Q1. Consider the following LTI systems with input–output relationships given below:

System 1: $y_1[n] = \sum_{k=-\infty}^{n+2} x_1[k]$, System 2: $y_2[n] = \sum_{k=-\infty}^{n-2} x_2[k]$.

(a) Find the impulse responses of these systems.

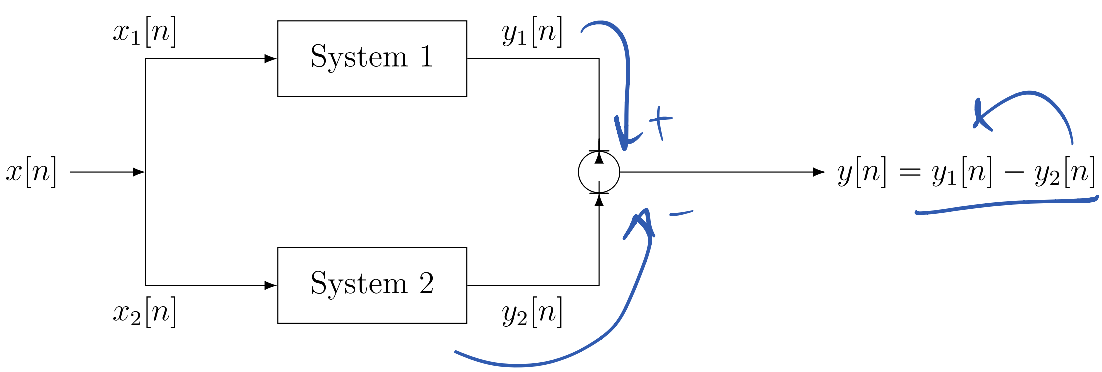
(b) Are these systems

(i) Causal?

(ii) Stable?

Justify your answers.

(c) These two systems are connected in parallel to form the system given below.



(i) Find the impulse response of this system.

(ii) Is this system

(a) Causal?

(b) Stable?

Justify your answers.

a) $y_1[n] = \sum_{k=-\infty}^{n+2} x_1[k]$ $y_2[n] = \sum_{k=-\infty}^{n-2} x_2[k]$

$h_1[n] = \sum_{k=-\infty}^{n+2} \delta[k] \rightarrow \delta[0]=1$ $h_2[n] = u[n-2]$

1

$n+2 \geq 0$ $h_1[n] = 1$

o.w 0

$h_1[n] = u[n+2]$

A system is causal iff $h[n]=0$ $n<0$

b) Causal, Stable?

$\delta[n] \xrightarrow{S_1} u[n+2] \rightarrow$ Not causal

$u[2]$

not bounded

$\delta[n] \xrightarrow{S_2} u[n-2] \rightarrow$ Causal

past values

Stability BIBO

$x[n] = u[n]$

1

$n \geq 0$

$y_1[n] = \underbrace{x[n]}_{u[n]} * \underbrace{h_1[n]}_{u[n+2]}$

$\sum_{k=-\infty}^{n+2} u[k] = n+2+1 = n+3$

$k \geq 0$ 1

$k=0$ $1+1+1$

$k=1$

$\longrightarrow y_1[n] = n+3$ not bounded

$n \rightarrow \infty$

$y_2[n] = \sum_{k=-\infty}^{n-2} u[k] = n-1 \rightarrow$ not bounded

b) $h_1[n] - h_2[n] = h[n]$

$u[n+2] - u[n-2] = h[n] \rightarrow$ Not causal

future value

$x[n] = u[n]$

$n+3-(n-1)$

$=4$

$y[n] = \sum_{k=-\infty}^{n+2} u[k] - \sum_{k=-\infty}^{n-2} u[k]$

0 $n+3$

$$y[n] = \begin{cases} 0 & n<-2 \\ n+3 & -2 \leq n < 2 \\ 4 & 2 \leq n \end{cases}$$

