \ \mathcal{A}_2 = \{i \in \mathcal{A} : r_i < \sum_{j \in \mathcal{N}_{-i}} a_{ji}\} \tag{9}$$

be a disjoint partition of $\mathcal{A}$. The set of states in $\mathcal{A}_1$ and $\mathcal{A}_2$ are called *advantaged* and *disadvantaged* states of $\mathcal{A}$, respectively.

Using this definition, the two Nash profiles of Theorem 3 can be symbolically described by the following matrix representations in which equality and inequality signs in submatrices signify the ordering relation among the symmetric entries of a matrix:

$$\mathcal{N}_1 \ \left[\ = \ \right], \quad \begin{matrix} \mathcal{N}_1 \\ \mathcal{N}_2 \end{matrix} \left[\begin{array}{c|c} = & \geq_{with} > \\ \hline \leq_{with} < & = \end{array} \right].$$

Here, $\mathcal{N}_1$ is the set of advantaged and $\mathcal{N}_2$, the disadvantaged states. These symbolic matrices are the resource allocation matrices with permuted rows and columns to bring $\mathcal{N}_1$ and $\mathcal{N}_2$ together at each partition. In the second matrix, $\mathcal{N}_2$ is shown to be security-wise dominated by $\mathcal{N}_1$, leading to the partitioned-equilibrium. Now, imagine a disadvantaged state $i \in \mathcal{N}_2$ such that its every bilateral security is negative or zero giving $J_i = \mathcal{N}_{-i}$. Then, $v_i(a) = r_i - \sum_{j \in \mathcal{N}_{-i}} a_{ji}$ so that no further improvement in $v_i(a)$ is possible by a choice of $a_i \in A_i$. The utility $v_i(a)$ of state i is then independent of a_i. On the other hand, if $j \in \mathcal{N}_1$, then as soon as all $s_{jk} \geq 0$, the advantaged state j has no incentive to change its allocations. It follows that, in a partitioned equilibrium, the advantaged states $\mathcal{N}_1$ lack any incentive to change their allocations and the disadvantaged states $\mathcal{N}_2$ are not able to increase their utility by changing their allocations. Therefore, they end up in a partitioned-equilibrium.

3.2 Minimizing the Largest Bilateral Insecurity

Continuing to assume that the states are primarily concerned with their insecurities, let us replace (8) with

$$u_i(a) = \min_{j \in J_i} \{a_{ij} - a_{ji}\}, \ a \in A, \tag{10}$$

where $u_i(a) = 0$ if $J_i = \emptyset$ and J_i is given by (7). The utility (10) is now the lowest bilateral security of state-i and, by itself, it may also not be sufficient for a state to feel secure. A state may be a little insecure against each of the other $n-1$ states, resulting in a large negative value that is unacceptable in (8). As in the case of (8), a state trying to minimize its total insecurity measured by (10) is a modest state because, first, it disregards its positive bilateral securities and is satisfied as long as it achieves a nonnegative utility function and, second, because it is only concerned with its worst bilateral security while ignoring, among others, its (any type of) total insecurity. We now proceed to solve the strategic game with utilities (10).

Definition 4. An allocation profile is a *dominant-equilibrium*[6] if there exists a disjoint partition $\mathcal{N} = \mathcal{N}_1 \cup \mathcal{N}_2$ such that (i) $\mathcal{N}_1$ either a singleton or an alliance that is internally at bilateral-equilibrium, (ii) $\mathcal{N}_2$ is either a singleton or a coalition, and (iii) for every $k \in \mathcal{N}_2$, it holds that $\sum_{j \in \mathcal{N}_1} a_{jk} > r_k$ and

$$a_{kj} = \begin{cases} a_{jk} + \frac{1}{|\mathcal{M}_k|}(r_k - \sum_{i \in \mathcal{M}_k} a_{ik}) \text{ if } j \in \mathcal{M}_k, \\ 0 \text{ if } j \notin \mathcal{M}_k, \end{cases}$$

where the index set $\mathcal{M}_k \subset \mathcal{N}_1$ is defined by the property "$j \in \mathcal{M}_k$ if and only if $\frac{1}{|\mathcal{M}_k|}(\sum_{i \in \mathcal{M}_k} a_{ik} - r_k) \leq a_{jk}$".

The index set $\mathcal{M}_k$ is needed by state-k to determine which members of $\mathcal{N}_1$ have significant allocations against it thereby deserving its attention. State-k apportions its whole resource equally among the members of $\mathcal{M}_k$; the members $\mathcal{N}_1 - \mathcal{M}_k$ get zero allocation from k.[7] In Definition 4.(iii), for every $k \in \mathcal{N}_2$, we have $a_{kj} \leq a_{jk}$ for all $j \in \mathcal{N}_1$ with at least one strict inequality. By (ii), this implies that in any dominant-equilibrium, $\mathcal{N}_1$ is a hegemonic-alliance. Since $a_{kj} \leq a_{jk}$ for all $j \in \mathcal{N}_1$ and $k \in \mathcal{N}_2$, with strict inequality for at least one j, it follows that every dominant-equilibrium is a partitioned-equilibrium. The converse is not true. The matrices R_4 to R_6 are instances of partitioned but not dominant-equilibria. The matrix R_1 in (2), on the other hand, gives a dominant-equilibrium with $\mathcal{N}_1 = \{1, 3\}$, $\mathcal{N}_2 = \{2, 4\}$.

Theorem 3. *An allocation profile is a Nash equilibrium of the n-person strategic game with utilities (10) if and only if it is either a bilateral-equilibrium or a dominant-equilibrium.*

If state-1 is a hegemon, then by Theorem 1, a bilateral-equilibrium does not exist. By Theorem 2, the only Nash equilibria are then dominant-equilibria in which state-1 is a member of the hegemonic-alliance $\mathcal{N}_1$. Note that even in a nonhegemonic system, alliances that are hegemonic always exist and, hence, dominant-equilibria are always among the Nash equilibria. The two Nash profiles of Theorem 3 can be described by the following matrix representations, where $\mathcal{N}_1$ is the set of advantaged and $\mathcal{N}_2$, disadvantaged states:

$$\mathcal{N}_1 \begin{bmatrix} = \end{bmatrix}, \quad \begin{matrix} \mathcal{N}_1 \\ \mathcal{N}_2 \end{matrix} \left[\begin{array}{c|c} = & \geq_{with} > \\ \hline \leq_{with} < & 0 \end{array}\right].$$

[6]In the sense of an equilibrium in which some states "security-wise" dominate other states, *not* a weakly dominating or strictly dominating strategy.

[7]The fact that $\mathcal{M}_k$ is always a nonempty subset of $\mathcal{N}_1$ and a procedure to compute it are given in Remark A.2 of the Appendix.

3.3 Minimizing the Distance to Security

While the magnitude of utility (10) is a type of total insecurity of state-i, the magnitude of its following alternative

$$t_i(a) = -\sqrt{\sum_{j\in J_i}(a_{ij} - a_{ji})^2},\ a \in A \tag{11}$$

is also a kind of total insecurity. State-i maximizing (11) is, in effect, minimizing the *distance* to its security. Here, "security" is understood as an allocation profile that satisfies $s_{ij} \geq 0,\ j \in \mathcal{N}_{-i}$. (We talk about "distance" because the magnitude of (11) is reminiscent of the Euclidean norm of $\{s_{ij} :\ j \in \mathcal{N}\}$.) A state trying to minimize its total insecurity measured by (11) is still a modest state as it disregards its positive bilateral securities and because it underrates its individual bilateral securities, as was the case for the utility (8).

Both (11) and (8) are formed by some type of an "averaging" operation from negative bilateral securities. They hence substantially differ from (10) which simply focuses on the smallest bilateral security. Interestingly however, the Nash solutions of the games indicate in retrospect that, the notions of individual security defined via (10) and (11) are closely tied. They yield the same set of equilibria and this set is narrower than that obtained by (8).

Theorem 4. *An allocation profile is a Nash equilibrium of the n-person strategic game with utilities (11) if and only if it is either a bilateral-equilibrium or a dominant-equilibrium.*

To appreciate the difference in equilibria that (8) and (11) (or (8) and (10)) lead to, suppose that each bilateral security of state-i is nonpositive so that $J_i = \mathcal{N}_{-i}$. In case of (8), this will be the case if and only if $v_i(a) = r_i - \sum_{j\in\mathcal{N}_{-i}} a_{ji}$ and no further improvement in $v_i(a)$ would be possible by a choice of $a_i \in A_i$. The utility $v_i(a)$ will become independent of a_i. However, $t_i(a)$ and $u_i(a)$ will still depend on a_i. A further tuning of allocations by state-i would in general give a better utility $t_i(a)$ or $u_i(a)$.

3.4 Maximizing the Smallest Bilateral Security

Let us now consider less modest states than before and suppose that they are not only concerned with their insecurities but also with the level of their (positive) bilateral securities. We then need to define a notion of individual security which takes into account all of $\{s_{ij} :\ j \in \mathcal{N}\}$ rather than its subset $\{s_{ij} :\ j \in J_i\}$. It turns out that among the utilities $v_i(a), u_i(a), t_i(a)$, only $u_i(a)$ is amenable to an extension.[8] Suppose the individual security of state-i is

$$w_i(a) = \min_{j\in\mathcal{N}_{-i}} \{a_{ij} - a_{ji}\},\ a \in A. \tag{12}$$

This utility represents state-i's most vulnerable position, its smallest bilateral security; be it may negative, zero, or positive. In contrast to (10), a state with all its bilateral securities positive

[8]The utilities (11) and (8) complement (10) mathematically, since they are the L_2 and L_1 analogues of the L_∞-type function (10). Their extensions to $\{s_{ij} :\ j \in \mathcal{N}\}$ do lead to meaningful games but do not have interpretations as individual security. For instance, the function $\hat{v}_i(a) = -\sum_{j\in\mathcal{N}_{-i}} |a_{ij} - a_{ji}|$ defines an n-person game. However, $\hat{v}_i(a)$ penalizes bilateral securities and insecurities alike so that it is not an acceptable definition of individual security.

(so that J_i of (7) is empty) would still act to improve the value of $w_i(a)$.

A state preferring to define its individual security by (12) rather than (10) may be cast as "ambitious" as it does not stop at being secure. This term, however, should be applied with care since a "cautious" or "alert" state, although not ambitious, may like to leave a *safety margin* and prefer a positive bilateral security to zero security simply because it provides a margin against miscalculations; miscalculations in resource values or perhaps in best response functions. Further, as we will see below in Section 5.2, the nonuniqueness of Nash equilibrium actually forces states to be cautious as it gives way to a mistaken perception of intentions of other states. States may not be sure if others are allocating towards an equilibrium or a disequilibrium, which calls for as large a margin of security as one is able to ensure. We will, nevertheless, still refer to states that employ the utility (12) as ambitious keeping in mind that the adjective may not be quite appropriate.

Theorem 5. (Özgüler, Güner, and Alemdar (2000)) *Consider the n-person strategic game with utilities $w_i(a)$ of (12). If there is no hegemon, then an allocation profile is a Nash equilibrium if and only if it is a bilateral-equilibrium. Otherwise, it is a Nash equilibrium if and only if all states place full allocation against the hegemon and the hegemon apportions its resource among the rest so as to have uniformly the same positive bilateral securities.*[9]

The two Nash profiles of Theorem 5 are

$$\mathcal{N}_1 \ \left[\ =\ \right], \quad \begin{matrix} \mathcal{N}_1 \\ \mathcal{N}_2 \end{matrix} \left[\begin{array}{c|c} o & > \\ \hline < & 0 \end{array} \right],$$

where $\mathcal{N}_1$ is the set of advantaged and $\mathcal{N}_2$, disadvantaged states. Whenever there is a hegemon, say state-i, there is a unique Nash equilibrium, which is actually a dominant-equilibrium obtained with $\mathcal{N}_1 = \{i\}$, $\mathcal{N}_2 = \mathcal{N}_{-i}$ and the hegemon ensuring that all its bilateral securities are uniformly the same. In the absence of a hegemon, every Nash equilibrium in the security game is a bilateral-equilibrium of Theorem 1 and *vice versa*. If there is a near-hegemon or if $n \leq 3$, then there is a unique Nash equilibrium; otherwise, there are infinitely many equilibria. It is important to observe that, although infinitely many, this set is in general much smaller than the set of equilibria that results from any of the utilities $v_i(a), u_i(a)$, or $t_i(a)$.

