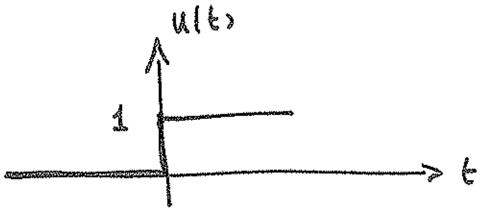
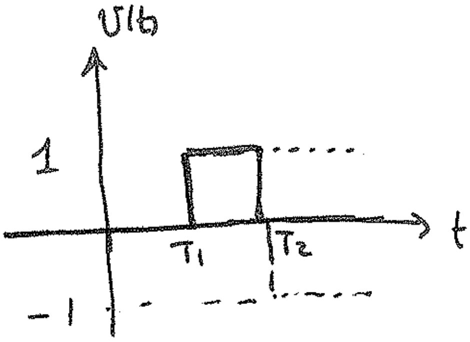
Chapter 5
Signal Waveforms

Unit Step $\quad u(t) = \begin{cases} 0 & t < 0 \\ 1 & t \geq 0 \end{cases}$

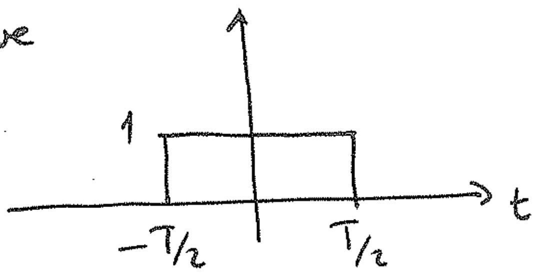


Pulse : $v(t) = u(t - T_1) - u(t - T_2)$
(Gate function)

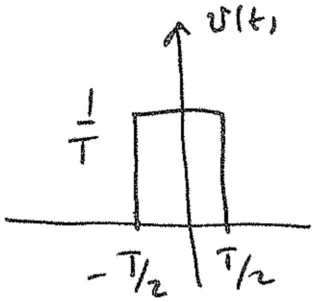


When $T_1 = -T/2 \quad T_2 = T/2$

we have



$$v(t) = \frac{1}{T}\left(u(t + T/2) - u(t - T/2)\right)$$

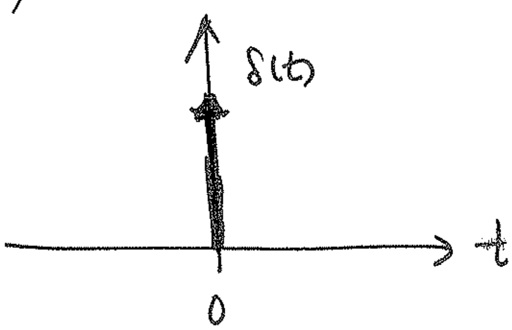


$$\int_{-\infty}^{+\infty} v(t)\, dt = 1$$

impulse

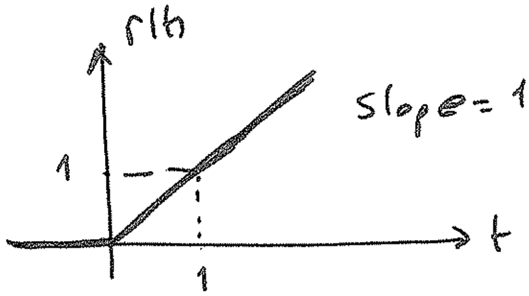
$$\lim_{T \to 0} \frac{1}{T}\left(u(t + T/2) - u(t - T/2)\right) = \delta(t)$$

$$\int_{-\infty}^{t} \delta(t)\, dt = u(t) \quad \text{or} \quad \delta(t) = \frac{du(t)}{dt}$$

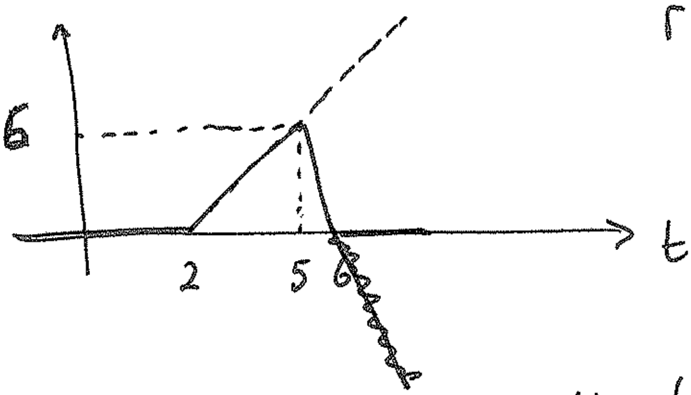


Unit ramp function:

$$r(t) = \begin{cases} 0 & t < 0 \\ t & t \geq 0 \end{cases}$$



Triangular Waveform can be obtained from ramp



$$r(t) = \left(r_1(t) - r_2(t)\right) u(6-t)$$

$$r_1(t) = 2\, r(t-2)$$

$$r_2(t) = 8\, r(t-5)$$

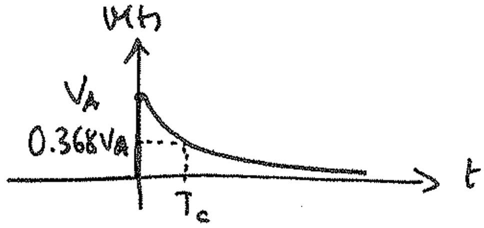
$$r(t) = \left(r_1(t) - r_2(t)\right) \left[u(t-2) - u(t-6)\right]$$

