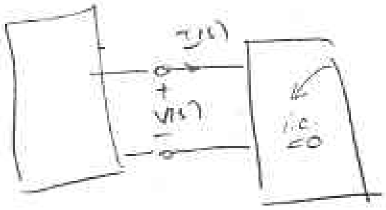
S - DOMAIN ANALYSIS



$Z(s) = \frac{V(s)}{I(s)}$ → impedance in s domain
a network fn. have initial conditions are set to zero.

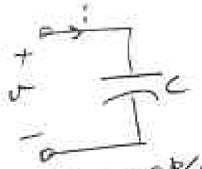
⇕

$Y(s) = \frac{I(s)}{V(s)}$ admittance in s domain
a network fn.

$\boxed{Z = \frac{1}{Y}}$

⇒ $Z_R(s) = R$

$Y_R(s) = \frac{1}{R}$



$Y_C(s) = sC$

$Z_C(s) = \frac{1}{sC}$

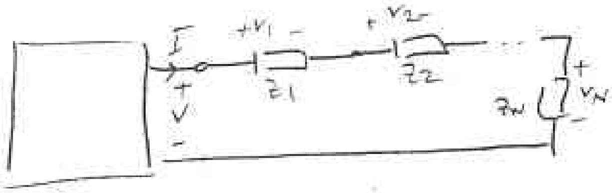
$I(s) = sC\,V(s)$

$Z_L(s) = sL$

$Y_L(s) = \frac{1}{sL}$

$V(s) = sL\,I(s)$

Series Connection :

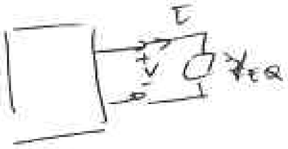


$V = V_1 + \dots + V_N$
$I = I_1 = \dots = I_N$
$V_j = Z_j I_j$

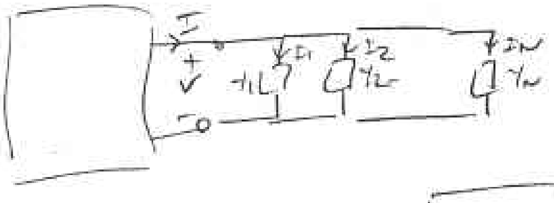
$V = (Z_1 + \dots + Z_N) I$

$Z_{EQ} = \frac{V}{I} = Z_1 + \dots + Z_N.$

≡



* PARALLEL CONNECTION

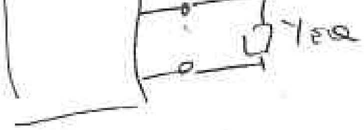


$I = I_1 + \dots + I_N$
$V = V_1 = \dots = V_N$
$I_j = Y_j V_j$

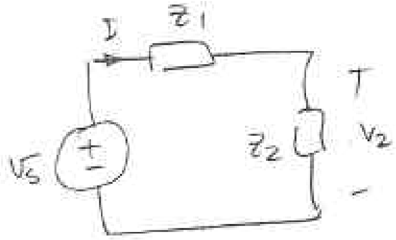
$I = (Y_1 + \dots + Y_N) V$

$\boxed{Y_{EQ} = Y_1 + \dots + Y_N}$

≡

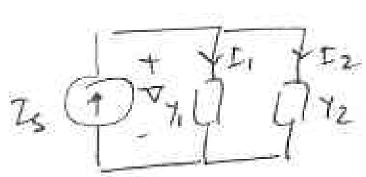


VOLTAGE DIVISION



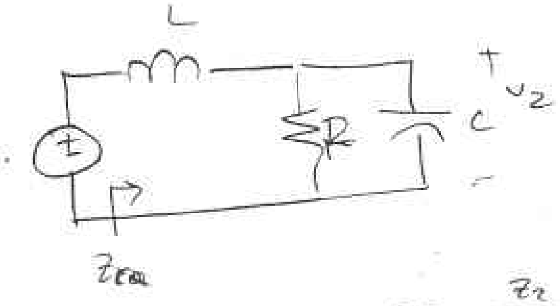
$$\left.\begin{array}{l} V_S = (Z_1+Z_2)I \\ V_2 = Z_2 I \end{array}\right\} \quad \boxed{V_2 = \frac{Z_2}{Z_1+Z_2} V_S}$$

CURRENT DIVISION



$$\left.\begin{array}{l} I_S = (Y_1+Y_2)V \\ I_1 = Y_1 V \end{array}\right\} \quad \boxed{I_1 = \frac{Y_1}{Y_1+Y_2} I_S = \frac{Z_2}{Z_1+Z_2} I_S}$$

