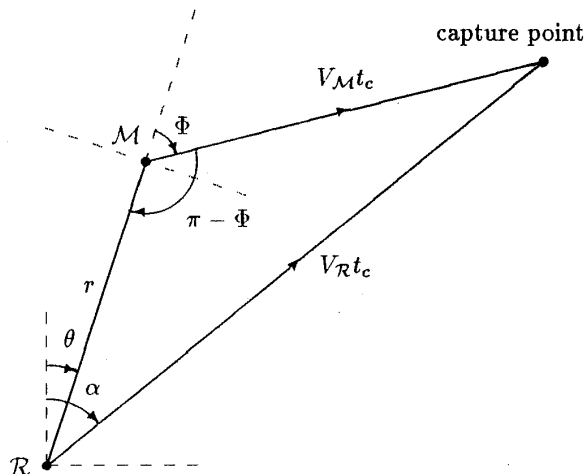


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I. INTRODUCTION

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We characterize prey capture performance for different strategies by computing their mean capture time and capture probability. In Sec. II. a lower bound for the mean capture time is derived for a linearly moving prey. Sec. III. summarizes the different phases of prey capture for our sonar system. Simulation studies are described in Sec. V. The results are presented and interpreted in Sec. VI. A brief description of the robotic system in our laboratory is given in Sec. VII.

In this section, we derive a lower bound on the time to capture a linearly moving prey. The game-theoretical approach [2] assumes that the pursuer (bat) has complete knowledge about the evader

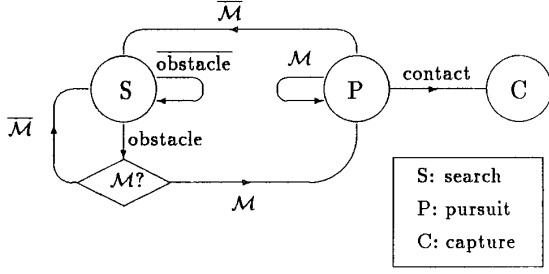


Fig. 2. State diagram for the motion of R .

(moth) at all times and at all points in space. Let us then consider a moth \mathcal{M} moving along a linear trajectory at speed $V_{\mathcal{M}}$ as shown in Fig. 1. Suppose that \mathcal{R} first detects \mathcal{M} at range r and azimuth θ . The orientation of moth's linear path measured with respect to the θ direction is assumed to be a random variable Φ , uniformly distributed in the interval $[-\frac{\pi}{2}, \frac{\pi}{2}]$. This restriction limits the system to moths drifting *away* from the bat and eliminates the favorable condition when \mathcal{M} has a velocity component towards \mathcal{R} . With complete information about \mathcal{M} , i.e. r, θ, Φ and $V_{\mathcal{M}}$, the bat wants to intercept \mathcal{M} as quickly as possible to minimize its energy expenditure [3]. For a given set of parameters, this defines to a *unique* linear trajectory with orientation α . From the geometry of Fig. 1, one can solve for α and t_c to get:

$$\alpha = \theta + \sin^{-1} \left[\frac{V_{\mathcal{M}}}{V_{\mathcal{R}}} \sin \Phi \right] \quad (1)$$

$$t_c = \frac{r \left[\frac{V_{\mathcal{M}}}{V_{\mathcal{R}}} \cos \Phi + \sqrt{1 - \left(\frac{V_{\mathcal{M}}}{V_{\mathcal{R}}} \right)^2 \sin^2 \Phi} \right]}{V_{\mathcal{R}} \left[1 - \left(\frac{V_{\mathcal{M}}}{V_{\mathcal{R}}} \right)^2 \right]} \quad (2)$$

Eq. (2) indicates that prey capture will occur in finite time only if $V_{\mathcal{M}} < V_{\mathcal{R}}$. Taking the expected value of (2) over Φ in the interval $[-\frac{\pi}{2}, \frac{\pi}{2}]$, we find

$$E_{\Phi}[t_c] = \frac{2r \left[\frac{V_{\mathcal{M}}}{V_{\mathcal{R}}} + \mathcal{E} \left(\frac{\pi}{2}, \frac{V_{\mathcal{M}}}{V_{\mathcal{R}}} \right) \right]}{\pi V_{\mathcal{R}} \left[1 - \left(\frac{V_{\mathcal{M}}}{V_{\mathcal{R}}} \right)^2 \right]} \quad (3)$$

where

$$\mathcal{E} \left(\frac{\pi}{2}, \frac{V_{\mathcal{M}}}{V_{\mathcal{R}}} \right) = \int_0^{\frac{\pi}{2}} \sqrt{1 - \left(\frac{V_{\mathcal{M}}}{V_{\mathcal{R}}} \right)^2 \sin^2 x} dx \quad (4)$$

is the complete elliptic integral of the second kind [4] with $\frac{V_{\mathcal{M}}}{V_{\mathcal{R}}} < 1$. $E_{\Phi}[t_c]$ is observed to be independent