Step - Ramp - Impulse → lead to singularity functions

$$u(t) = \int_{-\infty}^{t} \delta(x)\,dx \qquad r(t) = \int_{-\infty}^{t} u(x)\,dx \qquad \Bigg\| \qquad \delta(t) = \frac{du(t)}{dt} \qquad u(t) = \frac{dr(t)}{dt}$$

Exponential Waveform:

$$v(t) = V_A\, e^{-t/T_c}\, u(t)$$



T_c : time constant

V_A : amplitude

$$\frac{dv(t)}{dt} = -\frac{1}{T_c}\, v(t) \qquad\qquad v(0) = V_A$$

$$\left.\frac{dv(t)}{dt}\right|_{t=0} = -\frac{1}{T_c}\, V_A \qquad\qquad v(T_c) = V_A\, e^{-1} = 0.368\, V_A$$

$$v(5T_c) = V_A\, e^{-5} = 0.0067\, V_A$$

$v(t) \le 1\%$ of $v(0)$ $\qquad t \geq 5T_c$

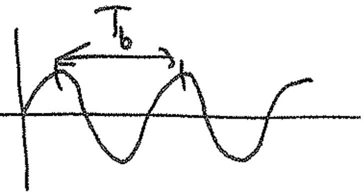
$v(t) \le 2\%$ of $v(0)$ $\qquad t \geq 4T_c$

$$\frac{v(t+\Delta t)}{v(t)} = e^{-\Delta t / T_c}$$ → indep. of amplitude and time

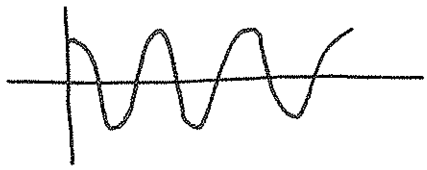
$$T_c = \frac{\Delta t}{\ln\left(\frac{v(t)}{v(t+\Delta t)}\right)}$$ → estimate T_c from graph.

Sinusoidal Signal

$B \sin(\omega_b t)$

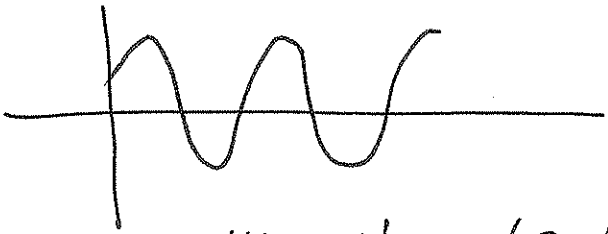


$A \cos(\omega_a t)$



$$\frac{2\pi}{T_b} = \omega_b$$

Shifted signal :



$$V(t) = V_A \cos\left(\frac{2\pi}{T_0}(t - T_s)\right)$$

ϕ : phase shift

$$= V_A \cos\left(\frac{2\pi t}{T_0} - \underbrace{\frac{2\pi T_s}{T_0}}_{\phi}\right)$$

$$= V_A \cos(\omega_0 t - \phi)$$

$$\cos(x+y) = \cos(x)\cos(y) - \sin(x)\sin(y)$$

$$V_A \cos(\omega_0 t - \phi) = V_A \cos(\omega_0 t) \cdot \cos(-\phi) - V_A \sin(\omega_0 t)\sin(-\phi)$$

$$= \underbrace{V_A \cos(-\phi)}\ \cos(\omega_0 t) - \underbrace{V_A \sin(-\phi)}\ \sin(\omega_0 t)$$

$$v(t) = a \cos(\omega_0 t) + b \sin(\omega_0 t)$$

$$V_A = \sqrt{a^2 + b^2}$$

$a = V_A \cos(-\phi)$ $\quad$ $b = -V_A \sin(-\phi)$

$a = V_A \cos(\phi)$ $\quad$ $b = V_A \sin(\phi)$

$$\frac{b}{a} = \tan(\phi)$$

$f_0 = \dfrac{1}{T_0}$: cyclic frequency

$\omega_0 = 2\pi f_0$ angular frequency

Properties of Sinusoids $V_A \cos(\omega_0 t - \phi) = v(t)$

1) Periodic $v(t+T_0) = V_A \cos(\omega_0(t+T_0) - \phi) = V_A \cos(\omega_0 t - \phi + \omega_0 T_0)$