We see comparing Theorems 2-5 that if all states adopt the less modest, the more ambitious motive of maximizing their smallest bilateral securities, then they are not necessarily better off. In a nonhegemonic world, imagine a hegemonic alliance $\mathcal{N}_1$ of states who dominate the rest of the states $\mathcal{N}_2$. If all states in $\mathcal{N}_1$ adopt the same motive (12), then, perhaps contrary to expectations of $\mathcal{N}_1$, a world of no domination where every state is equally secure would result. This somewhat unexpected result is the consequence of the fact that the members of $\mathcal{N}_1$ would not stop at a dominant-equilibrium even if they have achieved it. Their greed would drive them to change their allocations still further in order to have as large bilateral securities as possible, which in turn would force them to increase their allocations against their allies.

[9] If state-1 is a hegemon, then a Nash equilibrium is an allocation profile in which $a_{1j} = r_j + \frac{1}{n-1}(r_1 - \sum_{t=2}^{n} r_t)$, $a_{j1} = r_j, a_{ij} = 0$, $i, j = 2, ..., n$.

Example 2. Consider the sequence of allocation profiles shown:

$$\begin{bmatrix} 0 & 0.04 & 0.55 & 0.41 \\ 0.04 & 0 & 0.41 & 0.35 \\ 0.52 & 0.38 & 0 & 0 \\ 0.38 & 0.32 & 0 & 0 \end{bmatrix} \rightarrow \begin{bmatrix} 0 & 0.06 & 0.54 & 0.40 \\ 0.04 & 0 & 0.41 & 0.35 \\ 0.52 & 0.38 & 0 & 0 \\ 0.38 & 0.32 & 0 & 0 \end{bmatrix} \rightarrow \begin{bmatrix} 0 & 0.06 & 0.54 & 0.40 \\ 0.07 & 0 & 0.39 & 0.34 \\ 0.52 & 0.38 & 0 & 0 \\ 0.38 & 0.32 & 0 & 0 \end{bmatrix}$$

$$... \rightarrow \begin{bmatrix} 0 & 0.095 & 0.523 & 0.382 \\ 0.085 & 0 & 0.391 & 0.324 \\ 0.52 & 0.38 & 0 & 0 \\ 0.38 & 0.32 & 0 & 0 \end{bmatrix} \rightarrow ... \rightarrow \begin{bmatrix} 0 & 0.10 & 0.52 & 0.38 \\ 0.10 & 0 & 0.38 & 0.32 \\ 0.52 & 0.38 & 0 & 0 \\ 0.38 & 0.32 & 0 & 0 \end{bmatrix}.$$

Starting with the dominant-equilibrium of the initial matrix, where the hegemonic alliance is $\mathcal{N}_1 = \{1, 2\}$, state-1 acts to improve its utility (12) triggering state-2 to improve its utility in turn. The world ends up at a bilateral-equilibrium after states 1 and 2 go through the following sequence of utilities:

$$w_1(a) : 0 \rightarrow 0.02 \rightarrow -0.01 \rightarrow ...0.002 \rightarrow ... \rightarrow 0.000,$$
$$w_2(a) : 0 \rightarrow -0.02 \rightarrow 0.02 \rightarrow ... - 0.01 \rightarrow ... \rightarrow 0.000.$$

Initially "better-off" states $\{1, 2\}$ end up with equal bilateral securities against all.

4 Mixed Payoffs

In all strategic n-person games considered in the previous section, we have assumed that the utility, or preference functions, of all players are homogeneously the same, that is, that they are "of the same type". All states, we assumed, adopt the same notion of individual security. We did not go into what happens when the payoffs are mixed[10]. Also motivated by the debates on whether rational actor assumption is an integral part of neorealist theory or not, Mearsheimer (2009), we now examine some cases in which a group of states have a different notion of individual security than the rest, or even, cases in which not all states are rational so that they may not act strategically while some others do. In the context of strategic games, the meaning that must be attributed to an agent being irrational is that it is a "spoilsport" and does not obey the rules of the game. Thus, in our context, irrational is a state allocating arbitrarily, without conscious employment of any notion of payoff. Naturally, an irrational player is still constrained by its capacity (resource value) and the structural laws (nonnegative allocations, no internal allocation, and sum of its allocations equaling its total resource) so that there is a limit to the damage or good it may cause in its own security or in the security of others.

4.1 Mixed Payoffs Leading to Bilateral-Equilibrium

It is of course no coincidence that bilateral-equilibrium is among the Nash solutions of all four games we have looked at in Section 3. Let $p_i : A \rightarrow \mathbf{R}$ denote any of the utilities (8), (10), or

[10]Not to be confused with "mixed strategy."

(11). Then,

$$p_i(a) \leq 0, \;\; \forall\, i \in \mathcal{N}, \;\; \forall\, a \in A, \tag{13}$$

and hence, any $a^* \in A$ such that $p_i(a^*) = 0 \;\; \forall\, i \in \mathcal{N}$ is a Nash solution. A bilateral-equilibrium allocation, i.e., $a^0_{ij} = a^0_{ji}$, $\forall\, i \neq j$, is such that for every $i \in \mathcal{N}$, $p_i(a^0) = 0$ so that, $a^0 \in A$ must be a Nash solution for any of the utilities (8), (10), or (11).

Any other set of payoffs, uniformly adopted by states or not, would lead to a bilateral-equilibrium a^0 as a Nash solution provided they satisfy (13) and provided they all have value zero at a^0. For example, in the three-state game in which states $1, 2$, and 3 adopt the utilities (8), (10), and (11), respectively, bilateral-equilibrium will be *among* the Nash equilibria.

We now ask what properties of $p_i : A \to \mathbf{R}$ may yield (13) and $p_i(a^0) = 0 \;\; \forall\, i \in \mathcal{N}$ for a bilateral-equilibrium allocation $a^0 \in A$. Let $s_i := (a_{ij} - a_{ji})_{j \in \mathcal{N}_{-i}}$, the collection of the bilateral securities of state-i. If utility of state-i is based on bilateral securities only, then $p_i(a)$ can be regarded as a function $p_i(s_1, ..., s_n)$ of $(s_j)_{j \in \mathcal{N}}$. By definition, p_i is a *homogeneous function of* s_i if there exists a real number r such that $p_i(s_1, ..., s_{i-1}, \alpha s_i, s_{i+1}, ..., s_n) = \alpha^r p_i(s_1, ..., s_{i-1}, s_i, s_{i+1}, ..., s_n)$ for every $\alpha \in \mathbf{R}$.

Fact. *If every $p_i(s_1, ..., s_n) \leq 0$ for any $(s_j)_{j \in \mathcal{N}}$ and if every p_i is a homogeneous function of s_i, then bilateral-equilibrium is a Nash solution.*

Note that the kind of utilities covered here form a much larger class than utilities $v_i(a), u_i(a)$, or $t_i(a)$ since dependence not only on s_i but also on s_j for $j \in \mathcal{N}_{-i}$ is allowed. An example of such utilities is a weighted combination of say v_i's:

$$p_i(s_1, ..., s_n) := \sum_{j=1}^{n} k_{ij} v_i(a),$$

where $v_i(a)$ is given by (8) and $k_{ij} \geq 0$ are arbitrary but fixed real numbers. Such a utility would be adopted by a state that keeps an eye not only on its own bilateral securities but also on some other states'.

4.2 Irrational or Modest among Ambitious

Continuing to examine mixed payoffs, let us now consider a world in which some states adopt the more ambitious utility $w_i(a)$ of (12) while the rest maximize the modest payoffs (8), (10), or (11); or, some states are not strategic players after all. To investigate whether an ambitious state would always be individually more secure than modest or irrational states, we now incorporate the irrational actor assumption as well as nonhomogeneous payoffs into the framework of the strategic games of Section 3. Let $\mathcal{A} \subset \mathcal{N}$ be a nonempty set of states such that each $i \in \mathcal{A}$ maximizes $w_i(a)$. We do not postulate any strategic action (by way of having preferences) on the remaining states in $\mathcal{B} := \mathcal{N} - \mathcal{A}$.

Theorem 6. *If states in a nonempty subset $\mathcal{A}$ of $\mathcal{N}$ maximize $w_i(a)$ of (12), then a Nash equilibrium profile fits into*[11] *one of the four allocation profiles: (1) $\{a_{ij} = a_{ji},\; i \in \mathcal{A},\; j \in \mathcal{N}_{-i}\}$,*

[11] The Nash equilibrium that results has actually more structure since according to the best response function (27) of (12), the allocations in $\mathcal{A}$ obey (24) that imposes more constraints on allocations of i through $\mathcal{M}_i$.

(2) $\{a_{ij} = a_{ji} = 0,\ \{i,j\} \subset \mathcal{A}$ *and* $a_{ij} < a_{ji},\ i \in \mathcal{A},\ j \in \mathcal{B}\}$, *(3)* $\{\mathcal{A} = \{i\}$ *and* $a_{ij} > a_{ji},\ j \in \mathcal{N}_{-i}\}$, *(4)* $\{\mathcal{A} = \{i\} \cup \mathcal{A}',\ a_{il} > a_{li},\ l \in \mathcal{A} - \{i\}$, *and* $a_{jk} < a_{kj},\ j \in \mathcal{A}',\ \ k \in \mathcal{B}\}$

These profiles are described by the following symbolic matrix representations, where $\mathcal{A}_a$ and $\mathcal{A}_d$ denote the set of advantaged and disadvantaged states in $\mathcal{A}$:

$$\begin{array}{c} \mathcal{A}_a \\ \mathcal{B} \end{array} \left[\begin{array}{c|c} = & = \\ \hline = & ? \end{array}\right], \quad \begin{array}{c} \mathcal{A}_d \\ \mathcal{B} \end{array} \left[\begin{array}{c|c} 0 & < \\ \hline > & ? \end{array}\right], \quad \begin{array}{c} \mathcal{A}_a \\ \mathcal{B} \end{array} \left[\begin{array}{c|c} o & > \\ \hline < & ? \end{array}\right], \quad \begin{array}{c} \mathcal{A}_a \\ \mathcal{A}_d \\ \mathcal{B} \end{array} \left[\begin{array}{c|c|c} o & > & > \\ \hline < & 0 & < \\ \hline < & > & ? \end{array}\right].$$

In the presence of irrational players, whether a state of $\mathcal{A}$ is in advantaged or disadvantaged category is by pure chance. This is because the states in $\mathcal{A}_a$ and $\mathcal{A}_d$ are determined by the allocations of irrational players $\mathcal{B}$. Thus, in all four profiles, the security scheme inside $\mathcal{B}$ is somewhat arbitrary, constrained only by the resource magnitudes and the spontaneous impulses of irrational states. According to the result of Theorem 6, first, a Nash equilibrium may result in which $\mathcal{A}_a$ and $\mathcal{B}$ ("ambitious and advantaged" and "irrational") enjoy zero insecurities against each other and in which zero insecurity also prevails inside $\mathcal{A}_a$. Or second, a Nash equilibrium in which all ambitious states are disadvantaged and are dominated by the irrational may result. In this second Nash profile, the set $\mathcal{B}$ of irrational states is a hegemonic-alliance. In the third Nash profile, there is only one ambitious state and that state is advantaged. The allocations of this state (that may or may not be a hegemon) strictly dominate those of $\mathcal{B}$. Finally, there is again one advantaged ambitious state, one or several disadvantaged, and some irrational states. The Nash profile then is one in which all ambitious *and* disadvantaged $\mathcal{A}_d$ are dominated by both $\mathcal{A}_a$ and $\mathcal{B}$ while $\mathcal{A}_a$ also dominates $\mathcal{B}$.

It is not true that, in a Nash equilibrium, the irrational states will always be dominated. Neither is it true that some of them will always be dominating. In the last three profiles, all possible domination schemes are present for the irrational states.

Let us now suppose that members of $\mathcal{B}$ choose their allocations to maximize $v_i(a)$ of (8) so that they are no longer irrational but are modest strategic players against the ambitious players of $\mathcal{A}$.