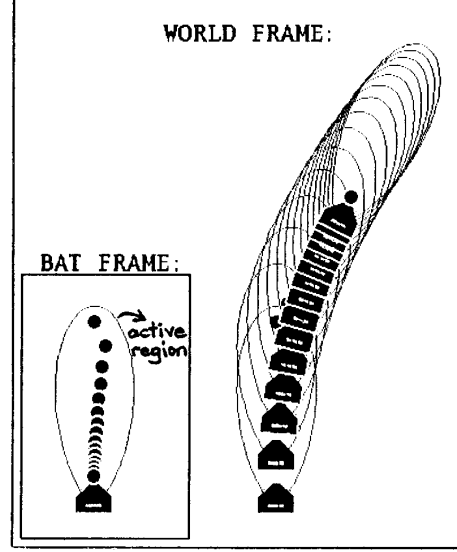


Fig. 3. A simulation example.

of θ and maximum when $\Phi = 0$. This analytical result is shown in Fig. 4 as a function of $\frac{V_{\mathcal{M}}}{V_{\mathcal{R}}}$, and will be compared to our simulation results employing limited information.

III. PHASES OF PREY CAPTURE

\mathcal{R} 's prey capture strategy consists of three modes in analogy to the bat foraging behavior [5]. The state diagram is shown in Fig. 2. The initial state is the *search mode* S where \mathcal{R} scans the environment in a stop-and-go pattern at equal time intervals T_s . This type of scan is observed frequently in nature and is called *saltatory search* [6]. At each stop, \mathcal{R} performs a rotation of the sensor system to cover the maximum area possible. The most efficient length for the distance traveled between two scans is related to both the direction of the move and the shape of the volume being searched: to maximize the efficiency of the search, \mathcal{R} moves just far enough to avoid rescanning a previously searched area while minimizing unsearched area [6]. Each time \mathcal{R} senses an obstacle within its active region, it checks to see if it is \mathcal{M} or another obstacle by *dual-thresholding* the echo amplitude, analogous to a bat [7]. The lower threshold is set to avoid spurious noise, and the higher threshold is set to eliminate the echoes from walls/corners that are much larger than those observed from \mathcal{M} . When \mathcal{M} is detected, \mathcal{R} switches to the *pursuit mode* P.

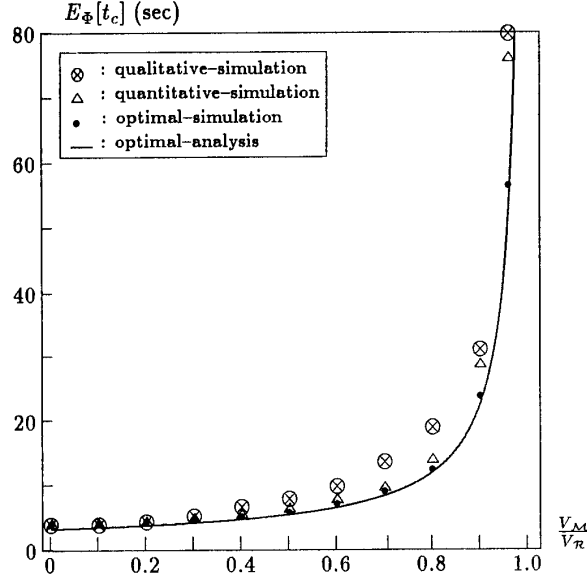


Fig. 4. Mean capture time vs. $\frac{V_M}{V_R}$ for $V_R=50$ cm/s, $T_s=1$ s, $r=200$ cm, $\theta=0^\circ$, and $\beta=10^\circ$.

In the pursuit mode, \mathcal{R} moves to reduce the range r and azimuth θ to zero. When r and θ are sufficiently small, \mathcal{M} is considered captured and the process terminates in C. However, it is not unusual for \mathcal{M} to move out of the active region after it has been detected (not *all* moths are caught by bats). In this case, \mathcal{R} returns to S by rotating in the direction where \mathcal{M} was last seen.

