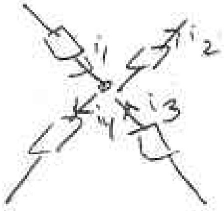
CHP. 10 S-DOMAIN CIRCUIT ANALYSIS

1 Let $i_1(t), \dots, i_b(t), v_1(t), \dots, v_b(t)$ branch currents / voltages

$I_j(s) = \mathcal{L}\{i_j(t)\}$, $V_j(s) = \mathcal{L}\{v_j(t)\}$ $j = 1, \dots, b$.

Likewise for node voltages.

KCL:

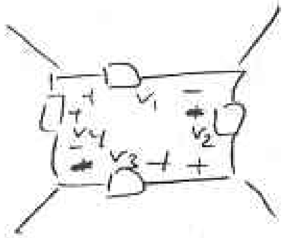


$-i_1(t) + i_2(t) - i_3(t) + i_4(t) = 0$ take Laplace

$\Rightarrow \boxed{-I_1(s) + I_2(s) - I_3(s) + I_4(s) = 0}$

$\Rightarrow$ KCL retains its form

KVL:



$v_1(t) - v_2(t) + v_3(t) - v_4(t) = 0$

$\Rightarrow \boxed{V_1(s) - V_2(s) + V_3(s) - V_4(s) = 0}$

$\Rightarrow$ KVL retains its form.

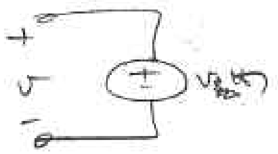
BRANCH RELATIONS:

Resistor.

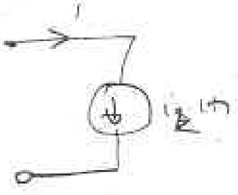
$v(t) = R i(t) \Rightarrow \boxed{V(s) = R I(s)}$

Resistor retains its form.

Sources:

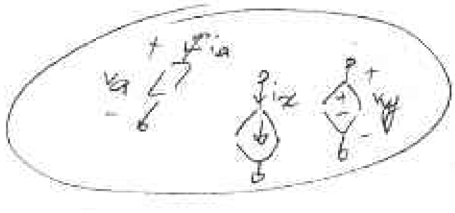


$v(t) = v_g(t) \Rightarrow V(s) = V_E(s)$ ← known



$i(t) = i_k(t) \Rightarrow I(s) = I_k(s)$ ← known.

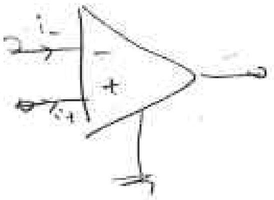
In general: any algebraic relation retains its form:



$v_y(t)=k\,i_a(t) \Rightarrow V_y(s)=k\,I_a(s)$

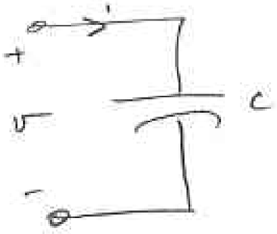
$i_x(t)=\alpha\, v_a(t) \rightarrow I_x(s)=\alpha\, V_a(s)$

...



$$\left.\begin{array}{l} i_-=0 \\ i_+=0 \\ v_+=v_- \end{array}\right\} \Rightarrow \begin{array}{l} I_-=0 \\ I_+=0 \\ V_+=V_- \end{array}$$

CAPACITOR:



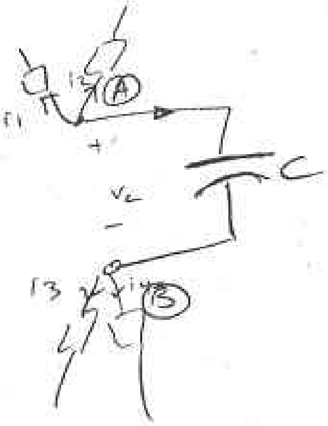
$$i=C\frac{dv}{dt} \Rightarrow \boxed{I(s)=sCV(s)-Cv(0)}$$

(for node eqn's)

$$\boxed{V(s)=\frac{1}{sC}I(s)+\frac{1}{s}v(0)}$$

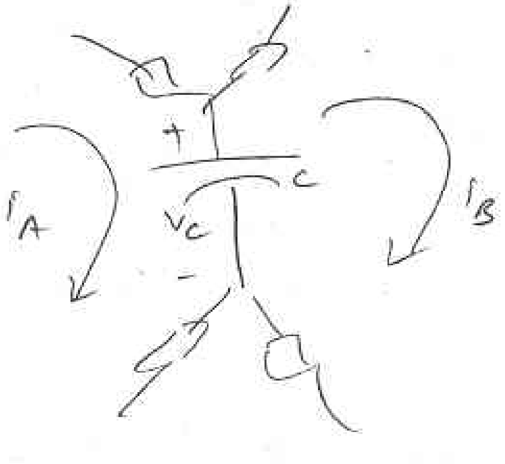
(for mesh eqn's)

example:



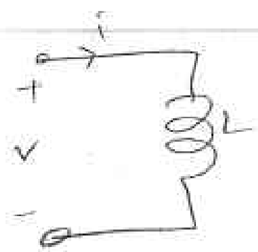
(A) $I_1+I_2+sC\left(V_A(s)-V_B(s)\right)-Cv_C(0)=0$

(B) $I_3(s)+I_4(s)-\left[sC\left(V_A(s)-V_B(s)\right)-Cv_C(0)\right]=0$



mesh A: $\dots+\frac{1}{sC}\left[I_A(s)-I_B(s)\right]+\frac{1}{s}v_C(0)+\dots=0$

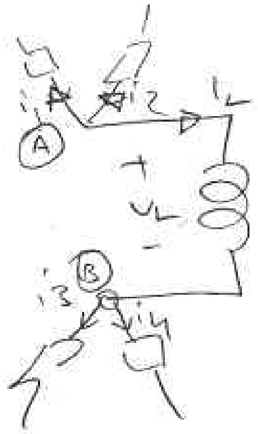
mesh B: $\dots-\left[\frac{1}{sC}\left[I_A(s)-I_B(s)\right]+\frac{1}{s}v_C(0)\right]+\dots=0$



$v = L\frac{di}{dt} \Rightarrow$

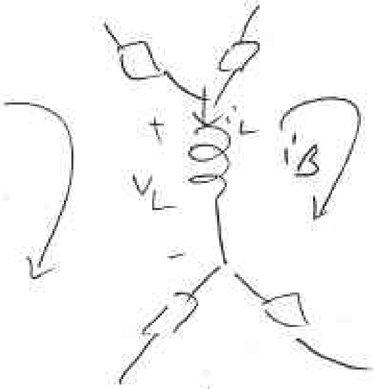
$$\boxed{V(s) = sL\,I(s) - L\,i(0)}$$

$$\boxed{I(s) = \frac{1}{sL}V(s) + \frac{1}{s}i(0)}$$



$$(A)\quad I_1 + I_2 + \left\{\frac{1}{sL}\left[V_A(s) - V_B(s)\right] + \frac{1}{s}i_L(0)\right\} \cdots = 0$$

$$(B)\quad I_3 + I_4 - \left\{\frac{1}{sL}\left[V_A(s) - V_B(s)\right] + \frac{1}{s}i_L(0)\right\} \cdots = 0$$



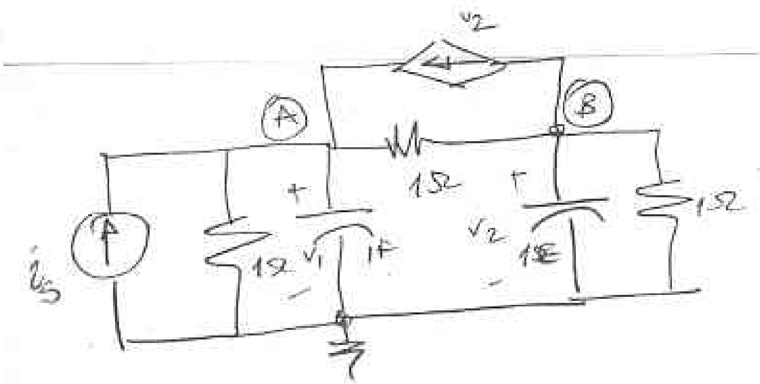
mesh A $\Rightarrow \cdots + \left[sL\left(I_A(s) - I_B(s)\right) - L\,i_L(0)\right] + \cdots = 0$

mesh B $\Rightarrow \cdots - \left[sL\left(I_A(s) - I_B(s)\right) - L\,i_L(0)\right] \cdots = 0$

10-4

NODE ANALYSIS:

* write KCL at nodes
* Express each current in terms of voltages if you can. If for a certain branch, current cannot be expressed in terms of voltages, leave it as an unknown
* add the branch relation for such branches in the list of equations.



$v_{c1}(0) = 2V$

$v_{c2}(0) = 1V$

$i_s(t) = u(t)$ (unit step)

Ⓐ $\Rightarrow$ $I_s = V_A/1 + [sV_A - 2] + \frac{V_A - V_B}{1} - V_B$

$$\boxed{\frac{1}{s} = (s+2)V_A - 2V_B - 2} \Rightarrow \boxed{(s+2)V_A - 2V_B = \frac{2s+1}{s}}$$

Ⓑ) $-V_B + \frac{V_B}{1} + [sV_B - 1] + \frac{V_B - V_A}{1} = 0 \Rightarrow$ $\boxed{-V_A + (s+3)V_B = 1}$

$-V_A = (s+3)V_B - 1 \Rightarrow$ $(s+2)(s+3)V_B - (s+2) - 2V_B = \frac{2s+1}{s}$

$\Rightarrow (s^2 + 5s + 4)V_B = \frac{2s+1}{s} + (s+2) = \frac{s^2+4s+1}{s}$

$V_B = \frac{s^2+4s+1}{s(s^2+5s+4)} = \frac{s^2+4s+1}{s(s+1)(s+4)} = \frac{k_1}{s} + \frac{k_2}{s+1} + \frac{k_3}{s+4}$