this system is Bounded, so stable

Q2. The impulse response of an LTI system is given as

$$h(t) = e^{-2t}u(t).$$

Let $x(t)$ be an input signal defined as

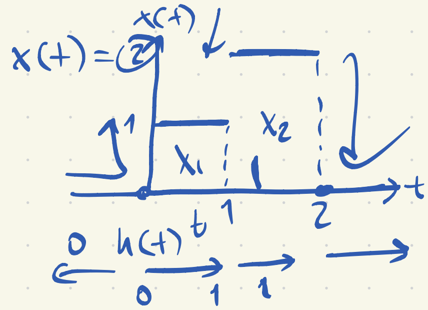
$$x(t) = \begin{cases} 0, & t < 0, \\ 1, & 0 \leq t < 1, \\ 2, & 1 \leq t < 2, \\ 0, & 2 \leq t. \end{cases}$$

(a) Evaluate the output $y(t)$ of this LTI system for the input $x(t)$ using the convolution integral (clearly indicate the regions for which the convolution integrals are valid).

(b) Write $x(t)$ in terms of the superposition of shifted unit step signals.

(c) Find the unit step response $g(t)$ for the given LTI system.

(d) Use $g(t)$ to check your answer in part (a).



$$y(t) = \int_{-\infty}^{\infty} \underbrace{x(\tau)}_{1} \cdot h(t-\tau)\, d\tau$$

$$y(t) = \begin{cases} 0 & t<0 \\ \frac{1}{2}(1-e^{-2t}) & 0<t<1 \\ 1-\frac{e^2+1}{2} \cdot e^{-2t} & 1<t<2 \\ \frac{1}{2}(2e^4-e^2-1) \cdot e^{-2t} & 2<t \end{cases}$$

R1. $1>t>0$

$$\int_0^t e^{-2(t-\tau)} \cdot d\tau =$$

$$\int_0^t e^{-2t} \cdot e^{+2\tau} d\tau = e^{-2t} \underbrace{\int_0^t e^{+2\tau} d\tau}_{\left.\frac{e^{2\tau}}{2}\right|_0^t} = e^{-2t}\left(\frac{e^{2t}}{2} - \frac{1}{2}\right) = \frac{1}{2} - \frac{1}{2} e^{-2t} \checkmark$$

R2 $2>t>1$

$$\int_0^1 e^{-2(t-2)} d\tau + \int_1^t 2 \cdot e^{-2(t-2)} d\tau = 1 - \frac{e^2+1}{2} \cdot e^{-2t}$$

R.3 $$\int_0^1 e^{-2(t-2)} d\tau + \int_1^2 2 \cdot e^{-2(t-2)} d\tau = \frac{1}{2}(2 e^4 - e^2 - 1) \cdot e^{-2t}$$

b) $x(t) = \underbrace{u(t)}_{x_1} + \underbrace{u(t-1)}_{x_2} - \underbrace{2u(t-2)}_{x_3}$ $\quad \tau > t$

c) $g(t) = h(t) * u(t)$ $\quad \displaystyle\int_{-\infty}^{\infty} e^{-2\tau} u(\tau) \cdot u(t-\tau) d\tau$

$u(t)$

$$= \int_0^t e^{-2\tau} d\tau = \frac{1}{2}(1-e^{-2t}) \cdot u(t) = g(t)$$

$g(t-1)$

d) $y = y_1 + y_2 - 2y_3$

$$\left.\begin{aligned} y_1 &= \frac{1}{2}\left(1-e^{-2t}\right)u(t) \\ y_2 &= 1/2\left(1-e^{-2(t-1)}\right)u(t-1) \\ y_3 &= 1/2\left(1-e^{-2(t-2)}\right)u(t-2) \end{aligned}\right\} y$$

Q3. The discrete-time input signal $x[n]$ is passed through a pair of series-connected LTI systems with impulse responses $h_1[n]$ and $h_2[n]$. The signals $y[n]$ and $z[n]$ are obtained at the outputs of System 1 and System 2 as shown in Figure 1.