Theorem 7. *If states in a nonempty subset* $\mathcal{A}$ *of* $\mathcal{N}$ *maximize* $w_i(a)$ *of (12) and* $\mathcal{B} = \mathcal{N} - \mathcal{A}$ *maximize* $v_i(a)$ *of (8), then a Nash equilibrium profile is either a (1) bilateral-equilibrium or fits into one of the allocation profiles: (2)* $\{a_{ij} = a_{ji} = 0,\ \{i,j\} \subset \mathcal{A}$, *and* $a_{ij} < a_{ji},\ i \in \mathcal{A},\ j \in \mathcal{B}$, *and* $a_{kl} = a_{lk},\ \{k,l\} \subset \mathcal{B}\}$, *(3)* $\{\mathcal{A} = \{i\}$, *and* $a_{ij} > a_{ji},\ j \in \mathcal{N}_{-i}$, *and* $a_{kl} = a_{lk},\ \{k,l\} \subset \mathcal{B}\}$, *(4)* $\{\mathcal{A} = \{i\},\ a_{ij} = a_{ji},\ j \in \mathcal{N}_{-i}$, *and* $a_{kl} \geq a_{lk},\ \{k,l\} \subset \mathcal{B}\}$ *with at least one strict inequality* $a_{kl} > a_{lk}$ *in the last profile.*

These four profiles are described by the following matrix representations:

$$\begin{array}{c} \mathcal{A}_a \\ \mathcal{B}_a \end{array} \left[\begin{array}{c|c} = & = \\ \hline = & = \end{array}\right], \quad \begin{array}{c} \mathcal{A}_d \\ \mathcal{B}_a \end{array} \left[\begin{array}{c|c} 0 & < \\ \hline > & = \end{array}\right], \quad \begin{array}{c} \mathcal{A}_a \\ \mathcal{B}_d \end{array} \left[\begin{array}{c|c} o & > \\ \hline < & = \end{array}\right], \quad \begin{array}{c} \mathcal{A}_a \\ \mathcal{B}_a \\ \mathcal{B}_d \end{array} \left[\begin{array}{c|c|c} o & = & = \\ \hline = & = & \geq_{with}> \\ \hline = & \leq_{with}< & = \end{array}\right].$$

It is seen that the world is at bilateral-equilibrium in the first and the fourth Nash profiles, except possibly inside $\mathcal{B}$, where internally only a partitioned-equilibrium may be present. In the second Nash profile $\mathcal{B}$ is a hegemonic-alliance, all ambitious states are disadvantaged, and they are dominated by $\mathcal{B}$; this profile is a dominant-equilibrium. The third Nash equilibrium is a partitioned-equilibrium in which the only ambitious state of the world (that may or may not be a hegemon) dominates all the modest states.

Theorem 8. *If states in a nonempty subset $\mathcal{A}$ of $\mathcal{N}$ maximize $w_i(a)$ of (12) and $\mathcal{B} = \mathcal{N} - \mathcal{A}$ maximize $u_i(a)$ or $t_i(a)$ of (10), (11), then a Nash equilibrium profile is either a bilateral-equilibrium or a dominant-equilibrium.*

These profiles are described by the following matrix representations:

$$\begin{matrix}\mathcal{A}_a\\ \mathcal{B}_a\end{matrix}\left[\begin{array}{c|c} = & = \\ \hline = & = \end{array}\right], \quad \begin{matrix}\mathcal{A}_d\\ \mathcal{B}_a\end{matrix}\left[\begin{array}{c|c} 0 & < \\ \hline > & = \end{array}\right], \quad \begin{matrix}\mathcal{A}_a\\ \mathcal{B}_d\end{matrix}\left[\begin{array}{c|c} o & > \\ \hline < & 0 \end{array}\right].$$

Note that $\mathcal{B}$ is the hegemonic-alliance in the dominant-equilibrium of the second profile and the singleton $\mathcal{A}$ is the hegemon in the third.

Theorems 6-8 are extensions of Theorem 5 for $n \geq 2$ since, if $\mathcal{B} = \emptyset$, then all but the first Nash profiles are eliminated and the first profile is a bilateral-equilibrium.

5 Refinements of Equilibria

The nonuniqueness of Nash equilibrium necessitates either additional motives or a coordination process among states for convergence at a single equilibrium. Consider a modified utility function of a state (its individual security) in which bilateral securities are weighed according to the level of threat the opponent state poses; or, according to the level of democracy in the opponent state. Or, a weighing that takes geographical proximity of the opponent state into account. If each state *also* maximizes such an individual security in addition to one of (8), (10), (11), (12), then the sets of Nash profiles that will result would be a subset of the sets we have obtained, and further, they would be most likely much narrower thereby effecting a refinement. Alternatively, postulate of a central authority, cooperation, socialization, or similarity of world view among states would eliminate some Nash equilibria that have been identified and give a refinement. For instance, cooperation may bring states together at a Pareto efficient equilibrium whenever one exists. Our main purpose in examining such static refinements is to demonstrate how further considerations by states imposed upon the motive of security can cut down the set of possible equilibria.

A further possible refinement of equilibria may be obtained by dynamic processes. Recall that one acceptable interpretation for the notion of Nash equilibrium is that it captures a "steady state" of a game played repeatedly without any strategic links between plays, Osborne and Rubinstein (1994). In case of multiplicity of equilibria, a repeated play of the game, while refining, may never yield a single equilibrium and repetition may give, for instance, oscillatory steady-states between two or more different Nash equilibria. It is hence necessary, in a game with multiple equilibria, to examine closely how agents learn to play the game or to introduce a strategic link such as a "machine strategy" or a "dynamic adjustment process" in refining via any repeated version of the game; and then examine whether such mechanisms result at a unique equilibrium.

In this section, we first show that a bilateral-equilibrium is Pareto efficient and socially optimal in all the games defined by (8), (10), (11), and (12). Next, we indicate a severe consequence of the multiplicity of equilibria, namely, "uncertainty" among states. We then propose two more refinements of Nash equilibria, first, by imposing a "similarity of views" among states, and second, by examining some dynamic adjustments of allocations.

5.1 Pareto Efficient and Socially Optimal Equilibria

By definition, a profile $a \in A$ is *strongly Pareto efficient* if no other profile can lead to better utilities for all states with strictly better utility for at least one state. It is a *social optimum* if it maximizes the sum of all n utilities. The following result identifies Pareto efficient profiles and social optima as bilateral-equilibria in the absence of a hegemon.

Theorem 9. *Consider the n-person strategic game with utilities (8), (10), (11), or (12). Suppose there is no hegemon.* **(i)** *An allocation profile is strongly Pareto efficient with respect to (8), (10), or (11) if and only if it is a bilateral-equilibrium.* **(ii)** *A bilateral-equilibrium profile is strongly Pareto efficient with respect to (12).* **(iii)** *An allocation profile is a social optimum with respect to (8), (10), (11), or (12) if and only if it is a bilateral-equilibrium.*

It follows that if states are able to cooperate, then they may eliminate all partitioned or dominant equilibria (in cases of individual utilities (8), (10), and (11)), restrict themselves to a bilateral-equilibrium allocation, and they are not worse off. With respect to the utility (12), the set of strongly Pareto efficient profiles is much larger than the set of bilateral-equilibria. We note that, although a refinement (at least for some utility functions) at bilateral-equilibria occurs, such a level of cooperation is still not tight enough to refine at a unique equilibrium. Coordination that may follow by learning, or by a scheme to realize (implement) an equilibrium is still needed.

5.2 Multiple Equilibria and Uncertainty

In order to fix ideas about the difficulty the states face in coordinating at a particular Nash equilibrium, consider the system of four states and assume that no hegemon or a near hegemon exists. By Theorem 5, independent security maximization by each state implies a general behavior towards attaining a bilateral-equilibrium whereas nothing specific is predicted concerning the actual process of reaching a bilateral-equilibrium. Every choice of parameter values (x, y) in the triangle of Figure 2 gives a Nash equilibrium. Suppose that the four states, each targeting to reach an equilibrium, choose the following three corner points: states 1 and 2 choose $(x, y) = (0, m)$, state-3 chooses $(x, y) = (m, 0)$, and state-4 chooses $(x, y) = (0, 0)$. Under such choices, the formulae in (5) give $a_{13} < a_{31}$ and $a_{14} < a_{41}$ although bilateral equalities are attained in other allocations. Consequently, a Nash equilibrium is not reached in spite of the fact that each state chose its allocations towards achieving an equilibrium. A state's knowledge of all resource values in the system and its best response function is not sufficient to reach a unique point in the set of bilateral-equilibria.

The nonuniqueness of Nash equilibrium necessitates a coordination process to choose a unique point in the triangle of Figure 2. Burns (1957) and Waltz (1979, p. 135), by referring to the number of possible decisions in a system of n states, emphasized the importance of "seeking certainty" along with the basic motive of seeking security. Our model points to a more fundamental uncertainty. In the situation depicted in Figure 2, state-4 does not know whether the other states have chosen their allocations towards an equilibrium or a disequilibrium. The nonuniqueness of equilibrium may imply an uncertainty of motives of others or a "mistaken perception of intentions."

This uncertainty is strongly contingent upon the "size" of the set of equilibria. If in their attempts to maximize their security, states' efforts leads to cycling or fluctuations in allocations, then because the amplitude of such fluctuations determines the threat perception of the states, the narrower the set of equilibria the less would be uncertainty about true intentions.

5.3 Similar Views: Consistent Equilibrium

We now show that some equilibria in the security game can be eliminated, if the member states possess consistent views as to which of any given two states poses a greater security threat. We restrict the discussion to utility (12) and to $n = 4$, although similar results can be stated for other utilities and for larger n as well.

Suppose that all states are unanimous in their views as to which of any given two states is more threatening and allocate accordingly. This would mean that for every pair of states $\{k, l\} \subset \{1, 2, 3, 4\}$,

$$sign\{a_{ik} - a_{il}\} = sign\{a_{jk} - a_{jl}\}, \;\; \forall \;\; i, j \in \{1, 2, 3, 4\} - \{k, l\}. \tag{14}$$

Thus, $a_{ik} > a_{il}$ if and only if $a_{jk} > a_{jl}$ so that whenever state-i emphasizes state-k so does state-j and *vice versa*. While one natural cause of such an emphasis may be that state-k has more resource than state-l, there may as well be other reasons such as state-k being more hostile than state-l, state-l being geographically more distant than state-k, or state-k having a better offensive capability than state-l.[12]

Using (5) in (14) gives that parameters x and y are further constrained and all possible *consistent equilibria* are those obtained by parameters (x, y) inside the small trapezoid $ABCD$ in Figure 2. Depending upon a given resource distribution, this trapezoid might degenerate into a point or to a triangle. In our 4-state-system a unique equilibrium obtains if and only if $r_1 = r_2 = r_3$ or $r_2 = r_3 = r_4$. Thus, a unique consistent equilibrium is possible if at least three states have equal resources. If, $r_1 - r_2 \geq m$, $r_2 - r_3 \geq m/2$, $r_3 - r_4 \geq m$, then the triangle with corners $A, (0, 0), (m/2, 0)$ results; so that, as the resource imbalance gets larger, the set of possible consistent equilibria enlarges. Recall, however, that as the resource imbalance gets larger, state-1 is also approaching to being a near hegemon. Consequently, the set of possible profiles itself is shrinking, i.e., the triangle in Figure 2 is diminishing in size. In effect, for any given resource distribution, the set of consistent equilibria is a "small" set. *The perception of threat combined with the motive of security thus causes a sharper and much more focused allocation behavior.* We note on passing that, if the motive of security is *replaced with* the motive of responding to threat, then the narrowing down in the possible allocation profiles is actually very little. This indicates that, in the context of the model considered here, the motive of responding to threat, by itself, does not lead to distinctive modes of allocation behavior.

[12]How such differences cause insecurity among other states has been subjected to intensive examinations. On offense/defense debate see Jervis (1978) and Van Evera (1998) and on a formal treatment of the role of geography see Niou and Ordeshook (1989).

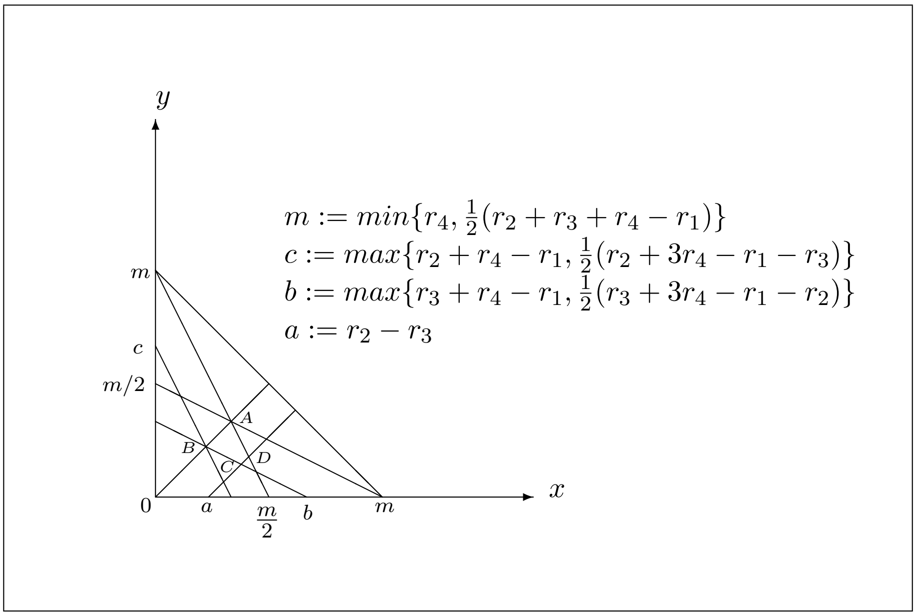

Figure 2: Region of Consistent Equilibria

5.4 A Convergent Dynamic Update Mechanism

Let us now look at the *process* of reaching an equilibrium closely and examine how a unique equilibrium point may be realized by states. If there are multiple Nash profiles or that there is a unique profile but the states initially are not able to choose the correct allocations, then the chance of all n-states meeting at the same equilibrium allocation profile at one shot is very slim. One needs to examine how agents learn as they play the game repeatedly (Fudenberg and Levine, 1998) or postulate an update mechanism that may eventually, at the steady state, converge to one particular Nash profile. It is easier for us to take a quick look at the second approach.

