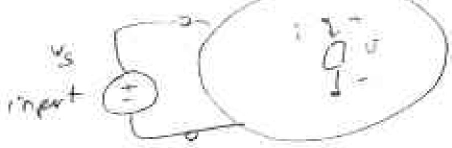
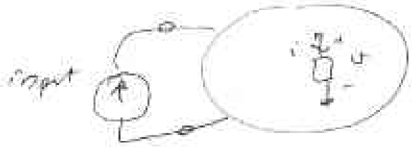
Chp 11: Network Function:





$$\text{Network Function} = \frac{\mathcal{L}\{\text{zero-state response of output}\}}{\mathcal{L}\{\text{input}\}}$$

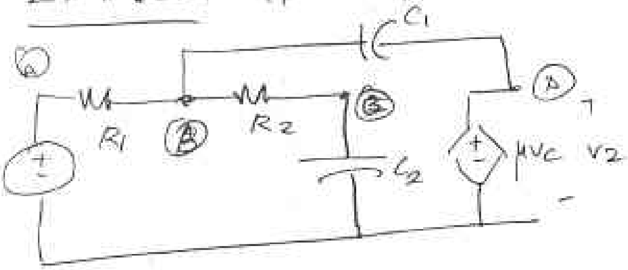
zero-state response: response (solution) when initial conditions are zero

- Methodology: Use Node or mesh analysis, use zero-initial conditions. Find the Laplace transform of the output in question.

$$H(s) = \frac{O(s)}{I(s)} \Rightarrow \boxed{O(s) = H(s) \cdot I(s)}$$

$\Rightarrow$ can find the output (zero-state response) when the input is specified.

EXAMPLE: (p. 548)



Find $\boxed{H(s) = \frac{V_2(s)}{V_1(s)}}$

$(v_{C1}(0) = 0!)$

B) $\Rightarrow$ $G_1(V_B - V_1) + G_2(V_B - V_C) + sC_1(V_B - V_D) = 0$

$(v_{C2}(0) = 0)$

C) $\Rightarrow$ $G_2(V_C - V_B) + sC_2 V_C = 0$

$V_D = \mu V_C$

$$\Rightarrow \boxed{\begin{aligned} (G_1 + G_2 + sC_1)V_B - (G_2 + \mu sC_1)V_C &= G_1 V_1 \\ -G_2 V_B + (G_2 + sC_2)V_C &= 0 \end{aligned}}$$

Find V_C in terms of V_1 $\rightarrow$ $V_2 = \mu V_C \Rightarrow$ find $H(s)$!

$$V_B = \frac{G_2 + sC_2}{G_2} V_C$$

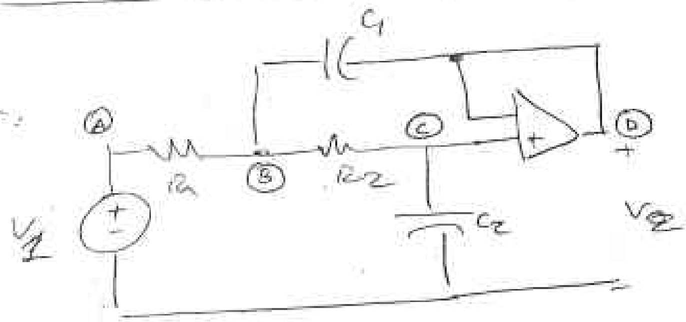
$$\Rightarrow \left[(G_1 + G_2 + sC_1) \frac{G_2 + sC_2}{G_2} - (G_2 + \mu s C_1) \right] V_C = G_1 V_1$$

$$\therefore V_C = \frac{G_1 G_2}{s^2 C_1 C_2 + s\left(C_1 G_2 (1-\mu) + C_2 (G_1 + G_2)\right) + G_1 G_2} \cdot \frac{R_1 R_2}{R_1 R_2} \cdot V_1$$

$$\boxed{V_C = \frac{1}{s^2 C_1 C_2 R_1 R_2 + s\left(C_1 R_1 (1-\mu) + C_2 (R_1 + R_2)\right) + 1} \cdot V_1}$$

$$\boxed{H(s) = \frac{\mu V_C}{V_1} = \frac{\mu}{s^2 C_1 C_2 R_1 R_2 + s\left(C_1 R_1 (1-\mu) + C_2 (R_1 + R_2)\right) + 1}}$$

<u>EXAMPLE:</u>



$$H(s) = \frac{V_2}{V_1} = ?$$

Note that $V_C = V_D$ $\Rightarrow$ previous situation with $\mu = 1$!

