A continuous-time plant has the transfer function

\[ G(s) = \frac{1 - s}{(1 + s)(1 + 10s)} \]

1. Design a continuous-time proportional and integral controller of the form

\[ C(s) = K \left(1 + \frac{1}{T_s s}\right) \]

such that the dominant poles of the resulting unity-feedback closed-loop system have

\[ \zeta = 0.75, \quad \omega_n = 5 \]

2. Compute the expected steady state error, overshoot, and settling time of the closed-loop system for a unit step reference input.

3. Obtain the pulse transfer function \( G_T(z) \) of the plant by keeping the sampling period as a parameter.

4. Discretize the controller in part (1) using pole-zero matching to obtain an equivalent discrete-time controller \( D(z) \).

5. Repeat parts (1-4).

6. Construct a Simulink model of the unity-feedback closed-loop system together with the controller in part (1). Obtain and plot the unit step response for 30 sec. Measure the steady state error, overshoot, and settling time; compare with the expected values; and explain the differences.

7. Find \( G_T(z) \) in part (3) and \( D(z) \) in part (4) for \( T = 2 \) sec.

8. Repeat part (6) for the closed-loop sampled-data system.

9. Iterate on \( D(z) \) to improve the performance of the closed-loop sampled-data system, and repeat part (6) for the final design.
SOLUTION

1. \[ C(s) = K \left( 1 + \frac{1}{T_1 s} \right) = \frac{K(1 + T_1 s)}{T_1 s} \]

Let \( T_1 = 10 \) so that

\[ C(s)G(s) = \frac{K(1 - s)}{10s(1 + s)} \Rightarrow H(s) = \frac{1 - s}{1 + (10K^{-1} - 1)s + 10K^{-1}s^2} \]

\( \zeta = 0.75, \omega_n = 0.5 \Rightarrow 1 + (10K^{-1} - 1)s + 10K^{-1}s^2 = 1 + 3s + 4s^2 \Rightarrow K = 2.5 \)

Thus

\[ C(s) = \frac{2.5(1 + 10s)}{s}, \quad H(s) = \frac{1 - s}{1 + 3s + 4s^2} \]

2. \( \zeta = 0.75 \Rightarrow M_p = e^{-\pi\zeta/\sqrt{1-\zeta^2}} = 2.8 \% \)

\( \zeta\omega_n = 0.375 \Rightarrow t_s = \frac{4.6}{0.375} = 12.3 \text{ sec} \)

and obviously \( e_{ss} = 0 \) due to the integral action of \( C(s) \)

3.

\[ G_T(z) = \frac{z - 1}{z} Z_T\left\{ \frac{1 - s}{s(1 + s)(1 + 10s)} \right\} \]

\[ = \frac{z - 1}{z} Z_T\left\{ \frac{1 + 2/9}{s + 1} - \frac{11/9}{s + 0.1} \right\} \]

\[ = \frac{(1 + \frac{2}{9}e^{-T} - \frac{11}{9}e^{-0.1T})z + (e^{-1.1T} - \frac{11}{9}e^{-T} + \frac{2}{9}e^{-0.1T})}{(z - e^{-T})(z - e^{-0.1T})} \]

4. \[ D(z) = K_D \frac{z - e^{-0.1T}}{z - 1} \]

We require that

\[ \lim_{s \to 0} \frac{C(s)}{s} = \lim_{s \to 1} D(z) = \lim_{s \to 0} \frac{(1 - e^{-0.1T})K_D}{e^{sT} - 1} = \frac{(1 - e^{-0.1T})K_D}{T} \]

Thus

\[ K_D = \frac{2.5T}{1 - e^{-0.1T}} \]

5. Already done.
6. The Simulink model for the continuous-time closed-loop system is shown in Fig. 1(a), and the simulation result in Fig. 2(a). From the output plot we measure

\[ M_p = 3.1 \%, \quad t_s = 14.0 \text{ sec}, \quad e_{ss} = 0 \]

Differences in \( M_p \) and \( t_s \) from the expected values are due to the presence of the zero in \( H(s) \).

7. \[ G_T(z) = \frac{0.0294(z + 4.33)}{(z - 0.1353)(z - 0.8187)} , \quad D(z) = \frac{2.758(z - 0.8187)}{(z - 1)} \]

8. The Simulink model for the sampled-data closed-loop system is shown in Fig. 1(b), and the simulation result in Fig. 2(b). From the output plot we measure

\[ M_p = 22.1 \%, \quad t_s = 22.6 \text{ sec}, \quad e_{ss} = 0 \]

Differences in \( M_p \) and \( t_s \) from the expected values are due to the large sampling period (\( \omega_s = \frac{\pi}{T} = 3.14 \approx 6\omega_n \)) in addition to the presence of the zero in \( H(s) \).

9. With some trial, we modify the sampled-data controller as

\[ D(z) \frac{1.8(z - 0.83)}{(z - 1)} \]

which guarantees a well-damed positive real closed-loop pole in addition to a conjugate pair. From the simulation result in Fig. 2(c) we observe that the response is considerably improved with

\[ M_p = 1.1 \%, \quad t_s = 15.2 \text{ sec} \]

\( t_s \) is close to that of the continuous-time controlled system, and \( M_p \) is even better.
Figure 1: Simulink Models

Figure 2: Simulation Results