Circuit Theory

Introduction

- **Circuit:** $\Longrightarrow$. An interconnection of electrical devices.

- **Theory:** $\Longrightarrow$. A set of axioms, definitions, rules, techniques, etc. to help us to understand the subject.

- Utilization of various forms of energy is very important for our daily life (e.g. thermal, mechanical, etc.). Electrical energy is probably the most significant among others. To utilize this energy we form physical circuits $\rightarrow$ power systems, radios, televisions, etc..These are formed by interconnecting electrical devices. Our aim is to **predict the electrical behaviour** of these systems. $\rightarrow$ To determine the electrical variables $\rightarrow$ current, voltage, electrical power, and less importantly charge and flux.

- To define **current**, we use the concept of **electrical charge** q. Unit of charge is **Coulomb**. Current is the rate of change of the electrical charge passing **through** a cross-section in a part of circuit, i.e. the **net** charge passing **through** from one side of the cross-section to the other side in unit time :

- $i = \frac{dq}{dt} \rightarrow$ unit of current : Ampere $\rightarrow A = \frac{Coulomb}{Time} = \frac{C}{sec}$

- Note that current can be positive or negative.

- To define **voltage**, we use the concept of **electrical energy** w. The unit of energy is Joule. The voltage is a potential difference between two points

A and B. The voltage between A and B is the change in the energy when a unit charge is moved from A and B. This is equivalent to the electrical work done against the electrical field to move a unit charge from A and B.

- $v = \frac{dw}{dq} \rightarrow$ unit of voltage : Volt $\rightarrow V = \frac{Joule}{Coulomb} = \frac{J}{C}$

- Note that voltage can be positive or negative.

- **electrical power** p is the rate of change of electrical energy with respect to time.

- $p = \frac{dw}{dt} = \frac{dw}{dq}\frac{dq}{dt} = vi \rightarrow$ unit of power : Watt $\rightarrow W = Volt \times Amp.$

- Note that since voltage and current can be positive or negative, so is the power.

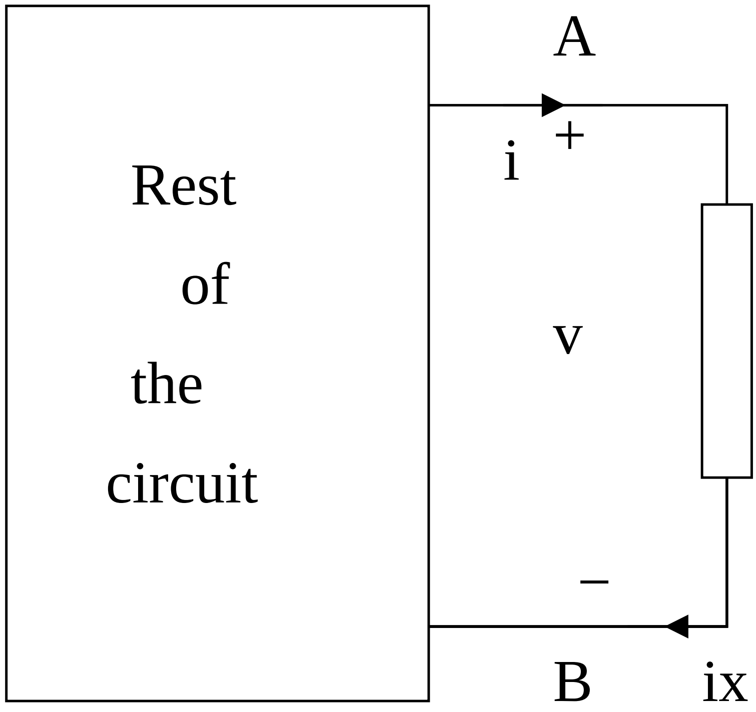
- **Basic Terminology :**

- **Terminals :** The joints where devices are accessible from outside. We apply electrical excitations to terminals and measure electrical variables at terminals.

- **Electrical Circuits :** Arbitrary interconnections of electrical devices through their terminals.

- **Nodes :** Any junction in the circuit where the terminals are connected together, or any isolated terminal. If two terminals are connected by an electrical wire only, they form a single node.

- **Connected circuit :** Any node can be reached from any other node by traversing a path through circuit elements.