$$\boxed{H(s) = \frac{1}{s^2 C_1 C_2 R_1 R_2 + s\, C_2 (R_1 + R_2) + 1}}$$

<u>ex.</u> $R_1 = R_2 = 1\,\Omega$ $C_1 = 1F$ $C_2 = 1/2\,F$.

$$H(s) = \frac{1}{s^2 \cdot \frac{1}{2} + s\,\frac{1}{2} \cdot 2 + 1} = \frac{2}{s^2 + 2s + 2}$$

if $v_1(t) = 1(t)$ find $v_2(t)$.

$$V_1(s) = \frac{1}{s}$$

$$V_2(s) = H(s) \cdot V_1(s) = \frac{2}{s^2 + 2s + 2} \cdot \frac{1}{s} = \frac{2}{s(s - (-1+j))(s - (-1-j))}$$

$$V_2(s) = \frac{k_1}{s} + \frac{k_2}{s-(-1+j)} + \frac{k_2^*}{s-(-1-j)}$$

$$k_1 = s\,V_2(s)\Big|_{s=0} = 1 \qquad k_2 = (s-(-1+j))\,V_2(s)\Big|_{s=-1+j} = \frac{1}{\sqrt{2}}\,e^{-j225}$$

$$v_0(t) = \left(1 + \frac{2}{\sqrt{2}}\,e^{-t}\cos(t-225^\circ)\right)u(t).$$

* NETWORK FUNCTIONS AND IMPULSE RESPONSE (11.3)

Let input be an impulse
$\Rightarrow$ zero-state response in this case is called impulse response.

$$H(s) = \frac{\mathcal{L}\{\text{zero-state-response}\}}{\mathcal{L}\{\text{input}\}} \Rightarrow H(s) = \frac{\mathcal{L}\{\text{impulse-response}\}}{\mathcal{L}\{\text{impulse}\}} = \mathcal{L}\{\text{impulse response}\}$$

$$\Rightarrow \boxed{\mathcal{L}\{\text{impulse-response}\} = H(s)}$$

$$h(t) = \mathcal{L}^{-1}\{H(s)\} = \text{impulse response}$$

$\Rightarrow$ CONVOLUTION

form of $H(s)$ = ?

Combined constraints:

KCL: $A\,I(s) = 0$

KVL: $B\,V(s) = 0$

node $M(s)\,V(s) + N(s)\,I(s) = U(s)$

$M(s)$: form of $sC \Rightarrow$ polynomial in s

$N(s)$: form of $sL \Rightarrow$ polynomial in s

$$\Rightarrow \underbrace{\begin{bmatrix} 0 & A \\ B & 0 \\ M(s) & N(s) \end{bmatrix}}_{T(s)} \begin{bmatrix} V(s) \\ I(s) \end{bmatrix} = \begin{bmatrix} 0 \\ \vdots \\ U(s) \end{bmatrix}$$

$T(s)$: entries are polynomial in s.

$$\begin{bmatrix} V(s) \\ I(s) \end{bmatrix} = \frac{1}{\det T(s)} \begin{bmatrix} \text{cofactor} \\ \text{matrix} \end{bmatrix} \begin{bmatrix} 0 \\ \vdots \\ U(s) \end{bmatrix}$$

polynomial in s

polynomial in s

$$\Rightarrow \boxed{H(s) = \frac{\text{polynomial in } s}{\text{polynomial in } s} = \frac{a_m s^m + a_{m-1} s^{m-1} + \dots + a_1 s + a_0}{b_n s^n + b_{n-1} s^{n-1} + \dots + b_1 s + b_0}}$$

* RELATION WITH SINUSOIDAL STEADY STATE. (11.5)

call the source as $u(t)$

call the output as $y(t)$

$$H(s) = \frac{Y(s)}{U(s)} \Rightarrow \boxed{Y(s) = H(s) \cdot U(s)} \quad \text{(zero-state response)}$$

Let us assume that $u(t)$ is sinusoidal:

$$u(t) = U_m \cos(\omega t + \phi) = Re\{U_m e^{j\phi} e^{j\omega t}\} = Re\{\hat{u} e^{j\omega t}\}$$

$$= \frac{1}{2} \hat{u} e^{j\omega t} + \frac{1}{2} \bar{\hat{u}} e^{-j\omega t}$$

$$\mathcal{L}\{u(t)\} = \frac{1}{2} \hat{u} \cdot \frac{1}{s - j\omega} + \frac{1}{2} \bar{\hat{u}} \frac{1}{s + j\omega}$$

$$Y(s) = H(s) \cdot \left\{ \frac{1}{2} \hat{u} \frac{1}{s - j\omega} + \frac{1}{2} \bar{\hat{u}} \frac{1}{s + j\omega} \right\}$$

$$= \frac{1}{2} H(s) \hat{u} \cdot \frac{1}{s - j\omega} + \frac{1}{2} H(s) \bar{\hat{u}} \cdot \frac{1}{s + j\omega},$$