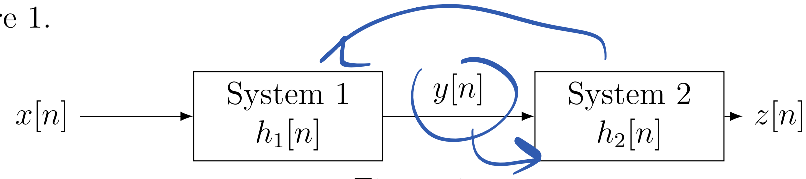


Figure 1

(a) System 1 is known to be causal, and its input–output relation is given by

$$y[n] - \frac{1}{2}y[n-1] = x[n].$$

The system is excited with a unit impulse input $x[n] = \delta[n]$. Find the output $y[n] = h_1[n]$.

(b) The impulse response of System 2 is given as

$$h_2[n] = \delta[n] + 3\delta[n-1].$$

Obtain the input–output relation of System 2, i.e., the relation between the signals $y[n]$ and $z[n]$. $z[n] = y[n] * h_2[n]$

(c) The series connection of the two systems in Figure 1 can equivalently be represented as in Figure 2 as a single LTI system (System 3). Find the impulse response $h_3[n]$.

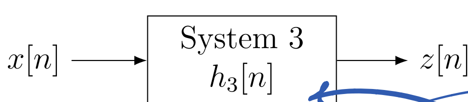


Figure 2

$h_1[n] * h_2[n]$

$y[n] = x_1[n] * h_1[n]$

$z[n] = y[n] * h_2$

$z[n] = x_1[n] * \underbrace{(h_1[n] * h_2[n])}_{h_3[n]}$

a) $y_1[n] - \frac{1}{2} y[n-1] = x[n]$ $\quad \uparrow \delta[n]$

$$\frac{dy(t)}{dt} + y(t) = x(t)$$

$\rightarrow Ce^{rt}$

* Definition of the derivative is the rate of change from one point to another.

$h_1[n] = z^n$ $\quad\quad h[n] = 0 \quad n<0$

$$z^n - \frac{1}{2} z^{n-1} = \delta[n] \quad n \neq 0$$
$$= 0$$

$$h[n] = (1/2)^n u[n]$$

$$z^n\left(1 - \frac{1}{2z}\right) \quad z = 1/2$$

b) $h_2[n] = \delta[n] + 3\delta[n-1] \rightarrow z[n] = y[n] + 3y[n-1]$

$$x[n] * \delta[n] = x[n]$$
$$x[n] * \delta[n-1] = x[n-1]$$

c) $h_3[n] = h_1[n] * h_2[n]$

$$= (1/2)^n u[n] * (\delta[n] + 3\delta[n-1])$$

$$h_3[n] = (1/2)^n u[n] + 3(1/2)^{n-1} u(n-1)$$

Q4.

(a) Given that $x[n]$ is an odd sequence, prove that

$$\sum_{n=-\infty}^{\infty} x[n] = 0.$$

(b) The input–output relation of a system is given as

$$y(t) = \mathrm{Re}\{e^{j2\pi t} x(t)\},$$

where $x(t)$ can be complex–valued.

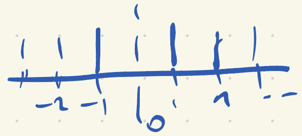
(i) Does this system satisfy the additivity property? Justify. Does this system satisfy the homogeneity (or scaling) property for the complex scalar and input case? Justify. Is this system linear?

(ii) Is this system time–invariant? Justify.

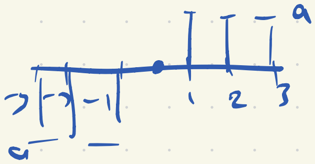
(iii) Is this system stable? Justify.

(iv) Is this system causal? Justify.

a) Even: $x[n] = x[-n]$



Odd: $\underline{x[n] = -x[-n]}$



$x[0] = -x[-0]$

$x[0] = -x[0]$ $\underline{x[0]=0}$

$$\sum_{k=-\infty}^{\infty} x[k] = 0, \quad = \sum_{k=-\infty}^{-1} x[k] + 0 + \sum_{k=1}^{\infty} x[k] \quad + = 0$$

$$\sum_{k=-\infty}^{-1} -x[-k] \rightarrow \sum_{m=1}^{\infty} -x[m]$$

$-k = m$

b) $\underline{\text{additivity}}$

$x_1 + x_2$

$\underbrace{x_1(t) + x_2(t)}_{x(t)} \rightarrow \underline{y_1(t)} + \underline{y_2(t)}$

$$y(t) = Re\{e^{j2\pi t} x(t)\}$$

$$Re\{e^{j2\pi t}(x_1(t) + x_2(t))\}$$

$$Re\{e^{j2\pi t}x_1(t) + e^{j2\pi t}x_2(t)\} = \underbrace{Re\{e^{j2\pi t}x_1(t)\}}_{y_1(t)} + \underbrace{Re\{e^{j2\pi t}x_2(t)\}}_{y_2(t)} \checkmark$$