- **Reference Directions :** Two nodes are required for voltage, and a terminal is needed for a current. If, after calculations voltage/current turns out to be **positive** → the chosen reference directions are right, otherwise wrong.

- We will use **passive sign convention**. → $p = vi > 0$ means the device is **receiving power** from the rest of the circuit, $p = vi < 0$ means the device is **giving power** to the rest of the circuit.

- $i = i_x$? Later, we will show that this is the case.

- Example 1.3, p. 9-10

- $D1 : v_1 = 100\ V, p_1 = -1\ W,\ D2 : i_2 = 5\ mA, p_2 = 0.5\ W,\ D3 : v_3 = 25\ V, i_3 = 5\ mA,\ D4 : v_4 = 75\ V, p_4 = 0.75\ W,\ D5 : v_5 = -75\ V, i_5 = 5\ mA.$

- $D1 : i_1 = p_1/v_1 = -10\ mA,\ p_1\ <\ 0 \rightarrow D1$ is giving power.

- $D2 : v_2 = p_2/i_2 = 100\ V,\ p_2\ >\ 0 \rightarrow D2$ is receiving power.

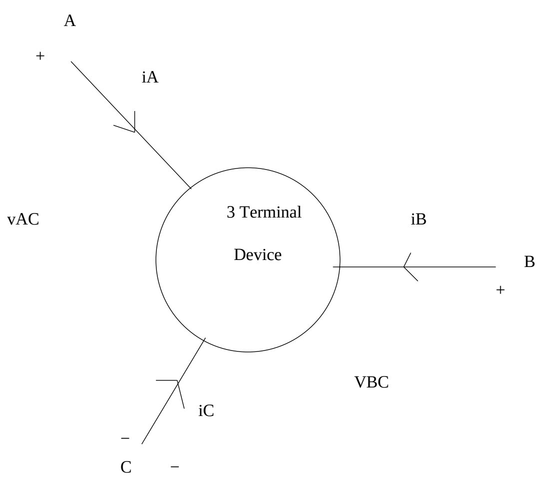
- $D3 : p_3 = v_3 i_3 = 125\ mW,\ p_3\ >\ 0 \rightarrow D3$ is receiving power.

- $D4 : i_4 = p_4/v_4 = 10\ mA,\ p_4\ >\ 0 \rightarrow D4$ is receiving power.

- $D5 : p_5 = v_5 i_5 = -375\ mW,\ p_5\ <\ 0 \rightarrow D5$ is giving power.

- Note that $p_1 + p_2 + \ldots + p_5 = 0 \rightarrow$ Conservation of power $\rightarrow$ Tellegens's Theorem.

- Note that not all devices have 2 terminals. We have 3 terminal (e.g. transistor), or many terminal (e.g. op-amps, integrated circuits etc..) as well.



- **Devices and Models**

- Physical devices are what we have in the lab, e.g. resistors, transistors, capacitors, motors, etc...

- To solve/predict the behaviour, we need **mathematical relations** → device model.

- Devices may behave differently in different conditions. → setup/consider the conditions → Collect experimental data/use physical rules → Get mathematical relations.

- As conditions change, models may change. So the same device may have **different models** under different physical conditions/considerations.

    - Satellite in orbit, many body problems in physics..

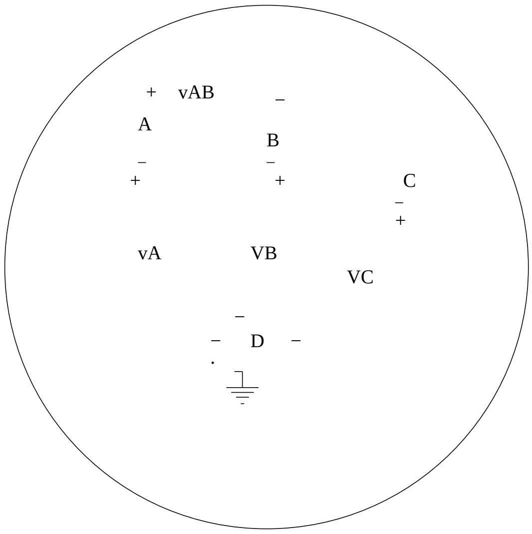
    - Transistors, diodes under different operating conditions...