IV. RELATIONSHIP OF SENSING AND CONTROL

Since the measurements of \mathcal{M} 's location influence the actions of \mathcal{R} , prey capture can be viewed in terms of a control problem [8]. To accomplish prey capture, there are a variety of options for controlling the action of the pursuer \mathcal{R} (and possibly \mathcal{M} for sensory equipped moths). Each option has its own trade-off in performance and complexity. Two different control strategies for the pursuit mode are described next that use different levels of information. Both methods are memoryless, assume no knowledge of \mathcal{M} 's motion, and extract information sequentially from the environment.

A. Qualitative Information

To address the purposive imaging problem [9, 10], the minimal information required to achieve prey capture was determined. This result is important for understanding how to implement a practical, economic and efficient robot system that is to accom-

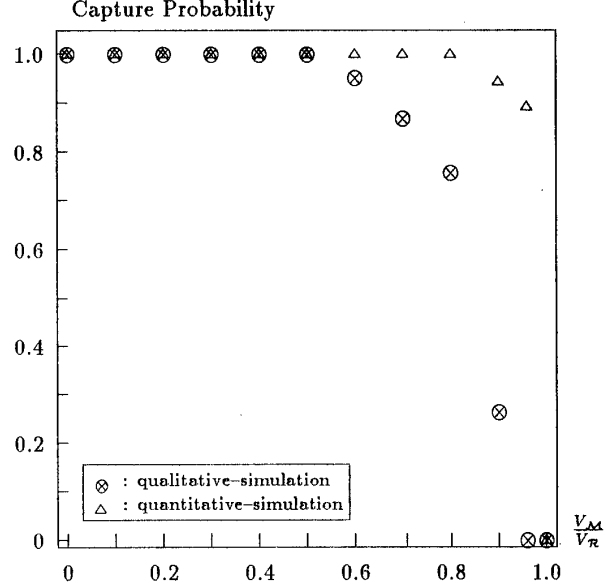


Fig. 5. Capture probability vs. $\frac{V_M}{V_R}$ for $V_R=50$ cm/s, $T_s=1$ s, $r=200$ cm, $\theta=0^\circ$ and $\beta=10^\circ$.

plish a particular task. Successful prey capture by \mathcal{R} that is faster than \mathcal{M} was previously demonstrated by employing the direction θ of \mathcal{M} relative to \mathcal{R} [2, 10]. Our results indicate the binary information that \mathcal{M} is either to the right or left of \mathcal{R} 's line-of-sight is sufficient. The nonlinear response by \mathcal{R} is then a rotation by a fixed angle β to the right or left. This minimum-information/minimum-response *bang-bang* strategy is the least sophisticated working method of accomplishing capture that we have investigated. We will compare the cost of having only this information in Sec. VI.

B. Quantitative Information

With this strategy, \mathcal{R} moves toward the location of \mathcal{M} , determined from the most recent echoes which produce a delayed estimate of \mathcal{M} 's position due to the echo travel time. \mathcal{R} makes a rotation that centers the beam on \mathcal{M} 's current location and steps forward. This is a more reasonable model given the information available in the bat brain [7].

V. SIMULATION STUDIES

To test the significance of available information, prey capture was simulated and experimentally verified. A program that models the physical operation of the actual sonar system has been developed on a VAX 3100. Dynamic equations for the motion of the

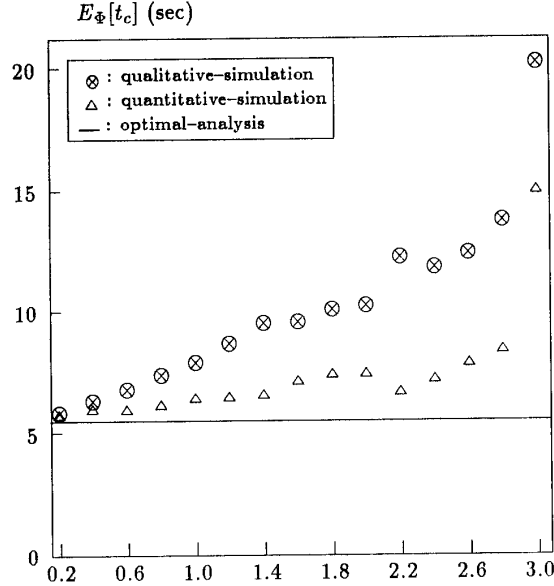


Fig. 6. Mean capture time vs. T_s for $V_R=50$ cm/s, $V_M=25$ cm/s, $r=200$ cm, $\theta=0^\circ$ and $\beta=10^\circ$.