EXAMPLE p.493



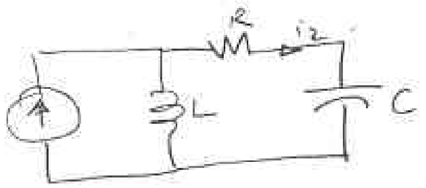
$\Rightarrow$ I.C. $=0$

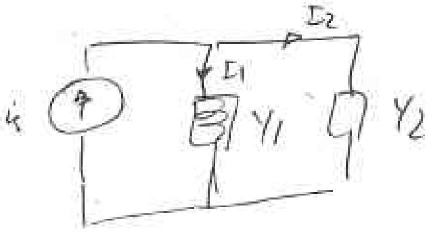
$$Z_1 = sL \qquad Z_2 = \frac{R\cdot 1/sC}{R+1/sC} = \frac{R}{sCR+1}$$

$$Z_1+Z_2 = sL + \frac{R}{sCR+1} = \frac{s^2LCR+sL+R}{sCR+1}$$

$$V_2 = \frac{Z_2}{Z_1+Z_2} V_S = \frac{R/sCR+1}{\frac{s^2LCR+sL+R}{sCR+1}} V_S = \frac{R}{s^2LCR+sL+R} \cdot V_S$$

EXERCISE p.494





$$Z_2 = R + \frac{1}{sC} = \frac{sCR+1}{sC} \qquad Y_2 = \frac{sC}{sCR+1}$$

$$Z_1 = sL \rightarrow Y_1 = 1/sL$$

$$Z_1+Z_2 = sL + \frac{sCR+1}{sC} = \frac{s^2LC+sCR+1}{sC}$$

$$I_2 = \frac{Y_2}{Y_1+Y_2} I_S = \frac{Z_1}{Z_1+Z_2} I_S$$

$$\boxed{I_2 = \frac{sL}{\frac{s^2LC+sCR+1}{sC}} = \frac{s^2LC}{s^2LC+sCR+1} I_S}$$

SUPERPOSITION IN S DOMAIN:

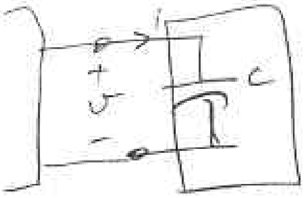
any solution = zero-state solution + zero-input solution

zero-state solution = initial conditions are zero, only input is effective

zero-input solution = inputs are zero, only initial conditions are effective.

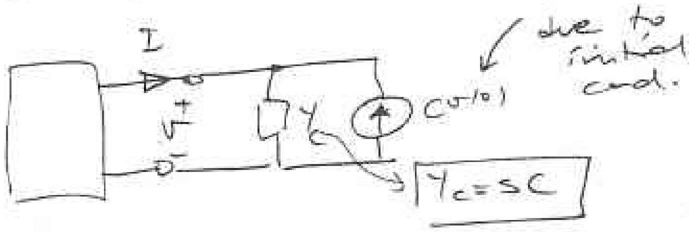
HOW TO DEAL WITH THE INITIAL CONDITIONS:

- capacitor



$i = C\dfrac{dv}{dt} \Rightarrow I(s) = sCV(s) - Cv(0)$

$\Rightarrow$

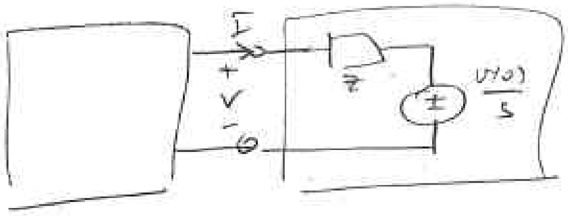


in time domain

in s domain

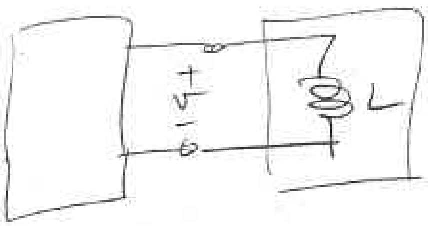
or equivalently we may use $\boxed{V(s) = \dfrac{1}{sC} I(s) + \dfrac{v(0)}{s}}$