$\omega_0 T_0 = 2\pi$

2) Additive : $a_1 \cos(\omega_0 t) + a_2 \cos(\omega_0 t) = (a_1 + a_2) \cos(\omega_0 t)$

3) Derivative: $\frac{d}{dt} \cos(\omega_0 t + \phi) = - \sin(\omega_0 t + \phi)$

Example :

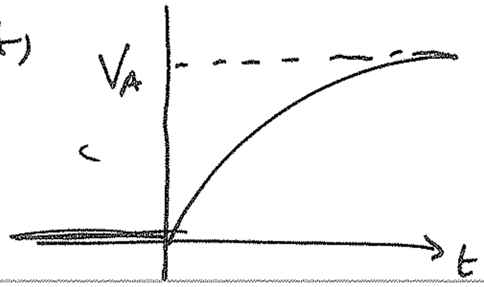
$$V_A = V_m \cos(\omega_0 t)$$
$$V_B = V_m \cos\left(\omega_0 t - \frac{2\pi}{3}\right) = V_m\left[-\frac{1}{2}\cos(\omega_0 t) + \frac{\sqrt{3}}{2}\sin(\omega_0 t)\right]$$
$$V_C = V_m \cos\left(\omega_0 t - \frac{4\pi}{3}\right) = V_m\left[-\frac{1}{2}\cos(\omega_0 t) - \frac{\sqrt{3}}{2}\sin(\omega_0 t)\right]$$

$$V_A + V_B + V_C = 0$$

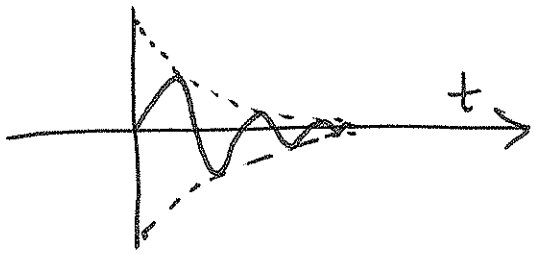
Composite Waveform

Ex

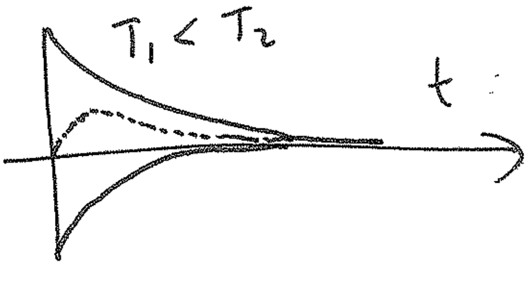
$$v(t) = V_A\left(1 - e^{-t/T_C}\right)u(t)$$



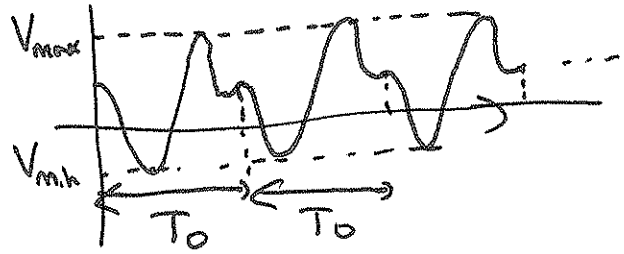
$$v(t) = V_A e^{-t/T_C} \sin(\omega_0 t)\, u(t)$$



$$v(t) = V_A\left[e^{-t/T_1} - e^{-t/T_2}\right]u(t)$$



Periodic Signals properties



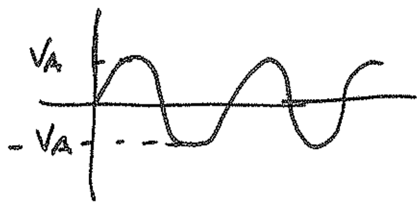
peak value $= V_p = \max\{|V_{max}|, |V_{min}|\}$

peak to peak value $V_{pp} = V_{max} - V_{min}$

Average value $= V_{avg} = \frac{1}{T_0} \int_t^{t+T_0} v(x)\,dx$ $\quad$ t: arbitrary can be 0

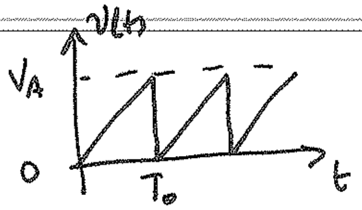
root-mean square value $V_{rms} = \sqrt{\frac{1}{T_0} \int_t^{t+T_0} [v(x)]^2 dx}$

Exercise: Show that



for $v(t) = V_A \sin(\omega_0 t)$

$$V_{RMS} = \frac{V_A}{\sqrt{2}}$$



$\Rightarrow$ $V_{RMS} = \frac{V_A}{\sqrt{3}}$