The simplest approach is to update the allocations a_i of each state-i iteratively by making use of the best response of i: With $a_i(k)$ denoting the allocation vector of state-i at the k-th step of iterations, we simply choose

$$a_i(k+1) = a_i^*(k) := B_i^w(a_{-i}(k)), \tag{15}$$

where $a_{-i}(k)$ is the collection of the profiles of all states but i *at time instant* k and $B_i^w(a_{-i}(k))$ denotes the best response of state-i to $a_{-i}(k)$ with respect to utility w. Unfortunately, it is easy to show that this scheme may fail to converge to a Nash equilibrium and exhibits oscillations. Oscillatory behavior also occurs even in the unique equilibrium case of $n = 3$ when states start at different initial allocations. In (15), the states update their allocations at every step according to their best response functions. It has been observed that if some states keep their allocations unchanged for a few steps, then convergence at a unique equilibrium is more often attained. In fact, experiments with the modified version of (15) below has been reported in Sezer and Özgüler (2006) to always converge to a Nash equilibrium allocation. This convergent iterative scheme is given by $a_i(k+1) = \alpha a_i(k) + (1-\alpha)a_i^*(k)$ for some $0 < \alpha < 1$ and updates the allocations slower than (15). Although no convergence proof exists for this discrete update scheme, its continuous version, $\dot{a}_i = f_i(a_i, a_{-i}), i \in \mathcal{N}$ for some suitable nonlinear function f_i (that has uniformly the same form for all $i \in \mathcal{N}$), is proved in Sezer and Özgüler (2006) to converge to a Nash equilibrium for every initial condition.

These initial attempts at deriving dynamic laws for realizing a unique equilibrium profile shows that (i) convergence requires slow updating of allocations and that (ii) in view of the severe assumptions needed in order to be able to obtain a convergent update scheme, realizing an equilibrium point is intrinsically difficult unless a considerable refinement of Nash equilibria is first effected.

6 Implications

Some implications of Theorems 2-9 are sharp enough to be listed right away:

i. Certain modest notions of individual security, like $v_i(a)$, may lead to the domination of some states, even relatively strong ones (Example: A partitioned-equilibrium in which a hegemonic-alliance is disadvantaged, allocation matrix R_6.)

ii. A bilateral-equilibrium profile is a common Nash profile with respect to many notions of individual security. In a nonhegemonic world, cautious states, that adopt a more ambitious notion of individual security, will always end up in a bilateral-equilibrium (Theorems 2-8).

iii. Being ambitious, rather than modest, will not necessarily make states better off. Their greed will lead them from a dominant-equilibrium, in which they dominate, to a bilateral-equilibrium, in which they are as secure as the rest of the states (Example 2).

iv. A bilateral-equilibrium is very frail because, in it, all bilateral securities are exactly zero. Any slight perturbation in the allocation scheme is sure to disrupt equilibrium and make many states insecure (Definition 1).

v. Structure and motive of security combined lead to indigenous formations of tight-alliances and coalitions (Examples: Partitions of Theorems 2, 3, and 4. Some Nash profiles in Theorems 6-8. Strongest state approaching the status of a near-hegemon in Theorem 1).

vi. In a world without a hegemon, if all states are cautious and ambitiously maximize their individual securities $w_i(a)$, then a bilateral-equilibrium, which is Pareto efficient and a social optimum, will prevail (Theorems 5, 9).

We now look closer at some plausible Nash profiles under a number of salient structural constraints in the nontrivial cases $n > 2$. In Theorems 2-8, certain Nash equilibria will be eliminated due to an imposed structural constraint. Certain others will be possible but not very likely to occur because in a randomly chosen allocation profile a weak (comparably small resource value) state will most probably be disadvantaged and a strong (large resource value) state will most probably be advantaged. Nash equilibria in which a weak state is disadvantaged and in which a strong state is advantaged may thus be labeled as *plausible.*

Going outside the scope of the static theory, the main disadvantage of a Nash profile that is not a bilateral-equilibrium may be that a sustained security dominance would be a source of constant worry for the dominated states. A world in which many dominated states exist for a prolonged period may be laden with crisis. In a dynamic sense, bilateral-equilibrium (although still precarious) may be preferable to partitioned- or dominant-equilibrium, or to other Nash profiles like those in Theorems 6-8 in which some states remain dominated.

6.1 One Spoilsport in the World

It is interesting to observe that, one modest state can survive without being dominated in a nonhegemonic world of ambitious states, because bilateral-equilibrium is the only possible Nash profile. In fact, if all states employ w_i but one, then $\mathcal{B}$ is a singleton and in Theorems 7 and 8, only the first and second profiles are possible. The second profile is obtained only if the modest state is a hegemon. It follows that the bilateral-equilibrium of the first profile is the only possibility in a world without a hegemon.

If there is one irrational state among ambitious, then in Theorem 6, the first Nash profile is now a bilateral-equilibrium since $\mathcal{B}$ is a singleton. The other possibilities are the second and last profiles as the third drops by $n > 2$. The second profile is possible only if the irrational state is a hegemon. In the last profile, there is one ambitious and advantaged state and all the other ambitious states are disadvantaged. Such an allocation profile is plausible in case one ambitious state is very strong and the next stronger state is the irrational one. Thus, a strong irrational state among ambitious is likely to yield an equilibrium in which many (last profile) or all (second profile) other states are insecure. Of course, a bilateral-equilibrium is still a possibility in such a world and it is the only likely one if the irrational state is not strong.

6.2 Bipolar World

Suppose there are two states with large resource values and $n-2$ small states. Neither of the two strong states is a hegemon. It is possible, but not assumed, that the two strong states constitute a hegemonic-alliance. One prominent feature of a bipolar world is a significant reduction in the set of possible bilateral-equilibria, which has the consequence that a bipolar world suffers less from uncertainty because the fluctuations about the equilibrium allocations would be limited in size.[13]

If all states uniformly adopt a notion of utility v_i, u_i, t_i, or w_i, then Theorems 2-5 anticipate a partitioned, dominant, or a bilateral-equilibrium to prevail. In a partitioned and dominant equilibrium, it is very likely that the two strong states will be security-wise dominating the rest of the states because, even in case they allocate a considerable portion of their resources against each other, their remnant resources will still be sufficient to ensure security against all smaller states. In the partitioned-equilibrium of utility v_i, there is a possibility that one or both of the strong states are disadvantaged, which leads to a profile that leaves one or both dominated. However, the plausible partitioned-equilibrium profile would be the one in which both states are advantaged and constitute $\mathcal{A}_a$ to security-wise equate or dominate the rest of the states. For a dominant-equilibrium, two strong states must constitute a hegemonic-alliance and the invisible hand brings the small states together in a coalition so that they allocate no resources against

[13]The reduction in bilateral-equilibria follows by the hypothesis that $r_3, ..., r_n$ are much smaller than $r_1 \approx r_2$ and by the characterization in the Remark A.1 of Appendix.

each other. This may be argued to leave the bilateral-equilibrium as the more likely Nash profile in case of utilities u_i and t_i in addition to w_i (where it is the only possible Nash profile).

If the two strong states are cautious and adopt the ambitious utility w_i, then Theorems 5-8 would be relevant and those imply that a (near) bilateral-equilibrium is the most plausible Nash profile to prevail. The implication of Theorem 5 being clear, let us clarify this first for Theorem 6, in which smaller states are irrational. As there are two ambitious states the third allocation profile is dropped. The fact that all but two states are relatively weak would mean that, in a randomly chosen allocation profile by irrational states, the stronger states will be advantaged and constitute $\mathcal{A}_a$. This eliminates the Nash profiles *(2)* and *(4)* in Theorem 6 and leaves the first one, in which the world is at bilateral-equilibrium possibly except locally among the small (irrational) states themselves. If the smaller states are strategic players and adopt some notion of individual security, then at a minimum, we are in the domain of Theorem 7, and at a maximum, of Theorem 8, which both imply that bilateral-equilibrium is the most plausible Nash profile.

Turning things around, what would be the implications of two strong states being irrational, assuming that all small states maximize w_i? If there are two or more small states, then the third Nash profile of Theorem 6 is eliminated. If there is one advantaged state among the smaller ones, then the last profile of Theorem 6, in which both strong states are dominated by the advantaged small state, is a possibility but not likely. In addition to bilateral-equilibrium, the only other possible Nash profile is the second allocation scheme of Theorem 6. This scheme is a dominant-equilibrium with the strong (irrational) states forming a hegemonic-alliance and small states being dominated; departure from the dominant-equilibrium of Definition 4 is that the strong states may not be in an internal bilateral-equilibrium. Thus, the somewhat surprising result is that, being irrational is not likely to hurt the two strong states. More often than not, they are secure against the rest of the world.

The world in the cold-war era is often described as bipolar with its two poles USA and USSR. The description is in agreement with the usage here and, in fact, the resource levels of these two countries would qualify them as a near hegemonic-alliance or, more appropriately, as "hegemonic adversaries." If they did at all, the smaller states pursued very modest utilities of individual security, while USA and USSR adopted an ambitious one. This puts the cold-war equilibrium into the framework of Theorems 6 or 7, in which the only possible Nash profile left would be a bilateral-equilibrium (except possibly internally among irrational small states, if any). The Nash equilibrium of the cold war period was, hence, a bilateral-equilibrium. However, as was witnessed, the world experienced not the frail equilibrium of exactly zero bilateral securities but constant fluctuations about it. The fluctuations left one state or the other, including the two poles themselves, insecure for certain periods of time (sometimes significantly such as in the Cuban Missile Crisis, the Korean War, or the Yom Kippur War) but always recovering the equilibrium, only to be disturbed once again. Although it is outside the scope of the static theory developed here, it would not be a mistake to characterize the bilateral-equilibrium of the cold war era as a *stable equilibrium*, since perturbations about the equilibrium have always faded out. Further, because of the structure of a bipolar world, the multiplicity in bilateral-equilibria was small limiting the size of possible fluctuations from it. In this sense, the cold-war era was also a period of certainty.

6.3 Unipolar World

If there is a hegemon in an n-state world, then a bilateral equilibrium is not possible. A state is a hegemon in an n-state world if it can reciprocate and will still be left with excess resource even in case all remaining $n-1$ states direct all their resources against it. In view of the vigor of this definition, it is doubtful whether there has been any time in history when a world power could comfortably be called a hegemon. While it is easy to agree today that USA is a superpower and is the strongest state in the world, its resource level would hardly qualify it as a hegemon. In fact, in the imaginary case of all states in the world uniting and acting against it, the mere consequences of its isolation from the rest of the world would leave USA with little power, even if one ignores the size of the hard resources directed to it. A true hegemon is difficult to find today or in the past. What is usually meant by a unipolar world is not that there is a hegemon, but that there is a strong state in the world, called the superpower, having a resource level that is much larger than the next strong state's.

Keeping in mind that a true hegemon is scarce, let us still examine the consequences of a hegemonic world. In the absence of strategically acting actors, existence of a hegemon does not necessarily imply that it will security-wise dominate the rest of the states in every allocation profile (except when $n = 2$). Even a hegemon, overemphasizing one state, may be insecure against a large number of states. Let us, however, analyze a hegemonic world in a strategic context in view of Theorems 2-8.

Recall that in the partitioned-equilibrium of Theorem 2, the advantaged states will always be in a hegemonic-alliance that has the hegemon as a member; perhaps the only one. In the dominant-equilibrium of Theorems 3 and 4, the situation is the same with the additional constraint that the disadvantaged states form a coalition. In Theorem 5, the only hegemonic-alliance is the singleton of the hegemon and the remaining $n-1$ states are in a coalition.