Re $\cancel{j}$ R $\cancel{+j}$

$jj \rightarrow Re$

$\underline{\text{homogeneity}}$

$$a_c x(t) \longrightarrow a_c \cdot y(t) \longrightarrow Re\{e^{j2\pi t} \cdot a_c \cdot x(t)\}$$

$$\cancel{a_R + a_j}\, y(t) = a_R \cancel{+a_j}$$

for complex a homogeneity does not hold

$\rightarrow$ Also not linear for complex $\underline{a}$

$\underline{\text{time-invariance}}$

$$x(t-t_0) \rightarrow y(t-t_0)$$

$$Re\{e^{j2\pi t} \cdot x(t-t_0)\} \neq Re\{e^{j2\pi(t-t_0)} \cdot x(t-t_0)\} \rightarrow \text{not time-invariant}$$

Stability BIBO

$|y(t)| < c$

$\rightarrow \infty$

$\underbrace{|Re}_{a}\{\underbrace{e^{j2\pi t} x(t)}_{|a+ib|}\} < |e^{j2\pi t} x(t)| = \underbrace{|e^{j2\pi t}|}_{1} \underbrace{|x(t)|}_{B}$

$< |x(t)|$

y(t) is bounded

↳ stable ✓

Causality

$y(t_0) = Re\{e^{j2\pi t_0} x(t_0)\} \rightarrow$ causal

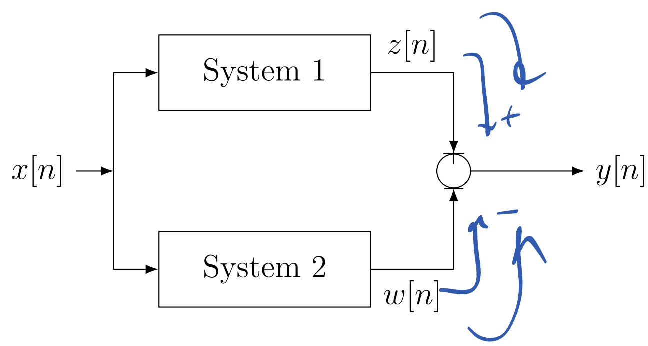
Q5. System 1 (called a "moving average filter") is an LTI system with the following input–output relation:

$$z[n] = \frac{1}{8} \sum_{m=n-7}^{n} x[m].$$

System 2 is another LTI system with impulse response

$$h_2[n] = \left(-\tfrac{1}{2}\right)^n u[n-1].$$

The overall system is shown below.



Answer the following questions about System 1, System 2 and the overall system above.

(a) Is System 2 memoryless, causal, and/or stable? Justify your answers.

(b) If $x[n] = \cos(\pi n/4) + \delta[n-1]$, find $w[n]$.

(c) Find the impulse response $h_1[n]$ of System 1, and express it using unit step signals.

(d) Find the impulse response of the overall system whose input is $x[n]$ and output is $y[n]$. Justify your answer.

a) $h[n] = (-1/2)^n u[n-1]$ → not memoryless

memory → it is causal

$n<1$

$h[n] = 0 \quad n<0$

for checking stability, you can look to the absolute summation of the impulse response.