$\Rightarrow$



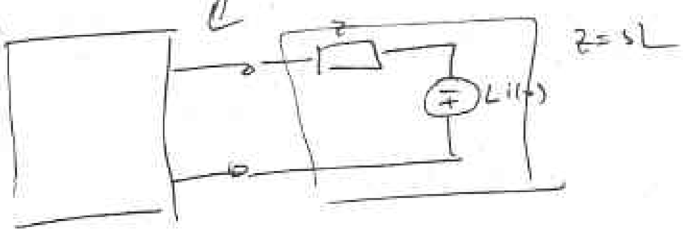
$Z = \dfrac{1}{sC}$

for inductor



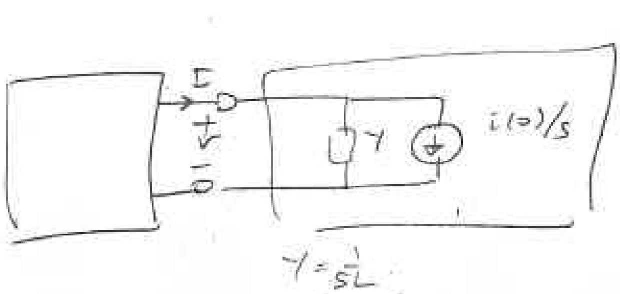
$v = L\dfrac{di}{dt} \Rightarrow \boxed{V(s) = sLI(s) - Li(0)}$

$\Downarrow$



or equivalently

$\boxed{I(s) = \dfrac{1}{sL} V(s) + \dfrac{1}{s} i(0)} \Rightarrow$

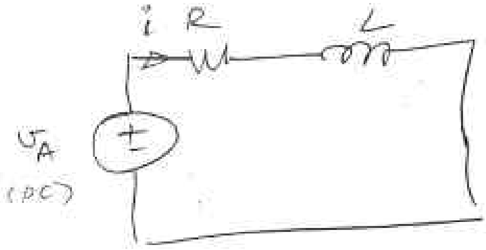


These transformations are known as initial condition transformations.

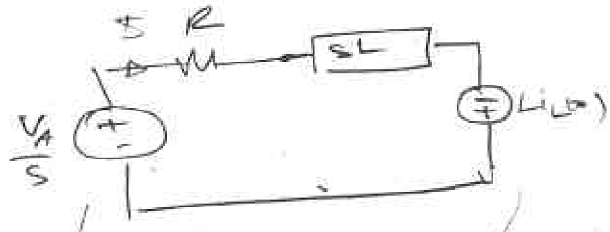
Application of superposition is as follows:

- Apply an appropriate initial condition transformation
- Treat initial conditions as equivalent independent sources in s-domain
- Apply the superposition to this circuit in s-domain
- The contributions of equivalent sources due to initial conditions give the zero-input response
- The contributions of independent sources alone give the zero-state response.

EXAMPLE: p. 498

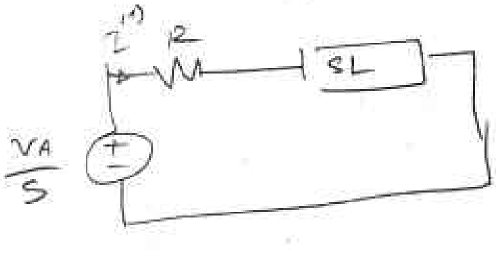


$i_L(0)$ is given $\Rightarrow$



as if 2 ind. sources.

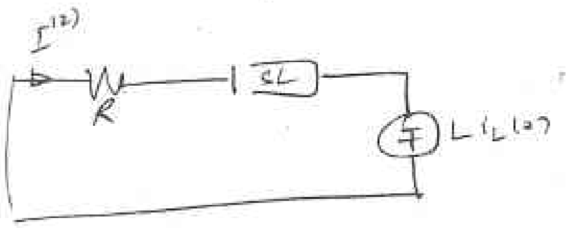
contribution of V_A:



$$I^{(1)} = \frac{V_A/s}{R+sL} = \frac{V_A/L}{s(s+R/L)}$$

This is zero-state solution.

contributions of $i_L(0)$



$$I^{(2)} = \frac{Li_L(0)}{R+sL} = \frac{i_L(0)}{s+R/L}$$

This is the zero-input solution.