If the hegemon adopts the ambitious utility w_i, then all but the third Nash profiles are eliminated in Theorems 6-8. Hence, whether the remaining states act strategically or not, the only possible Nash profile is the one in which the hegemon security-wise dominates every other state. The allocation scheme among other states, internally, may be anything from arbitrary to a bilateral-equilibrium to all-zero allocations. On the other hand, if the hegemon is one of the irrational states, then all Nash profiles of Theorem 6 are possible. The hegemon will be dominated or equated by the rational states if it directs most of its resource to one or more irrational states. The only plausible profile, however, is the second one of Theorem 6, in which the hegemon again strictly dominates all the rest.

These conclusions apply to a superpower as well, only the inequalities will be less pronounced. In a world in which most states are irrational or modest, a rational or irrational superpower will most likely dominate the rest of the world in a randomly chosen allocation profile. If we adopt the dynamic possibility indicated above that prolonged dominance is pregnant to crisis, then such a world would not be so safe. There is, however, one chance for a safer world. If at least one other strong state emerges and allocates, together with the superpower, cautiously according to w_i, then the most plausible Nash profile will be one of (near) bilateral-equilibrium, the first Nash profile in Theorems 6-8.

One more point need be mentioned. The set of available Nash profiles in a hegemonic world or a world with a superpower is not necessarily small because the partitioned or dominant equilibria, second or third profiles of Theorems 6-8, may each encompass a large number of allocation schemes. The level of certainty in a hegemonic world is, thus, not necessarily high. There is however one persistent certainty: whom all states are insecure against.

6.4 Multipolar World

Let us now suppose that there are three or more similarly sized states, which are stronger than the rest. If strong states are all ambitious, then they constitute $\mathcal{A}$. Let us only consider the case all weaker states make up $\mathcal{B}$. Thus, a multipolar world would be such that $\mathcal{A}$ is close to a hegemonic-alliance, that is, all resources of the strong states put together would be comparable to the rest of the resources in the world. The second Nash profiles in Theorems 6-8 are then eliminated. The third Nash profile of Theorems 6-8 also do not apply since there is more than one ambitious state. It follows that in a Nash profile, the world is either at a (near) bilateral-equilibrium or the last profile in Theorem 6 is obtained. This profile, however, is such that all but one state in $\mathcal{A}$ are disadvantaged, not likely to occur but still a possibility. Thus, similar to a bipolar world in which the two poles are ambitious, a multipolar world in which all the poles are ambitious would most likely reach bilateral-equilibrium whether the remaining states are rational or irrational. The difference from a bipolar world is in the size of the available Nash profiles. Also note that, if one or more strong states are irrational, then the last profile of Theorem 6 in which many states are insecure is also plausible.

One distinctive feature of a multipolar world is that, because there are many states of comparable resource levels, even when limited to bilateral-equilibrium, the set of possible equilibria is large. By the discussion in Section 5.2, the larger the set of Nash equilibria, the more difficult it gets for the states to meet at one equilibrium point. A multipolar world, is a less certain one than a bipolar world.

Suppose that the trend continues in the next decade so that the resource gap between the superpower USA and the stronger states China, Russian Federation, India, European powers, and Japan continues to close. Also suppose that there are new strong players like Brazil, Turkey, South Korea, Indonesia in the resource allocation game. The world would then be a true multipolar one with three or four very strong, six or seven strong, and many weak states. Assume that all very strong states and some strong states adopt the ambitious utility. The theory, then, implies that the most plausible Nash profile is a bilateral-equilibrium; an equilibrium that is very difficult to realize and to maintain. The multiplicity of possible equilibrium points and the many uncoordinated attempts to attain one will give way to large oscillations in bilateral securities and to many misconceptions about true intentions. We may expect an era in which foreign diplomacy will have to skilfully resolve many recurring conflicts and administer hard bargaining processes.

7 Conclusions

Contrary to many game-theoretic studies that put structure theory to test, our results offer strong support for the theory. The main source of this difference may be that the model here applies to peacetime. While one can debate whether the balancing imperative of Waltz was meant for peacetime as well, the widespread perception has clearly been that it applies to times of crisis. The reason is that the term "balance of power" has been applied to crisis environments in which states form and/or break tight alliances or coalitions (external means of balancing) and to the transitory phases of building resources (internal means). This study shows that a peacetime theory of international politics still exists and has a rich and complex build even when all states are assumed to have fixed resources (no transfer from one state to another, no internal change) and even without any postulate of alliance configurations. Power balancing in such a regime reduces to a moderate series of measures in ensuring security — a behavior that exhibits many features Waltz has envisioned.

The fact that a principle of harmony follows from the assumptions of self-help and the motive of security, under a wide range of security notions by states, seems interesting. It is, of course, a result in the realm of neorealism and open to both optimistic and pessimistic interpretations, depending on how one looks at it. Bilateral-equilibrium is, after all, a mode in which all states are equally secure and, at the same time, equally insecure. One can also point blank ignore it because self-help or security motive postulates are not acceptable to start with. This has, in any case, been the main argument of many critics of neorealism. Their thesis is that the postulate of self-help or the motive of security should be superseded with ideational factors. An alternative may be to build upon the security harmony, asking whether friendship can spontaneously flourish among creatures whom we graphically sketched in Figure 1 to cheerfully push each other.

The closest analogue of the principle of harmony of security is the first welfare theorem and both, in general, suffer from the multiplicity of equilibria. The difficulty of coordinating at one fixed equilibrium point is a latent source of confusion and struggle in international politics. Intranational institutions, democracy tradition, socialization, and leadership qualities may all be viewed as factors that contribute to the refinement of plausible Nash profiles. Their contribution may be in the form of helping states *learn* an equilibrium or in delineating dynamic adjustment rules that *realize* one. These are challenging problems for formal studies.

APPENDIX

In proving Theorems 2-9, the terminology and notation of Osborne and Rubinstein (1994) will be used. All n-person strategic games considered consist of the set $\mathcal{N}$ of players, the set of actions A_i of (1) available to player-i, and the utility function $u_i : \times_{i\in\mathcal{N}} A_i \rightarrow \mathbf{R}$ associated with player-i. For any profile $a = (a_j)_{j\in\mathcal{N}}$ and any $i \in \mathcal{N}$, we let a_{-i} to be the collection $a = (a_j)_{j\in\mathcal{N}_{-i}}$ of all profiles except i's. By (a_{-i}, a_i), we denote the profile $a = (a_i)_{i\in\mathcal{N}}$. By definition, a profile of actions $a \in \times_{i\in\mathcal{N}} A_i$ is a Nash equilibrium (Nash profile, Nash solution) if

$$u_i(a) \geq u_i(a_{-i}, a'_i) \;\; \forall\, i \in \mathcal{N}, \;\; \forall\, a'_i \in A_i, \tag{16}$$

Recall that the *best response function* of a player $i \in \mathcal{N}$ is $B_i(a_{-i}) = \{a_i \in A_i : u_i(a_{-i}, a_i) \geq u_i(a_{-i}, a'_i) \;\forall\; a'_i \in A_i\}$. In terms of $B_i(a_{-i})$, a profile a is a Nash equilibrium if and only if $a_i \in B_i(a_{-i})$ for every $i \in \mathcal{N}$.

Proof of Theorem 1. We give a proof based on the resource allocation matrix $R = [a_{ij}]_{i,j\in\mathcal{N}}$, with $a_{ii} = 0 \;\forall\; i \in \mathcal{N}$ and with i-th row-sum equal to r_i. Recall that an allocation profile is a bilateral-equilibrium if and only if it gives a symmetric resource allocation matrix.

In any symmetric resource allocation matrix, the row-sum of any row (say, r_i) is less than or equal to the sum of the row-sums of other rows ($\sum_{j\in\mathcal{N}_{-i}} r_j$). This is easy to see since, by symmetry and by $a_{ii} = 0$, the entries of row-i are the same as the entries of column-i, which are included in the total sum of the rows $\{1, ..., i-1, i+1, ..., n\}$. Hence, symmetry implies $r_i \leq \sum_{j \in \mathcal{N}_{-i}} r_j$ for any i so that if a bilateral-equilibrium exists, then there is no hegemon. Conversely, suppose there is no hegemon. If $n = 2$, then no hegemon means $r_1 = r_2$, which in turn gives $a_{12} = a_{21} = r_1$ and this is a (unique) bilateral-equilibrium. Proceeding by induction, assume that given any $\hat{r}_1, ..., \hat{r}_{n-1} > 0$ such that $\hat{r}_i \leq \sum_{j \neq i} \hat{r}_j$ for each $i \in \mathcal{N} - 1$, a fixed symmetric resource allocation matrix of size $n-1$ with i-th row-sum $\hat{r}_i$ exists. Given resources $r_1 \geq ... \geq r_n > 0$ such that $r_1 \leq r_2 + ... + r_n$, we construct a symmetric resource allocation matrix R_n with i-th row-sum r_i as follows. Let parameters $x_i,\ i \in \mathcal{N} - 2$, satisfy

$$0 \leq x_1 + x_2 + ... + x_{n-2} \leq \min\{r_n, \frac{r_2 + r_3 + ... + r_n - r_1}{2}\}, \text{ for } n \geq 4 \tag{17}$$

and let $x_1 = \frac{r_2 + r_3 - r_1}{2}$ when $n = 3$. One can easily verify, using (17) and the ordering $r_1 \geq ... \geq r_n > 0$ that, $\hat{r}_1 := r_1 - r_n + \sum_{j=1}^{n-2} x_j,\ \hat{r}_j := r_j - x_{j-1},\ j \in \mathcal{N} - 1$ satisfy $\hat{r}_i \leq \sum_{j=1, j \neq i}^{n-1} \hat{r}_j$ for each $i \in \mathcal{N} - 1$. Hence, by the induction hypothesis, there exists a symmetric resource allocation matrix R_{n-1} of size $n-1$ with i-th row-sum $\hat{r}_i$. Consider the column vector $X := [r_n - \sum_{j=1}^{n-2} x_j \;\; x_1 \; ... \; x_{n-2}]^T$ of size $n-1$ and the $n \times n$ matrix

$$\begin{bmatrix} R_{n-1} & X \\ X^T & 0 \end{bmatrix}. \tag{18}$$

It is straightforward to check that this is a symmetric resource allocation matrix with i-th row-sum equal to r_i for $i = 1, ..., n$ so that it represents a bilateral-equilibrium. This proves the first statement.

If $n \geq 4$, then the lower limit of the inequality (17) is zero and provided $r_2 + r_3 + ... + r_n - r_1 > 0$, or equivalently, provided no near-hegemon exists, there are infinitely many bilateral-equilibria since the parameters in (17) can then take infinitely many different values. Let us now observe that given any symmetric resource allocation matrix $R_n = [a_{ij}]$ of size n with i-th row-sum r_i, we have $a_{1n} = r_n - (x_1 + ... + x_{n-2})$ with $x_{i-1} := a_{in},\ i \in \mathcal{N} - 1$. Partitioning, R_n can be written in the form (18) for some symmetric resource allocation matrix R_{n-1} of size $n-1$ with i-th row-sum equal to $r_i - a_{in}$ and for the vector $x = [a_{1n} \; ... \; a_{(n-1)n}]^T$. The parameters x_j satisfy (18) since $r_n - \sum_{j=1}^{n-2} x_j = a_{1n} \geq 0$ and since, by the fact that R_{n-1} gives a bilateral-equilibrium, $r_1 - a_{1n} \leq \sum_{j=2}^{n-1} (r_j - a_{jn})$. Hence, every symmetric resource allocation matrix of size n can be expressed as in (18) for some symmetric resource allocation matrix R_{n-1} of resources $\hat{r}_i$ and parameters x_i satisfying (17). Now, uniqueness for $n = 2$ being clear, note that for $n = 3$, (17) gives that $x_1 = \frac{r_2 + r_3 - r_1}{2}$ so that there is a unique symmetric resource allocation matrix with entries (4). If $n \geq 4$ and $r_1 = r_2 + r_3 + ... + r_n$, then the upper limit of the inequality (17) is zero which gives in (18) that $x = [r_n \; 0 \; ... \; 0]^T$ and that the first row-sum of R_{n-1} is $r_1 - r_n = r_2 + ... + r_{n-1}$. This means that among $\hat{r}_i,\ i \in \mathcal{N} - \{1\}$ the first state with $\hat{r}_1 = r_1 - r_n$ is a near-hegemon. By induction, we get that, if there is a near-hegemon with resource $r_1 = r_2 + r_3 + ... + r_n$, then $R_n = [a_{ij}]$, where $a_{1k} = a_{k1} = r_k$ for $k \in \mathcal{N}_{-1}$ and $a_{ij} = 0$ otherwise, is the unique symmetric resource allocation matrix. □

Remark A.1. The proof above also suggests a procedure for $n \geq 4$ to obtain *the set of all bilateral-equilibria*. For any fixed set of parameters $x_i,\ i \in \mathcal{N} - 2$ picked according to

(17), construct all symmetric resource allocation matrices of size $n-1$ with resources $\hat{r}_1 := r_1 - r_n + \sum_{j=1}^{n-2} x_j,\ \hat{r}_j := r_j - x_{j-1},\ j \in \mathcal{N}-1$ and form R_n according to (18). Note that, by induction, there are a total of $(n-2)+(n-3)+...+2 = \frac{(n-1)(n-2)}{2} - 1$ free parameters which are constrained consecutively by the inequalities (17) written for $n, n-1, ..., 4$. A principle that follows by this characterization is that, *the set of all bilateral-equilibria is a "small set" if and only if either resource values $r_4, r_5, ..., r_n$ are* all *sufficiently small or the first state is close to being a near-hegemon.* △

We now turn to derive the best response functions for the utilities (8), (10), (11), and (12).