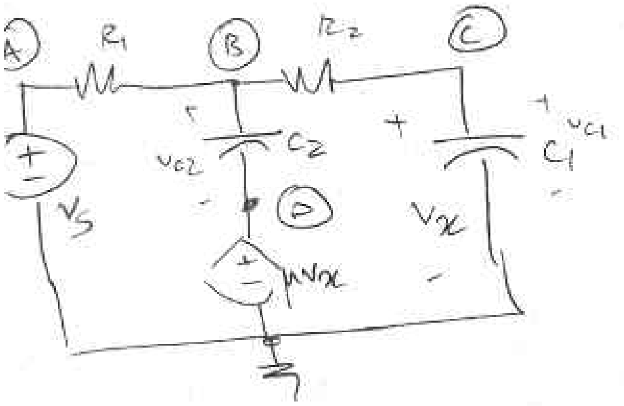
$k_1 = sV_B\Big|_{s=0} = 1/4$

$k_2 = (s+1)V_B\Big|_{s=-1} = \frac{1-4+1}{(-1)(3)} = \frac{-2}{-3} = 2/3$

$k_3 = (s+4)V_B\Big|_{s=-4} = \frac{16-16+1}{(-4)(-3)} = \frac{1}{12}$

$$\boxed{v_B(t) = v_{c2}(t) = \left(\frac{1}{4} + \frac{2}{3}e^{-t} + \frac{1}{12}e^{-4t}\right)u(t)}$$

Exercise (p. 510)



$$\underline{\underline{V_A = V_S}} \qquad \boxed{V_D = \mu V_x = \mu V_C}$$

B ⇒

$$G_1(V_B - V_A) + G_2(V_B - V_C) + sC_2\left[V_B(s) - V_D(s)\right] - C_2 v_{C2}(0) = 0$$

$$\boxed{(G_1 + G_2 + sC_2)\,V_B(s) - (G_2 + \mu C_2)\,V_C(s) = G_1 V_S + C_2 v_{C2}(0)}$$

C

$$G_2(V_C - V_B) + sC_1 V_C(s) - C_1 v_{C1}(0) = 0$$

$$\boxed{-G_2 V_B(s) + (G_2 + sC_1)\,V_C(s) = C_1 v_{C1}(0)}$$

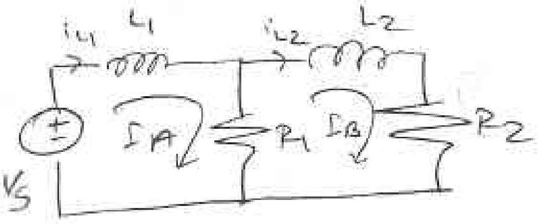
Find $V_B(s)$, $V_C(s)$ ⇒ $V_D(s) = \mu V_C(s)$ ⇒ Then can find any current/voltage.

⇒ $\mathcal{L}^{-1}\{\cdot\}$ ⇒ can find $\underline{i(t),\ v(t)}$

10.5 MESH EQUATIONS:

- Write KVL mesh eqn's
- Express each voltage in terms of mesh currents. If you can. If a branch voltage cannot be expressed in terms of mesh currents, leave it as an unknown
- Add the branch relations for such branches in the list of equations.

Example. (p. 520)



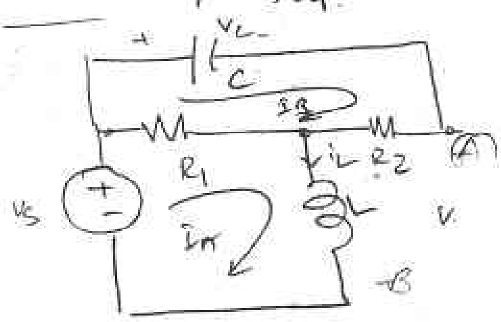
mesh A: $V_S = sL_1 I_A(s) - L_1 i_{L1}(0) + R_1(I_A - I_B)$

$$\Rightarrow \boxed{(sL_1 + R_1) I_A - R_1 I_B = V_s + L_1 i_{L1}(0)}$$

mesh B: $sL_2 I_B(s) - L_2 i_{L2}(0) + R_2 I_B + R_1(I_B - I_A) = 0$

$$\Rightarrow \boxed{-R_1 I_A + (sL_2 + R_1 + R_2) I_B = L_2 i_{L2}(0)}$$

ercise. p. 524.



$V_S = R_1(I_A - I_B) + sL I_A - L i_L(0)$

$$\Rightarrow \boxed{(R_1 + sL) I_A - R_1 I_B = V_S + L i_L(0)}$$

$\frac{1}{Cs} I_B + \frac{1}{s} v_C(0) + R_2 I_B + R_1(I_B - I_A) = 0$

$$\boxed{-R_1 I_A + \left(R_1 + R_2 + \frac{1}{Cs}\right) I_B = -\frac{1}{s} v_C(0)}$$

$\Rightarrow$ Find $I_A(s)$, $I_B(s)$ $\Rightarrow$ find any $I(s)$, $V(s)$

$\Rightarrow \mathcal{L}^{-1}\{\cdot\} \Rightarrow$ find $\underline{i(t)}$, $\underline{v(t)}$.