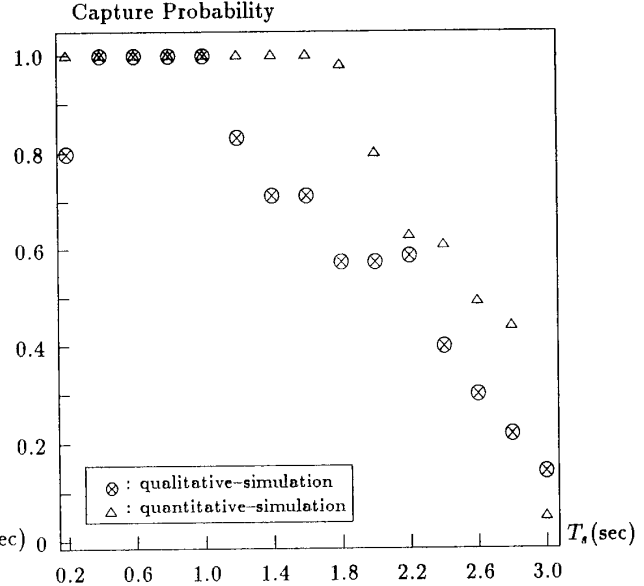


Fig. 7. Capture probability vs. T_s for $V_R=50$ cm/s, $V_M=25$ cm/s, $r=200$ cm, $\theta=0^\circ$ and $\beta=10^\circ$.

physical robot have been derived and implemented in the simulations.

A simulation example is shown in Fig. 3. The sonar system is illustrated by a rectangle on \mathcal{R} and its active region is shown in outline. The sequence of \mathcal{M} 's locations is shown as solid dots, added with each measurement. In the bat coordinate system, the relative positions of \mathcal{M} are shown as \mathcal{R} reacts to the sonar measurements and closes in for capture. The trajectories produced by the two control strategies are better observed in the latter system.

For accurate localization by the sonar system, the simulation is started when \mathcal{M} is located at the edge of \mathcal{R} 's active region along $\theta=0^\circ$ and starts fleeing at random orientation Φ . For qualitative information, the bang-bang rotation angle β was chosen to be 10° (close to its optimal value as will be shown). When \mathcal{M} escaped the active region, for both systems, \mathcal{R} responded by a saltatory search with $\pm 45^\circ$ rotations that cover the maximum possible area. If \mathcal{M} escaped out of the active region and was not detected within 12 cycles of saltatory search, \mathcal{M} was considered not captured.

One hundred realizations are generated to evaluate the penalty incurring on performance when different levels of information are extracted from the sensor system.

VI. PERFORMANCE EVALUATION RESULTS

A quantitative measure of the importance of information is provided by determining the corresponding cost in capture probability and mean capture time (when capture occurs) for each method. The simulation results are shown in Figs. 4,5,6 and 7. Obviously, the technique based on complete information, described in Sec. II, yields the highest capture probability and minimum capture time. We have observed that the following parameters are important for prey capture:

A. Relative Speed $\frac{V_M}{V_R}$

The speed of \mathcal{M} compared to \mathcal{R} is the most important factor in accomplishing prey capture. Mean capture time and capture probability are shown as a function of $\frac{V_M}{V_R}$ in Figs. 4 and 5. As expected, quantitative information gives better results than qualitative information, but there is a wide range of $\frac{V_M}{V_R}$ values for which they are comparable. The results indicate that moths with velocities below $0.5V_R$ are always captured with both methods. With quantitative information, moths moving as fast as $0.8V_R$ are successfully captured. For $V_M > 0.8V_R$, \mathcal{M} escapes out of the active region more often. For the qualitative system, when $V_M > 0.5V_R$, the rotation limited by β allows more moths to escape from the active region. The penalty of qualitative systems is not a significant increase in capture time but reduced