Remark A.2. We first examine the index set $\mathcal{M}_i$ of Definition 4, re-defined here for convenience: Given allocations $\{a_{ji} : j \in \mathcal{N}_{-i}\} \subset a_{-i}$ against state-i, let

$$j \in \mathcal{M}_i \ \textit{ if and only if } \ \frac{1}{|\mathcal{M}_i|}(\sum_{l\in\mathcal{M}_i} a_{li} - r_i) \leq a_{ji}. \tag{19}$$

If

$$\sum_{j\in\mathcal{N}_{-i}} a_{ji} \leq r_i, \tag{20}$$

then, $\mathcal{M}_i = \mathcal{N}_{-i}$ since with this index set the left hand side of the inequality in (19) is negative or zero. If, on the other hand,

$$\sum_{j\in\mathcal{N}_{-i}} a_{ji} > r_i, \tag{21}$$

then, let $a_{j_1 i} \geq a_{j_2 i} \geq ... \geq a_{j_{n-1} i},\ \{j_1, ..., j_{n-1}\} = \mathcal{N}_{-i}$ be an ordering of allocations against state-i. The index set will then be given by $\mathcal{M}_i = \{j_1, ..., j_{m(i)-1}\}$, where $m(i) \in [2, n]$ is the minimum integer satisfying

$$\frac{1}{m}(\sum_{t=1}^{m} a_{j_t i} - r_i) > a_{j_m i}, \tag{22}$$

with $a_{j_n i} := 0$. Note that if (21) holds and $\frac{1}{n-1}(\sum_{t=1}^{n-1} a_{j_t i} - r_i) \leq a_{j_{n-1} i}$, then $m(i)$ is equal to n and hence, again, $\mathcal{M}_i = \mathcal{N}_{-i}$ since the inequality in (19) will hold for every element of $\mathcal{N}_{-i}$. If (21) holds and $\frac{1}{n-1}(\sum_{t=1}^{n-1} a_{j_t i} - r_i) > a_{j_{n-1} i}$, then by the identity

$$\frac{1}{m}(\sum_{t=1}^{m} a_{j_t i} - r_i) - a_{j_m i} = \frac{m-1}{m}[\frac{1}{m-1}(\sum_{t=1}^{m-1} a_{j_t i} - r_i) - a_{j_{m-1} i}] + \frac{m-1}{m}(a_{j_{m-1} i} - a_{j_m i})$$

valid for $2 \leq m \leq n-1$, a minimum $m(i)$ exists and satisfies $m(i) \leq n-1$. If we define

$$c_i := \frac{1}{|\mathcal{M}_i|}(r_i - \sum_{j\in\mathcal{M}_i} a_{ji}), \tag{23}$$

then (20) clearly implies $c_i \geq 0$. On the other hand, (21) implies $c_i < 0$ since $\mathcal{M}_i = \{j_1, ..., j_{m(i)-1}\}$ and for all $2 \leq m \leq m(i)$, (22) holds. We note that given allocations $\{a_{ji} : j \in \mathcal{N}_{-i}\}$ against i, not necessarily ordered, a computation procedure of $\mathcal{M}_i$ is seen by the diagram in which $a_{1i} = 21, a_{2i} = 3, a_{3i} = 16, a_{4i} = 13, a_{5i} = 1, a_{6i} = 4, a_{7i} = 0$ and $r_i = 20$. Among the sequence

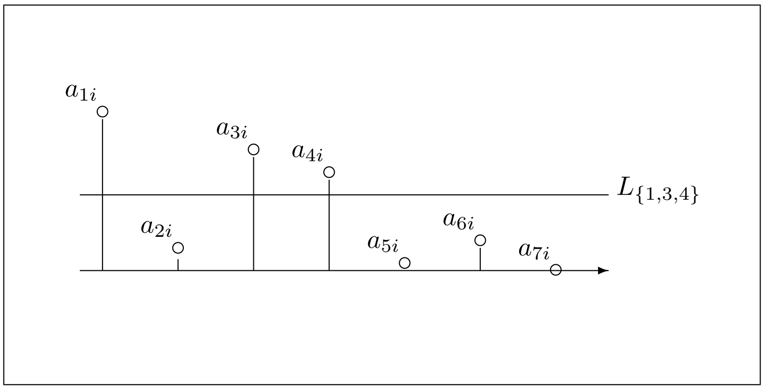

Figure 3: A procedure to compute $\mathcal{M}_i$

of index sets obtained by consecutively deleting $7, 5, 2, 6, 4, ...$ from $\mathcal{N}_{-i} = \{1, 2, 3, 4, 5, 6, 7\}$, at $\{1, 3, 4\}$, we obtain $L_{\{1,3,4\}} := [(21 + 16 + 13) - 20]/3 = 10 < 13 = a_{i4}$ and $L_{\{1,3,4\}} > a_{i6} = 4$ so that $\mathcal{M}_i = \{1, 3, 4\}$. $\triangle$

Let $B_i^p(a_{-i})$ denote the best response, with respect to utility p, of state-i to a_{-i}. Consider the allocation profile

$$a_{ij}^b := \begin{cases} a_{ji} + c_i & \text{for} \quad j \in \mathcal{M}_i \\ 0 & \text{for} \quad j \notin \mathcal{M}_i, \end{cases} \tag{24}$$

where c_i and $\mathcal{M}_i$ are defined by (23) and (19).

Lemma A.1. *The best response functions of state-i with respect to (8), (10), (11), and (12) are*

$$B_i^v(a_{-i}) = \{a_i \in A_i : \begin{cases} a_{ij} \geq a_{ji} \ \ \forall \ j \in \mathcal{N}_{-i} \text{ if } \sum_{j \in \mathcal{N}_{-i}} a_{ji} \leq r_i, \\ a_{ij} \leq a_{ji} \ \ \forall \ j \in \mathcal{N}_{-i} \text{ if } \sum_{j \in \mathcal{N}_{-i}} a_{ji} > r_i\}, \end{cases} \tag{25}$$

$$B_i^u(a_{-i}) = B_i^t(a_{-i}) = \{a_i \in A_i : \begin{cases} a_{ij} \geq a_{ji} \ \ \forall \ j \in \mathcal{N}_{-i} \text{ if } \sum_{j \in \mathcal{N}_{-i}} a_{ji} \leq r_i, \\ a_{ij} = a_{ij}^b \ \ \forall \ j \in \mathcal{N}_{-i} \text{ if } \sum_{j \in \mathcal{N}_{-i}} a_{ji} > r_i\}, \end{cases} \tag{26}$$

$$B_i^w(a_{-i}) = \{a_i \in A_i : a_{ij} = a_{ij}^b \ \ \forall \ j \in \mathcal{N}_{-i}.\} \tag{27}$$

Proof. Suppose (20) holds, i.e., $\sum_{j \in \mathcal{N}_{-i}} a_{ji} \leq r_i$. We first show that $B_i^v(a_{-i}) = B_i^u(a_{-i}) = B_i^t(a_{-i}) = \{a_i \in A_i : a_{ij} \geq a_{ji}\} =: B_i^*(a_{-i})$. If $a_i^* \in B_i^*(a_{-i})$, then $v_i(a_{-i}, a_i^*) = u_i(a_{-i}, a_i^*) =$

$t_i(a_{-i}, a_i^*) = 0$ while $v_i(a_{-i}, a_i) = u_i(a_{-i}, a_i) = t_i(a_{-i}, a_i) \le 0$ for any $a_i \in A_i$. Hence, a_i^* is indeed the best response. We next show that $B_i^w(a_{-i}) = \{a_i \in A_i : a_{ij} = a_{ij}^b\}$. Let $a_i^* \in B_i^s(a_{-i})$ so that, because (20) implies $\mathcal{M}_i = \mathcal{N}_{-i}$, we have $a_{ij}^* = a_{ji} + c_i$ for all $j \in \mathcal{N}_{-i}$ and $\min_{j\in\mathcal{N}_{-i}} (a_{ij}^* - a_{ji}) = c_i = \frac{1}{n-1}(r_i - \sum_{j\in\mathcal{N}_{-i}} a_{ji})$. If for some $a_i' \in A_i$, it holds that $\min_{j\in\mathcal{N}_{-i}} (a_{ij}' - a_{ji}) > \frac{1}{n-1}(r_i - \sum_{j\in\mathcal{N}_{-i}} a_{ji})$, then $a_{ij}' > a_{ji}) + \frac{1}{n-1}(r_i - \sum_{j\in\mathcal{N}_{-i}} a_{ji})$ for all $j \in \mathcal{N}_{-i}$ and, summing over j, we obtain $\sum_{j\in\mathcal{N}_{-i}} a_{ji}' = r_i > r_i$, a contradiction. It follows that $B_i^w(a_{-i})$ is also the best response if (20) holds.

Suppose (21) holds, i.e., $\sum_{j\in\mathcal{N}_{-i}} a_{ji} > r_i$. We show that for every best response a_i^* of state-i with respect to (8), (10), or (11), we must have $J_i^* = \mathcal{N}_{-i}$. Suppose $J_i^* \neq \mathcal{N}_{-i}$ so that there exists $l \in \mathcal{N}_{-i}$ for which $a_{il}^* > a_{li}$. Since by hypothesis $r_i - \sum_{j\in\mathcal{N}_{-i}} a_{ji} = \sum_{j\in\mathcal{N}_{-i}} (a_{ij}^* - a_{ji}) < 0$, there also exists $k \in \mathcal{N}_{-i}$ for which $a_{ik}^* < a_{ki}$. Let $I_i^* := \{j \in J_i^* : a_{ij}^* \le a_{ji}\}$. Let K be the number of elements in I_i^* and let I_i^{c*} denote the complement of I_i^* in $\mathcal{N}_{-i}$. Now consider $a_i' \in A_i$ defined by

$$
\begin{aligned}
&a_{ij}' := a_{ij}^* + \tfrac{1}{L}(a_{il}^* - a_{li}),\ \forall\, j \in I_i^*,\\
&a_{il}' := a_{il}^* - \tfrac{K}{L}(a_{il}^* - a_{li}),\\
&a_{ij}' := a_{ij}^*,\ \forall\, j \in I_i^{c*} - \{l\},
\end{aligned}
$$

for some large enough integer L such that $a_{ij}' \le a_{ji}$ for all $j \in I_i^*$ and such that $a_{il}' > a_{li}$. Thus, going from "star" to "prime", without changing signs, we increase the strictly negative bilateral securities, decrease one strictly positive security, and keep the rest of the bilateral securities the same. It follows that $J_i^* = J_i'$, which in turn is equal to $I_i^* \cup \{j \in J_i' : a_{ij}' = a_{ji}\}$. Now, $|a_{ij}' - a_{ji}| < |a_{ij}^* - a_{ji}|\ \forall\, j \in I_i^*$ implies that $v_i(a_{-i}, a_i') > v_i(a_{-i}, a_i^*)$, $u_i(a_{-i}, a_i') > u_i(a_{-i}, a_i^*)$, and $t_i(a_{-i}, a_i') > t_i(a_{-i}, a_i^*)$, which contradicts the fact that a_i^* is a best response of state-i. Hence, (21) implies that $J_i^* = \mathcal{N}_{-i}$ for any best response a_i^* with respect to utilities (8), (10), and (11). This gives in turn that,

$$
\begin{aligned}
v_i(a_{-i}, a_i^*) &= \sum_{j\in\mathcal{N}_{-i}} (a_{ij}^* - a_{ji}) = |\mathcal{M}_i| c_i - \sum_{j\notin\mathcal{M}_i} a_{ji},\\
u_i(a_{-i}, a_i^*) &= \min_{j\in\mathcal{N}_{-i}} (a_{ij}^* - a_{ji}) = c_i,\\
t_i(a_{-i}, a_i^*) &= -\sqrt{\sum_{j\in\mathcal{N}_{-i}} (a_{ij}^* - a_{ji})^2} = -\sqrt{|\mathcal{M}_i| c_i^2 + \sum_{j\notin\mathcal{M}_i} a_{ji}^2}.
\end{aligned}
$$

Let $a_i^* \in B_i^v(a_{-i})$. Suppose for some $a_i' \in A_i$, we have $v_i(a_{-i}, a_i') > v_i(a_{-i}, a_i^*)$, or equivalently, $\sum_{j\in J_i'} (a_{ij}' - a_{ji}) > |\mathcal{M}_i| c_i - \sum_{j\notin\mathcal{M}_i} a_{ji}$. But since, $J_i' = \mathcal{N}_{-i}$ as it should be for any best response, this gives $r_i - \sum_{j\in\mathcal{N}_{-i}} a_{ji} > |\mathcal{M}_i| c_i - \sum_{j\notin\mathcal{M}_i} a_{ji}$ so that $r_i - \sum_{j\in\mathcal{M}_i} a_{ji} > |\mathcal{M}_i| c_i$ contradicting the definition of c_i. Thus, $v_i(a_{-i}, a_i^*)$ is the best response.