Let $H(s) = \dfrac{n(s)}{d(s)} = \dfrac{n(s)}{(s-p_1)(s-p_2)\dots(s-p_n)}$

$$Y(s) = \underbrace{\frac{k_1}{s-p_1} + \cdots + \frac{k_n}{s-p_n}}_{\text{natural response}} + \underbrace{\frac{k}{s-j\omega} + \frac{k^*}{s+j\omega}}_{\substack{\text{forced response} \\ \text{(sinusoidal steady state solution)}}}$$

$$k = (s-j\omega)\,Y(s)\Big|_{s=j\omega} = \left(\frac{1}{2} H(s)\,\hat{u} + \frac{1}{2} H(s)\,\bar{\hat{u}} \cdot \frac{s-j\omega}{s+j\omega}\right)\Big|_{s=j\omega}$$

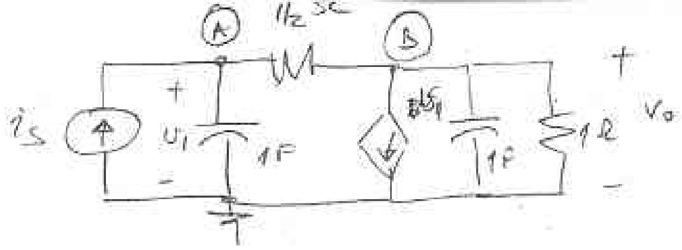
$$= \frac{1}{2} H(j\omega)\,\hat{u}$$

$$\Rightarrow \quad y(t) = \frac{1}{2} H(j\omega)\,\hat{u}\, e^{j\omega t} + \frac{1}{2}\overline{H(j\omega)}\,\bar{\hat{u}} \cdot e^{-j\omega t} + \sum_{i=1}^{n} k_i e^{p_i t}$$

$$= \underbrace{Re\left\{H(j\omega)\,\hat{u} \cdot e^{j\omega t}\right\}}_{\text{sinusoidal steady state response}} + \sum_{i=1}^{n} k_i e^{p_i t}$$

But $\underline{\underline{\text{then}}}$: output phasor $= H(j\omega) \cdot \hat{u}$

$$\Rightarrow \boxed{\text{transfer function in phasor} = \frac{\text{output phasor}}{\text{input phasor}} = H(j\omega) = H(s)\Big|_{s=j\omega}!}$$



$$I_s = sV_A + 2(V_A - V_B) = (s+2)V_A - 2V_B$$

$$0 = 2(V_B - V_A) + 3V_A + sV_B + V_B = -V_A + (s+3)V_B$$

$$\rightarrow \quad V_A = (s+3)V_B$$

$$I_s = [(s+2)(s+3) - 2]\,V_B = [s^2 + 5s + 6 - 2]\,V_B = (s^2+5s+4)\,V_B$$

$$\Rightarrow * \quad \boxed{H(s) = \frac{V_B}{I_s} = \frac{1}{s^2+5s+4}} \quad *$$

(*) $\quad i_s(t) = u(t) \Rightarrow \quad V_B(s) = H(s)\cdot \mathcal{L}\{i_s\} = \frac{1}{s^2+5s+4}\cdot\frac{1}{s}$

$$= \frac{1}{s(s+1)(s+4)} = \frac{k_1}{s} + \frac{k_2}{s+1} + \frac{k_3}{s+4}$$

$$k_1 = sV_B(s)\Big|_{s=0} = \frac{1}{4}$$

$$k_2 = (s+1)V_B(s)\Big|_{s=-1} = -\frac{1}{3}$$

$$k_3 = (s+4)V_B(s)\Big|_{s=-4} = \frac{1}{-4\cdot -3} = \frac{1}{12}$$

$$\boxed{v_o(t) = \left(\frac{1}{4} - \frac{1}{3}e^{-t} + \frac{1}{12}e^{-4t}\right) \quad t \geq 0}$$

ZERO-STATE RESPONSE

→ this is also the step response

→ impulse response = $\mathcal{L}^{-1}\{H(s)\}$

$$H(s) = \frac{1}{s^2+5s+4} = \frac{k_1}{s+1} + \frac{k_2}{s+4}$$

$$k_1 = \frac{1}{3} \qquad k_2 = -\frac{1}{3}$$

$$\boxed{h(t) = \frac{1}{3}e^{-t} - \frac{1}{3}e^{-4t}}$$

STEP RESPONSE:

- $Y(s) = H(s)\,U(s)$

step response: zero-state solution when $u(t) = 1(t) \Rightarrow U(s) = \frac{1}{s}$

$$Y(s) = \frac{1}{s}H(s) \quad \Rightarrow \quad y(t) = \int_0^t h(\tau)\,d\tau \iff \dot{y}(t) = h(t)$$

(generalized derivative)

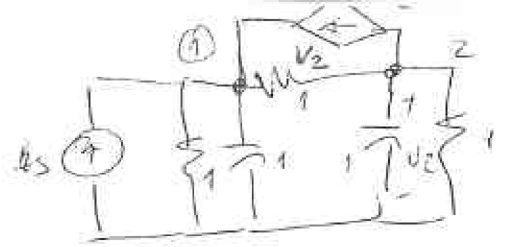
$h(t) = \mathcal{L}^{-1}\{H(s)\}$ $\Rightarrow$

= impulse response.

$$H(s) = \frac{2}{s^2+2s+2} = \frac{2}{[s-(-1+j)][s-(-1-j)]} = \frac{k}{s-(-1+j)} + \frac{k^*}{s-(-1-j)}$$

$$k = [s-(-1+j)]\,H(s)\Big|_{s=-1+j} = \frac{2}{-1+j-(-1-j)} = \frac{2}{2j} = \frac{1}{j} = -j = 1e^{-j90^\circ}$$

$$h(t) = \mathcal{L}^{-1}\{H(s)\} = 2e^{-t}\cos(t-90^\circ)$$



$$\begin{bmatrix} s+2 & -1 \\ -1 & s+2 \end{bmatrix}\begin{pmatrix} E_1 \\ E_2 \end{pmatrix} = \begin{pmatrix} I_s + E_2 \\ -E_2 \end{pmatrix}$$

$$\rightarrow \begin{bmatrix} s+2 & -2 \\ -1 & s+3 \end{bmatrix}\begin{pmatrix} E_1 \\ E_2 \end{pmatrix} = \begin{pmatrix} I_s \\ 0 \end{pmatrix} \rightarrow \Delta(s) = (s+2)(s+3) - 2 = s^2 + 5s + 6 - 2 = s^2 + 5s + 4$$

$$\begin{pmatrix} E_1 \\ E_2 \end{pmatrix} = \frac{1}{(s+1)(s+4)} \begin{bmatrix} s+3 & 2 \\ 1 & s+2 \end{bmatrix}\begin{pmatrix} I_s \\ 0 \end{pmatrix}$$

$$\boxed{E_2 = \frac{1}{(s+1)(s+4)} I_s} \qquad \boxed{H(s) = \frac{1}{(s+1)(s+4)}}$$

when $i_s(t) = \delta(t) \rightarrow I_s = 1$

$$E_2 = \frac{1}{(s+1)(s+4)} = \frac{k_1}{s+1} + \frac{k_2}{s+4}$$

$$k_1 = (s+1)E_2\Big|_{s=-1} = \frac{1}{3} \qquad k_2 = (s+4)E_2\Big|_{s=-4} = \frac{1}{-3}$$

$$\boxed{e_2(t) = \frac{1}{3}e^{-t} - \frac{1}{3}e^{-4t}}$$ impulse response

when $i_s(t) = 1(t) \rightarrow I_s = \frac{1}{s} \rightarrow$

$$E_2 = \frac{1}{s(s+1)(s+4)} = \frac{k_1}{s} + \frac{k_2}{s+1} + \frac{k_3}{s+4}$$

$$k_1 = sE_2\Big|_{s=0} = \frac{1}{4} \qquad k_2 = (s+1)E_2\Big|_{s=-1} = \frac{1}{-3} \qquad k_3 = (s+4)E_2\Big|_{s=-4} = \frac{1}{12}$$

$$\boxed{e_2(t) = \frac{1}{4} - \frac{1}{3}e^{-t} + \frac{1}{12}e^{-4t}}$$ step response

$\Rightarrow$ ~~step~~ $\boxed{\text{impulse response} = \frac{d}{dt}(\text{step response})}$

why?

$$\boxed{\begin{aligned} E_{2im} &= H(s) \\ E_{2st} &= \frac{1}{s}H(s) \end{aligned}}$$

~~step~~ impulse
step response! $\Rightarrow$ use integral rule.
$\Rightarrow E_{2st} = \int_0^t e_{2im}(\tau)d\tau$