- In circuit Theory, we consider **Device Models**. If the experimental results do not agree with the predicted results, we change the model...

- In Circuit Theory, we are interested in **electrical behaviour** of circuits, i.e. voltages, currents, electrical power..

- In Circuit Theory, we are interested in **external** behaviour of devices (i.e. measured through terminals), not the **internal** behaviour of the device → transistors, motors, etc...

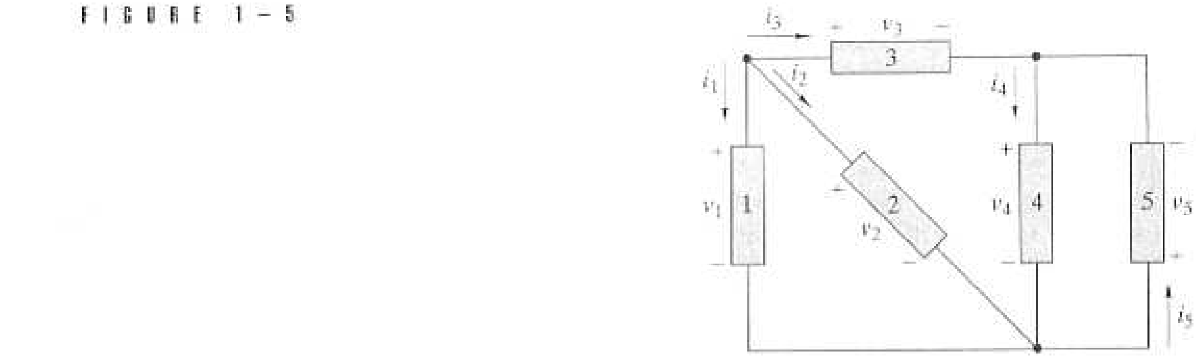
• Since voltage is defined as a **potential difference** between two points, to choose an **arbitrary reference point** to measure voltages is appropriate. This can be done in **any conservative field** which has a potential. → gravitation → Measuring height. → reference point → Sea level. We declare the potential of this **arbitrarily selected node** as zero, and this node as **Ground point**. This does not mean that this node is physicallly grounded.



• The voltage between any node and ground node is called **node voltage** → height of a point.

• Then we have e.g. $v_{AB} = v_A - v_B$. → Height difference.

FIGURE 1–5



SOLUTION:

(a) We use $p = vi$ to solve for the missing variable since two of the three circuit variables are given for each device.

Device 1: $i_1 = p_1/v_1 = -1/100 = -10$ mA $[p(t) < 0$, delivering power$]$

Device 2: $v_2 = p_2/i_2 = 0.5/0.005 = 100$ V $[p(t) > 0$, absorbing power$]$

Device 3: $p_3 = v_3 i_3 = 25 \times 0.005 = 0.125$ W $[p(t) > 0$, absorbing power$]$

Device 4: $i_4 = p_4/v_4 = 0.75/75 = 10$ mA $[p(t) > 0$, absorbing power$]$

Device 5: $p_5 = v_5 i_5 = -75 \times 0.005 = -0.375$ W $[p(t) < 0$, delivering power$]$

(b) Summing the device powers yields

$$p_1 + p_2 + p_3 + p_4 + p_5 = -1 + 0.5 + 0.125 + 0.75 - 0.375$$
$$= +1.375 - 1.375 = 0$$

This example shows that the sum of the power absorbed by devices is equal in magnitude to the sum of the power supplied by devices. A power balance always exists in the types of circuits treated in this book and can be used as an overall check of circuit analysis calculations. ■

Exercise 1–4

The working variables of a set of two-terminal electrical devices are observed to be as follows:

	DEVICE 1	DEVICE 2	DEVICE 3	DEVICE 4	DEVICE 5
v	+10 V	?	−15 V	+5 V	?
i	−3 A	−3 A	+10 mA	?	−12 mA
p	?	+40 W	?	+10 mW	−120 mW

Using the passive sign convention, find the magnitude and sign of the unknown variable and state whether the device is absorbing or delivering power.

Answers:

Device 1: $p = -30$ W (delivering power)
Device 2: $v = -13.3$ V (absorbing power)
Device 3: $p = -150$ mW (delivering power)
Device 4: $i = +2$ mA (absorbing power)
Device 5: $v = +10$ V (delivering power)