Let $a_i^* \in B_i^u(a_{-i})$. Suppose next that for some $a_i' \in A_i$, we have $u_i(a_{-i}, a_i') > u_i(a_{-i}, a_i^*) = c_i$, or equivalently, $\min_{j\in J_i'} (a_{ij}' - a_{ji}) = \min_{j\in\mathcal{N}_{-i}} (a_{ij}' - a_{ji}) > c_i$. This implies that $a_{ij}' - a_{ji} > c_i$ for all

$j \in \mathcal{N}_{-i}$. Summing each term over $j \in \mathcal{M}_i$, we have $\sum_{j\in\mathcal{M}_i} a'_{ij} > \sum_{j\in\mathcal{M}_i} a_{ji} + |\mathcal{M}_i|c_i = r_i$, which is again a contradiction. The same argument also applies to $w_i(a_{-i}, a^*i)$ so that both $u_i(a_{-i}, a_i^*)$ and $w_i(a_{-i}, a_i^*)$ are best responses.

Finally, let $a_i^* \in B_i^t(a_{-i})$ and suppose for some $a'_i \in A_i$, we have $t_i(a_{-i}, a'_i) > t_i(a_{-i}, a_i^*)$, or equivalently,

$$\sum_{j\in\mathcal{N}_{-i}} (a'_{ij} - a_{ji})^2 < \sum_{j\in\mathcal{N}_{-i}} (a^*_{ij} - a_{ji})^2. \tag{28}$$

Let $b_{ij} = a'_{ij} - a^*_{ij}$ so that $\sum_{j\in\mathcal{N}_{-i}} b_{ij} = 0$ and $b_{ij} = a'_{ij}$ for $j \notin \mathcal{M}_i$ by (24). The inequality (28) can then be written in terms of b_{ij} as $\sum_{j\in\mathcal{N}_{-i}} b_{ij}^2 + 2\sum_{j\in\mathcal{N}_{-i}} b_{ij}(a^*_{ij} - a_{ji}) < 0$, or using (24), $\sum_{j\in\mathcal{N}_{-i}} b_{ij}^2 + 2\sum_{j\in\mathcal{M}_i} b_{ij}c_i - 2\sum_{j\notin\mathcal{M}_i} b_{ij}a_{ji} < 0$. Since $-\sum_{j\in\mathcal{M}_i} b_{ij} = \sum_{j\notin\mathcal{M}_i} b_{ij} = \sum_{j\notin\mathcal{M}_i} a'_{ij}$, we can further write $\sum_{j\in\mathcal{N}_{-i}} b_{ij}^2 - 2\sum_{j\notin\mathcal{M}_i} a'_{ij}(c_i + a_{ji}) < 0$. Now, since $c_i + a_{ji} < 0$ for all $j \notin \mathcal{M}_i$ by (19) and (23), we have a contradiction $\sum_{j\in\mathcal{N}_{-i}} b_{ij}^2 < 0$. Thus, $t_i(a_{-i}, a_i^*)$ is also the best response.

Proof of Theorems 2 - 5. Let $a \in \times_{i\in\mathcal{N}} A_i$ be a profile that gives a bilateral-equilibrium. Then, for each $i \in \mathcal{N}$, $\sum_{j\in\mathcal{N}_{-i}} a_{ji} = r_i$ so that the allocation $a_i \in B^v(a_{-i}) \cap B^u(a_{-i}) \cap B^t(a_{-i})$. It is also true that $a_i \in B^w(a_{-i})$ since, by $\sum_{j\in\mathcal{N}_{-i}} a_{ji} = r_i$, $\mathcal{M}_i = \mathcal{N}_{-i}$, which gives $a_{ij}^b = a_{ji}$ for all $j \in \mathcal{N}_{-i}$ in (24). It thus follows that a bilateral-equilibrium profile a is a Nash solution for every utility (8), (10), (11), (12). Let a be a profile that gives a partitioned-equilibrium for the partition $\mathcal{N}_1 \cup \mathcal{N}_2$ of $\mathcal{N}$. If $i \in \mathcal{N}_1$, then $a_{ij} \geq a_{ji}$ for all $j \in \mathcal{N}_1 \cup \mathcal{N}_2$, with strict inequality for at least one $j \in \mathcal{N}_2$, implies that $r_i = \sum_{j\in\mathcal{N}_{-i}} a_{ij} > \sum_{j\in\mathcal{N}_{-i}} a_{ji}$. Similarly, if $i \in \mathcal{N}_2$, then $a_{ij} \leq a_{ji}$ for all $j \in \mathcal{N}_1 \cup \mathcal{N}_2$, with strict inequality for at least one $j \in \mathcal{N}_2$, implies that $r_i = \sum_{j\in\mathcal{N}_{-i}} a_{ij} < \sum_{j\in\mathcal{N}_{-i}} a_{ji}$. It follows that $a_i \in B^v(a_{-i})$ for every $i \in \mathcal{N}$, i.e., a partitioned-equilibrium is a Nash solution with respect to (8). Let a be a profile that gives a dominant-equilibrium for the partition $\mathcal{N}_1 \cup \mathcal{N}_2$ of $\mathcal{N}$. If $i \in \mathcal{N}_1$, then, by Definition 4.iii, $a_{ij} \geq a_{ji}$ for all $j \in \mathcal{N}_1 \cup \mathcal{N}_2$, with strict inequality for at least one $j \in \mathcal{N}_2$, implies that $r_i = \sum_{j\in\mathcal{N}_{-i}} a_{ij} \geq \sum_{j\in\mathcal{N}_{-i}} a_{ji}$. Thus, for each $i \in \mathcal{N}_1$, $a_i \in B^u(a_{-i}) = B^t(a_{-i})$. If $i \in \mathcal{N}_2$, then Definition 4.iii gives that $a_{ij} = a_{ij}^b$ for each $j \in \mathcal{N}_1$ so that $a_i \in B^u(a_{-i}) = B^t(a_{-i})$ for all $i \in \mathcal{N}_2$ as well. This shows that a dominant-equilibrium is a Nash solution with respect to (10) and (11).

We now show that every Nash solution for (8) must be either a bilateral-equilibrium or a partitioned equilibrium. Given such a Nash profile $a \in \times_{i\in\mathcal{N}} A_i$ with every $a_i \in B^v(a_{-i})$, let $\mathcal{N}_1 = \{i \in \mathcal{N} : r_i \geq \sum_{j\in\mathcal{N}_{-i}} a_{ji}\}$ and $\mathcal{N}_2 = \{i \in \mathcal{N} : r_i < \sum_{j\in\mathcal{N}_{-i}} a_{ji}\}$. Then, $\mathcal{N} = \mathcal{N}_1 \cup \mathcal{N}_2$ is a disjoint partition of $\mathcal{N}$ and $\mathcal{N}_1 \neq \emptyset$. It follows by (25) that for each $i \in \mathcal{N}_1$, $a_{ij} \geq a_{ji}$ $\forall j \in \mathcal{N}_{-i}$ which implies that $a_{ij} = a_{ji}$ $\forall \{i, j\} \subset \mathcal{N}_1$ so that $\mathcal{N}_1$ is internally at bilateral-equilibrium. Now, if $\mathcal{N}_2 = \emptyset$, then $\mathcal{N}_1 = \mathcal{N}$ and the allocation profile is a bilateral-equilibrium. If $\mathcal{N}_2 \neq \emptyset$, then by (25), for each $k \in \mathcal{N}_2$, $a_{kj} \leq a_{jk}$ $\forall j \in \mathcal{N}_{-i}$ implying that $a_{kl} = a_{lk}$ $\forall \{k, l\} \subset \mathcal{N}_2$. Thus, $\mathcal{N}_2$ is also internally at bilateral-equilibrium. Therefore, a is a partitioned-equilibrium.

To see that every Nash solution for (10) and (11) must be either a bilateral-equilibrium or a dominant-equilibrium, let $a \in \times_{i\in\mathcal{N}} A_i$ be such that $a_i \in B^u(a_{-i}) = B^t(a_{-i})$. Let $\mathcal{N}_1 = \{i \in$

$\mathcal{N} : r_i \geq \sum_{j \in \mathcal{N}_{-i}} a_{ji}\}$ and $\mathcal{N}_2 = \{i \in \mathcal{N} : r_i < \sum_{j \in \mathcal{N}_{-i}} a_{ji}\}$. Then, $\mathcal{N} = \mathcal{N}_1 \cup \mathcal{N}_2$ is a disjoint partition of $\mathcal{N}$ and $\mathcal{N}_1 \neq \emptyset$. By (26), it follows that for each $i \in \mathcal{N}_1$, $a_{ij} \geq a_{ji}$ $\forall j \in \mathcal{N}_{-i}$ which implies that $a_{ij} = a_{ji}$ $\forall \{i,j\} \subset \mathcal{N}_1$. If $\mathcal{N}_2 = \emptyset$, then $\mathcal{N}_1 = \mathcal{N}$ and the allocation profile is a bilateral-equilibrium. Suppose $\mathcal{N}_2 \neq \emptyset$. By (26), for each $k \in \mathcal{N}_2$, $a_{kj} \leq a_{jk}$ $\forall j \in \mathcal{N}_{-i}$ implying that $a_{kl} = a_{lk}$ $\forall \{k,l\} \subset \mathcal{N}_2$. Moreover, for each $k \in \mathcal{N}_2$, it holds that

$$a_{ki} = a_{ik}^b = \begin{cases} a_{ik} + c_k \text{ if } i \in \mathcal{M}_k, \\ 0 \text{ if } i \notin \mathcal{M}_k, \end{cases} \tag{29}$$

where c_k is as defined in (23). Since $a_{ki} = a_{ik}$ for all $i \in \mathcal{N}_2$ and since $c_k < 0$ for any $k \in \mathcal{N}_2$, it must be that $a_{ki} = 0$ for all $i \in \mathcal{N}_2$ so that $\mathcal{M}_k \subset \mathcal{N}_1$. We have thus shown that the allocation profile a obeys (i)-(iii) of Definition 4 provided $\mathcal{N}_2 \neq \emptyset$. Therefore, $a \in \times_{i \in \mathcal{N}} A_i$ is either a bilateral-equilibrium or a dominant-equilibrium as claimed.