$$I(s) = I^{(1)} + I^{(2)} = \underbrace{\frac{V_A/L}{s(s+R/L)}}_{\text{zero-state}} + \underbrace{\frac{i_L(0)}{s+R/L}}_{\text{zero-input}}$$

$$I(s) = \frac{k_1}{s} + \frac{k_2}{s+R/L} + \frac{i_L(0)}{s+R/L}$$

$$k_1 = s\,I^{(1)}\Big|_{s=0} = V_A/R$$

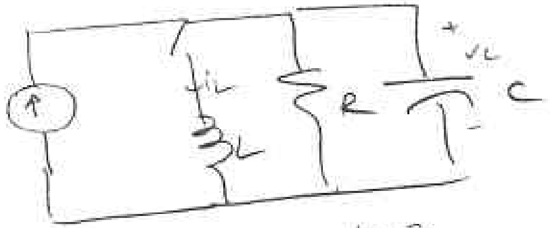
$$k_2 = (s+R/L)\,I^{(1)}\Big|_{s=-R/L} = -V_A/R$$

$$i(t) = \Big\{ \underbrace{\overbrace{\frac{V_A}{R}}^{\text{particular (forced)}} - \frac{V_A}{R} e^{-R/L\,t}}_{\text{zero-state}} + \underbrace{i_L(0)\, e^{-t R/L}}_{\text{zero-input}} \Big\} u(t)$$

homogeneous (natural)

Example (p. 498)

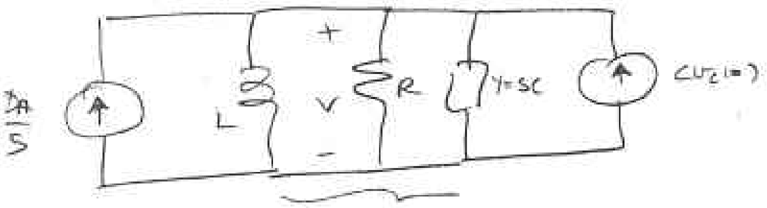
in steady-state C → open, L → short.



Time domain

$\Rightarrow$ $v_C(0) = R I_A$, $i_L(0) = 0$

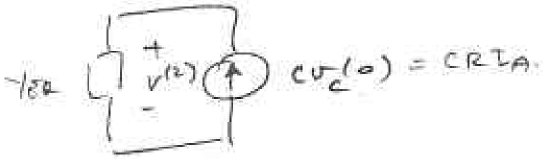
⇓



$$Y_{EQ} = \frac{1}{sL} + \frac{1}{R} + sC = \frac{s^2LCR + sL + R}{sLR}$$

$$V^{(1)} = \frac{I_A/s}{Y_{EQ}} = \frac{I_A/\cancel{s}\ \cancel{s}LR}{s^2LCR + sL + R} = \frac{I_A/C}{s^2 + s\frac{1}{CR} + \frac{1}{LC}}$$

zero-state response.

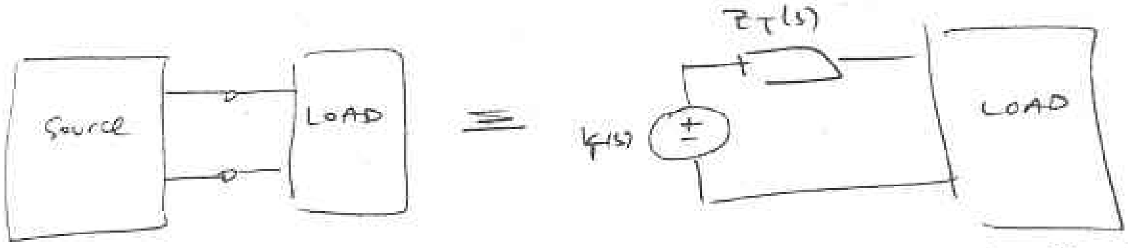


$$V^{(2)} = \frac{CRI_A}{Y_{EQ}} = \frac{CRI_A \cdot sLR}{s^2LCR + sL + R}$$

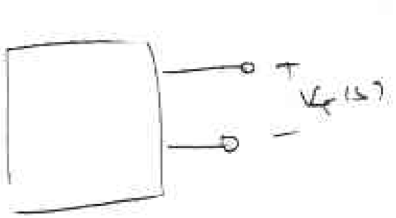
$$= \frac{RI_A s}{s^2 + s\frac{1}{CR} + \frac{1}{LC}} \Rightarrow \text{zero-input solution}$$

$$V(s) = V^{(1)} + V^{(2)} = \frac{RI_A s + I_A/C}{s^2 + s\frac{1}{CR} + \frac{1}{LC}}$$

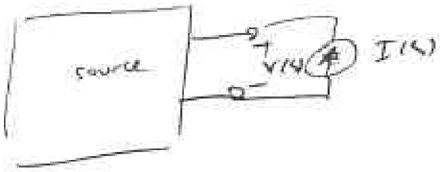
THEVENIN – NORTON EQUIVALENT CIRCUITS:



Finding $V_T(s)$ and $Z_T(s)$ are the same as before.