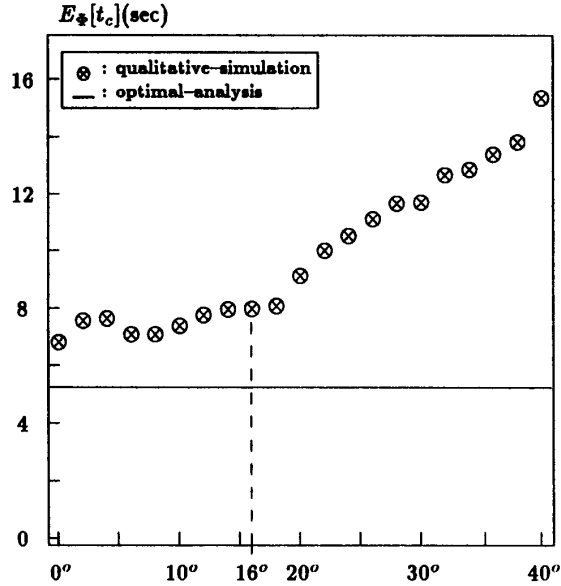


Fig. 8. Mean capture time vs. β for $V_R=50$ cm/s and $V_M=25$ cm/s.

capture probability.

B. Scan Interval

In bats, a new pulse is transmitted as soon as the echo from the previous pulse is detected and processed. Then, the scan interval T_s is equal to the TOF measurement plus the processing time of the echoes. Therefore, as with bats [7], the scan interval T_s decreases in proportion to the decrease in range. From \mathcal{R} 's perspective, the measurements of \mathcal{M} 's location appear more correlated and hence are more immune to noise effects.

In control terms, the time delay introduces a linear phase lag into the control loop, which, in turn, limits the open loop gain (bat maneuverability) for which the system is stable. As \mathcal{R} approaches \mathcal{M} , the slope of this phase diminishes, increasing the phase margin [11], thus allowing more gain and enhanced maneuverability for the bat.

To investigate the effect of the scan interval, T_s , was varied by adding a hypothetical processing delay to the delay due to TOF. The effect of varying T_s is shown in Figs. 6 and 7. With both methods, a reduction in T_s allows more feedback and reduces the effects of localization errors. Feedback is especially important for the bang-bang method which uses only qualitative information and does not perform an accurate rotation by θ . This explains further why the performance of this method deteriorates very quickly

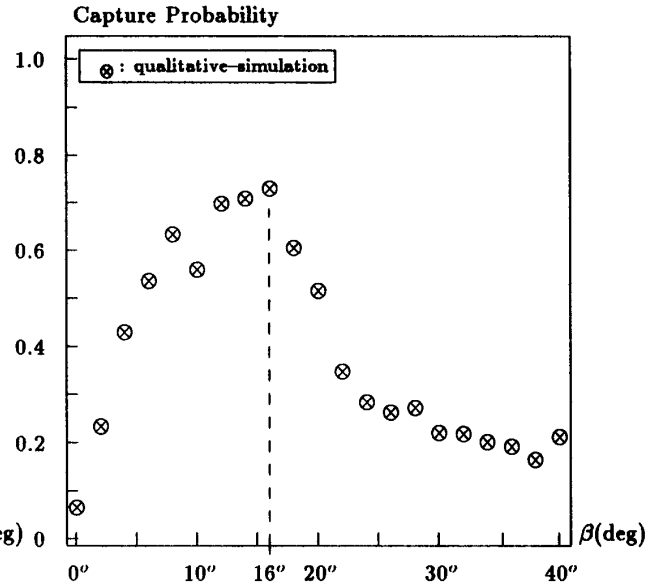


Fig. 9. Capture probability vs. β for $V_R=50$ cm/s and $V_M=25$ cm/s.

with increasing T_s .

C. Effect of β

The performance of the bang-bang algorithm for different values of β is shown in Figs. 8 and 9. For these results, the saltatory search angle was set equal to β , instead of the constant value of 45° used earlier. If β is too small (or too large), the system is overdamped (or underdamped). An intermediate value of $\beta=16^\circ$ yields the highest capture probability and yet reasonably small capture time. The smaller values of capture time around $\beta=0^\circ$ are due to the fact that only the 'easy' prey are captured.

VII. DESCRIPTION OF ROBOTS

Although the simulations are more flexible and efficient, real robots and the sensor system are essential to verify the assumptions. Experiments with the robots in our laboratory have indicated results similar to those of the simulations.

A. Description of ROBAT

A schematic illustration of the robotic system is shown in Fig. 10. The mobile robot \mathcal{R} is a position-controlled vehicle that carries the sonar system on-board with a maximum speed of $V_R=50$ cm/s. It consists of a triangular platform, placed on top of a passive front caster and two stepper motor wheels. The transducers are located high above the platform to eliminate the reflections off the platform and the