Finally, we show that every Nash solution of (12) must be a bilateral-equilibrium. suppose $a \in A$ is a Nash equilibrium so that every $a_i \in A_i$ obeys (27). Then, the sets $\mathcal{N}_1 = \{i \in \mathcal{N} : r_i \geq \sum_{j \in \mathcal{N}_{-i}} a_{ji}\}$ and $\mathcal{N}_2 = \{i \in \mathcal{N} : r_i < \sum_{j \in \mathcal{N}_{-i}} a_{ji}\}$ are such that $\mathcal{N} = \mathcal{N}_1 \cup \mathcal{N}_2$ is a disjoint partition of $\mathcal{N}$ and $\mathcal{N}_1 \neq \emptyset$. By (27), it follows that for each $i \in \mathcal{N}_1$,

$$a_{ij} - a_{ji} = \frac{1}{n-1}(r_i - \sum_{l \in \mathcal{N}_{-i}} a_{li}) \geq 0 \;\; \forall\, j \in \mathcal{N}_{-i}. \tag{30}$$

This implies that $a_{ij} = a_{ji}$ $\forall \{i,j\} \subset \mathcal{N}_1$. If $\mathcal{N}_2 = \emptyset$, then $\mathcal{N}_1 = \mathcal{N}$ and the allocation profile is a bilateral-equilibrium. Suppose $\mathcal{N}_2 \neq \emptyset$. By (27), for each $k \in \mathcal{N}_2$, $a_{kj} \leq a_{jk}$ $\forall j \in \mathcal{N}_{-i}$ implying that $a_{kl} = a_{lk}$ $\forall \{k,l\} \subset \mathcal{N}_2$. Moreover, since $a_{ki} = a_{ik}$ for all $i \in \mathcal{N}_2$ in (29) and since $c_k < 0$ for any $k \in \mathcal{N}_2$, it must be that $a_{ki} = 0$ for all $i \in \mathcal{N}_2$. Therefore, $\mathcal{M}_k \subset \mathcal{N}_1$ and $\mathcal{N}_2$ is a coalition. For a fixed $k \in \mathcal{N}_2$, let us sum each term in (30) over $i \in \mathcal{N}_1$ to get

$$\sum_{i \in \mathcal{N}_1} a_{ik} - \sum_{i \in \mathcal{N}_1} a_{ki} = \frac{1}{n-1}(\sum_{i \in \mathcal{N}_1} r_i - \sum_{i \in \mathcal{N}_1} \sum_{l \in \mathcal{N}_{-i}} a_{li})$$

which, using $a_{ij} = 0$ for all $i, j \in \mathcal{N}_2$, gives

$$(n-1)(\sum_{i \in \mathcal{N}_1} a_{ik} - r_k) = \sum_{i \in \mathcal{N}_1} r_i - \sum_{i \in \mathcal{N}_1} \sum_{l \in \mathcal{N}_{-i}} a_{li} = \sum_{i \in \mathcal{N}_2} (\sum_{l \in \mathcal{N}_{-i}} a_{li} - r_i).$$

The last equality is by the fact that the resource excess of the states in $\mathcal{N}_1$ (the second term) is equal to the resource deficiency of the states in $\mathcal{N}_2$ (the last term). If k is taken to be the state with the largest deficiency, then we get $(n-1-|\mathcal{N}_2|)(\sum_{i \in \mathcal{N}_1} a_{ik} - r_k) \leq 0$, which implies that either $|\mathcal{N}_2| = n-1$ or $\sum_{i \in \mathcal{N}_1} a_{ik} = r_k$. The latter is a contradiction since $k \in \mathcal{N}_2 = \{i \in \mathcal{N} : r_i < \sum_{j \in \mathcal{N}_{-i}} a_{ji}\}$ so that $\mathcal{N}_2$ must be empty, i.e., bilateral-equilibrium prevails. If, on the other hand, $|\mathcal{N}_2| = n-1$, or equivalently, $|\mathcal{N}_1| = 1$, then the member of $\mathcal{N}_1 = \{i\}$ is an almost hegemon or a hegemon standing against the *coalition* $\mathcal{N}_2$ with allocations $a_{ij} = \frac{1}{n-1}(r_i - \sum_{l \in \mathcal{N}_1} r_l)$ $\forall j \in \mathcal{N}_{-i}$ as claimed by Theorem 5. □

Proof of Theorem 6. Let $\mathcal{A} = \mathcal{A}_a \cup \mathcal{A}_d$ be a disjoint partition of $\mathcal{A}$ into advantaged and disadvantaged states. By the best response function (27) of utility $w_i(a)$, it follows that for each $i \in \mathcal{A}_a$, $a_{ij} - a_{ji} \geq 0 \ \ \forall\, j \in \mathcal{N}_{-i}$. Similarly, by (27), it follows that for all $j \in \mathcal{A}_d$, we have $a_{ij} = a_{ji} = 0$ for each $i \notin \mathcal{M}_i$ and $a_{ij} < a_{ji}$ for each $i \in \mathcal{M}_i$. If $|\mathcal{A}_d| > 1$, then strict inequality for some $j \in \mathcal{A}_d$ will lead to contradiction since it will imply that state $j \in \mathcal{A}_d$ has positive security. Hence, it must be that $a_{ij} = 0 = a_{ji}$ for all $i, j \in \mathcal{A}_d$ provided $|\mathcal{A}_d| > 1$. Hence, the resulting Nash equilibrium is such that

$$\begin{cases} a_{ij} = a_{ji}, & \{i,j\} \subset \mathcal{A}_a \\ a_{ij} \geq a_{ji}, & i \in \mathcal{A}_a,\ j \in \mathcal{A}_d \cup \mathcal{B} \\ a_{ij} = 0 = a_{ji}, & \{i,j\} \subset \mathcal{A}_d, \\ a_{ik} < a_{ki}, & i \in \mathcal{A}_d,\ k \in \mathcal{A}_a \cup \mathcal{B}, \end{cases} \tag{31}$$

where the first line drops if $\mathcal{A}_a = \emptyset$ or if $|\mathcal{A}_a| = 1$; the third line drops if $\mathcal{A}_d = \emptyset$ or if $|\mathcal{A}_d| = 1$. Note that, if $|\mathcal{A}_a| > 1$, $a_{ij} = a_{ji}$ for all $\{i,j\} \subset \mathcal{A}_a$ so that $c_i = 0$ in (23) and this implies that $a_{ij} = a_{ji}$ also for all $j \in \mathcal{A}_d \cup \mathcal{B}$, i.e., equality holds in the second line. But then, we must have $\mathcal{A}_d = \emptyset$, because otherwise there would be a contradiction with the fourth line in (31). Thus, if $|\mathcal{A}_a| > 1$, then the first allocation profile *(1)* is obtained. If $\mathcal{A}_a = \emptyset$, then the first and the second lines in (31) drop so that we obtain the second allocation profile *(2)*. If $|\mathcal{A}_a| = 1$ and $\mathcal{A}_d = \emptyset$, then the first and third lines in (31) drop giving the third profile *(3)* with "$\geq$" sign. By (27), the sign is either equality or "$>$" and equality is covered by the first profile; thus, the third profile *(3)* is obtained. Finally, if $|\mathcal{A}_a| = 1$ and $\mathcal{A}_d \neq \emptyset$, then due to the strict inequality in the fourth line of (31), we also have strict inequality in the second line, which gives the last profile *(4)*. □

Proof of Theorem 7. We use the proof of Theorem 6 and incorporate motive to irrational players there. Let us partition $\mathcal{B} = \mathcal{B}_a \cup \mathcal{B}_d$ into advantaged and disadvantaged states. The best response function (25) of utility $v_i(a)$ gives that for each $i \in \mathcal{B}_a$, $a_{ij} - a_{ji} \geq 0 \ \ \forall\, j \in \mathcal{N}_{-i}$ and for each $i \in \mathcal{B}_d$, $a_{ij} - a_{ji} \leq 0 \ \ \forall\, j \in \mathcal{N}_{-i}$. The allocation profile (31) is then modified as

$$\begin{cases} a_{ij} = a_{ji}, & \{i,j\} \subset \mathcal{A}_a \\ a_{ij} \geq a_{ji}, & i \in \mathcal{A}_a,\ j \in \mathcal{A}_d \cup \mathcal{B} \\ a_{ij} = 0 = a_{ji}, & \{i,j\} \subset \mathcal{A}_d, \\ a_{il} < a_{li}, & i \in \mathcal{A}_d,\ l \in \mathcal{A}_a \cup \mathcal{B}, \\ a_{kl} \geq a_{lk}, & k \in \mathcal{B}_a,\ l \in \mathcal{B}_d \cup \mathcal{A} \\ a_{lk} \leq a_{lk}, & k \in \mathcal{B}_d,\ l \in \mathcal{B}_a \cup \mathcal{A} \end{cases} \tag{32}$$

By the fifth and the sixth lines, we have that $\mathcal{B}_a$ and $\mathcal{B}_d$ are internally at bilateral-equilibrium. If $|\mathcal{A}_a| > 1$, then as above $\mathcal{A}_d = \emptyset$ and $a_{ij} = a_{ji}$ also for all $j \in \mathcal{A}_d \cup \mathcal{B}$. This gives that, a Nash equilibrium is either a bilateral-equilibrium or bilateral equality prevails everywhere except inside $\mathcal{B}$ with at least one state in $\mathcal{B}_a$ having positive bilateral security against a state in $\mathcal{B}_d$. This gives the first and the last Nash profiles *(1), (4)*. If $\mathcal{A}_a = \emptyset$, then as before the first and the second lines in (32) drop so that we obtain the second Nash profile *(2)*. If $|\mathcal{A}_a| = 1$ and $\mathcal{A}_d = \emptyset$, then as before the first and third lines in (32) drop giving the third profile *(3)*. Finally, if $|\mathcal{A}_a| = 1$ and $\mathcal{A}_d \neq \emptyset$, then due to the strict inequality in the fourth line of (32), we also have strict inequality in the second line, which would contradict the last line unless $\mathcal{B}_d = \emptyset$ and the fifth line unless $\mathcal{B}_a = \emptyset$, which is not possible by our assumption that $\mathcal{B} \neq \emptyset$. Thus, no Nash equilibrium results if $|\mathcal{A}_a| = 1$ and $\mathcal{A}_d \neq \emptyset$. □

Proof of Theorem 8. The best response function (26) of utility $u_i(a)$ and $t_i(a)$ gives that $\mathcal{B}_d$ is a coalition and internal allocations there are zero. This implies that the third profile is the dominant-equilibrium of Theorem 5. Moreover, by (26), the bilateral securities of a state in $\mathcal{B}_d$ is either uniformly strictly negative or uniformly zero. Since all states in $\mathcal{B}_d$ already have zero bilateral securities against the state $\mathcal{A}_a$, this implies that $\mathcal{B}_d = \emptyset$ and the fourth profile in Theorem 7 reduces to a bilateral equilibrium covered by the first profile. $\square$

Proof of Theorem 9. Let $p_i(a)$ denote one of (8), (10), (11), or (12). By definition, a profile $a \in A$ is strongly Pareto efficient if there is no $a' \in A$ for which $p_i(a') \geq p_i(a)$ for all $i \in \mathcal{N}$ with strict inequality for at least one $i \in \mathcal{N}$. A profile $a^* \in A$ is a social optimum if $\sum_{i=1}^{n} p_i(a^*)$ is maximal. Suppose there is no hegemon. **(i)** Let $f_i(a)$ denote any one of (8), (10), or (11). A bilateral-equilibrium a^b is such that $f_i(a^b) = 0$ for all $i \in \mathcal{N}$. Suppose, for some $a' \in A$, $f_i(a') \geq f_i(a^b) = 0$ for all $i = 1, ..., n$ with strict inequality for $k \in \mathcal{N}$. Since, by definition of the utility , $f_k(a') \leq 0$ for any $a' \in A$, this is a contradiction. Therefore, every bilateral-equilibrium is strongly Pareto efficient. Conversely, if some profile $a \in A$ is not a bilateral-equilibrium, then $a_{kj} < a_{jk}$ for some $k \neq j, ; \{k, j\} \in \mathcal{N}$, so that $f_k(a) < 0$, by definition of the utilities (8), (10), and (11) while $f_j(a) \leq 0$ for all $j \in \mathcal{N}$. But then, for a bilateral-equilibrium allocation a^b, we have $0 = f_i(a^b) \geq f_i(a)$ for all $i \in \mathcal{N}$ with strict equality for $i = k$, which implies that $a \in A$ is not strongly Pareto efficient. **(ii)** Now consider (12). Every Nash equilibrium is a bilateral-equilibrium $a^b \in A$ and $\min_i\{w_i(a)\} \leq 0$, $\forall\, a \in A$, since otherwise all bilateral securities in the system would be strictly positive. This will be contradicted for $i = k$ if it holds that for some $a' \in A$ and for some bilateral-equilibrium allocation a^b, $w_i(a') \geq w_i(a^b) = 0$ for all $i \in \mathcal{N}$ with strict inequality for some $k \in \mathcal{N}$. It follows that every a^b is strongly Pareto efficient. **(iii)** Since $f_i(a) \leq f_i(a^b) = 0$ for any profile a and a bilateral-equilibrium a^b, it is clear that social optima consists of bilateral-equilibria with respect to (8), (10), or (11). The utility w_i, on the other hand, is such that, if $w_i(a) > 0$ for some profile a, then by definition (12), there exists $j \in \mathcal{N}_{-i}$ for which $w_j(a) \leq w_i(a)$. This implies that $\sum_{i=1}^{n} w_i(a) \leq 0 = \sum_{i=1}^{n} w_i(a^b)$ for any profile $a \in A$. It follows that all social optima again consist of bilateral-equilibria. $\square$

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