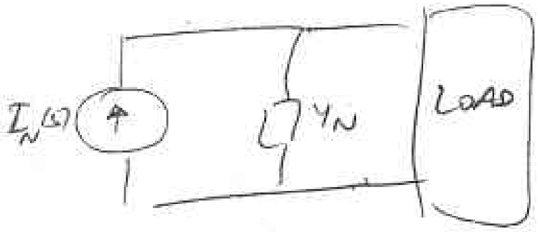
open circuit — keep all ind. sources inside the source part (possibly initial cond. as well)



set all ind. sources inside source part (including the initial cond.)

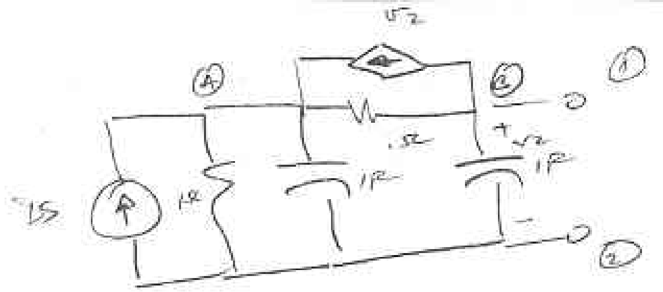
$$Z_T(s) = \frac{V(s)}{I(s)}$$

NORTON: (SIMILAR)



$$\boxed{V_T(s) = I_N(s)/Y_N}$$

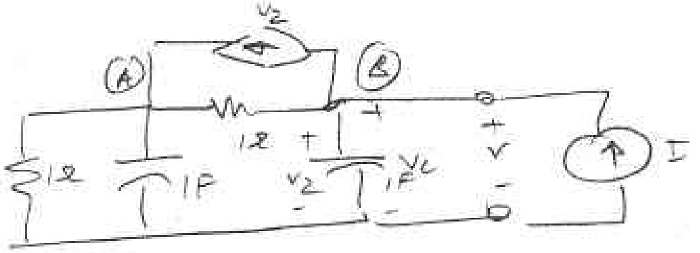
$$\boxed{Z_T = 1/Y_N}$$



$$I_S = (s+2)V_A - V_B - V_B = (s+2)V_A - 2V_B$$

$$0 = (s+1)V_B - V_A + V_B = (s+2)V_B - V_A \Rightarrow V_A = (s+2)V_B$$

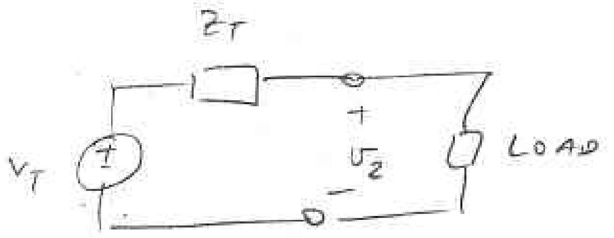
$$\Rightarrow \quad I_S = [(s+2)(s+2)-2]\,V_B = (s^2+4s+2)V_B \Rightarrow \boxed{V_B = \frac{I_S}{s^2+4s+2}}$$



$$(s+2)V_A - V_B - V_B = 0 \Rightarrow \boxed{V_A = \frac{2}{s+2}V_B}$$

$$I = (s+1)V_B - V_A + V_B = (s+2)V_B - V_A = (s+2)V - \frac{2}{(s+2)}V = \frac{s^2+4s+2}{s+2}V$$

$$Y_N = \frac{I}{V} = \frac{s^2+4s+2}{s+2} \qquad \boxed{Z_T = \frac{s+2}{s^2+4s+2}}$$



Let Load = 1Ω

$$\Rightarrow \quad V_2 = \frac{1}{1+Z_T}V_T$$

$$V_2 = \frac{1}{1+\frac{s+2}{s^2+4s+2}}\cdot\frac{I_S}{s^2+4s+2} = \frac{I_S}{s^2+5s+4} \qquad \boxed{\frac{V_2}{I_S} = \frac{1}{s^2+5s+4}} \checkmark$$