$$\sum_{k=-\infty}^{\infty} |(-1/2)^k \underbrace{u[k-1]}_{k\geq 1}| = \sum_{k=1}^{\infty} |(-1/2)^k| \Rightarrow \text{it will converge} \Rightarrow \text{it is summable} ✓$$

$$\frac{r}{1-r}$$

$H(j\omega)$

$$e^{st} \to \boxed{LTI} \to H(s).e^{st}$$

b) $x[n] = \cos(\pi/4\, n) + \delta[n-1]$

↳ write convolution and calculate

↳ we can realize that $\dfrac{e^{j\pi/4 n} + e^{-j\pi/4 n}}{2}$

$$H_2(j\omega) = \sum_{n=-\infty}^{\infty} h_2[n] e^{-j\omega n} = \underbrace{\sum_{n=1}^{\infty} (-1/2)^n e^{-j\omega n}}_{\frac{r}{1-r}} = \frac{-\frac{1}{2} e^{-j\omega}}{1+\frac{1}{2}e^{-j\omega}} = H(j\omega)$$

$\pi/4$

$$w[n] = H_2(j\omega).\cos(\pi/4\, n) + (-1/2)^n \underbrace{u[n-1] * \delta[n-1]}_{u[n-2]}$$

$$= H_2(j\pi/4) . \left[\frac{e^{j\pi/4 n} + e^{-j\pi/4 n}}{2}\right]$$

$$= \frac{-1/2\, e^{-j\pi/4}}{1+1/2\, e^{-j\pi/4}} \cdot \left[\frac{e^{j\pi/4 n} + e^{-j\pi/4 n}}{2}\right] +$$

c) $h_1[n] = \frac{1}{8} \sum_{m=n-7}^{n} \underbrace{\delta[m]}_{1}$ $\quad 7 \geq n \geq 0 \quad h_1[n] = 1/8$

o.w $\quad h_1[n] = 0$

$$\frac{1}{8}\left\{u[n] - u[n-8]\right\} = h_1[n]$$

d) $h_1[n] - h_2[n]$

$$\frac{1}{8}\left[u[n] - u[n-8]\right] - \left[(-1/2)^n u[n-1]\right] = h_3[n]$$ ✓

Q6. Consider a discrete-time (DT) LTI system whose impulse response is given by

$$h[n] = \begin{cases} \dfrac{1}{n}, & n \geq 1, \\ 0, & \text{otherwise.} \end{cases}$$

(a) By justifying briefly, determine whether this system is

(i) Stable,

(ii) Causal,

(iii) Memoryless.

Hint: Recall that the series $\sum_{n=1}^{\infty} \frac{1}{n^k}$ converges for $k \geq 2$ and diverges for $k = 1$.

(b) Find the output $y[n]$ when the following input signal $x[n]$ is given as input to the system, where α, β, γ are nonzero real numbers. Express $y[n]$ as a function of n, by properly indicating the intervals:

$$x[n] = \begin{cases} \alpha, & n = -1, \\ \beta, & n = 0, \\ \gamma, & n = 1, \\ 0, & \text{otherwise.} \end{cases}$$

(c) Now consider that the system is excited by the constant (and thus bounded) input signal $x[n] = \alpha$, where α is a nonzero real number. Find the output of the system. Comment on the result based on your answer to part (a).

$$h[n] = \begin{cases} 1/n & \text{if } n \geq 1 \\ 0 & \text{otherwise} \end{cases}$$

$\rightarrow \underbrace{y[n] = h[n] * x[n]}_{BO}$

$$\sum_{k=-\infty}^{\infty} h[k]\, x[n-k]$$

$\rightarrow$ Absolute summability of $h[n]$?

$$|y[n]| = \left| \sum_{k=-\infty}^{\infty} h[k] x[n-k] \right| \leq \sum_{k=-\infty}^{\infty} |h[k]| \cdot \underbrace{|x[n-k]|}_{M}$$

$$\leq \underbrace{\sum_{k=-\infty}^{\infty} |h[k]| M}_{\text{this must be bounded}}$$

$\sum_{n=1}^{\infty} \frac{1}{n} \rightarrow$ hint says it diverges

causal $h[n] = 0$ for $n < 0$

Memoryless systems $h[n] = c \cdot \delta[n]$?

$$\sum_{k=-\infty}^{\infty} \underbrace{h[k]}_{\delta[n]} x[n-k] \rightarrow x[n]$$

b) $y[n] = \sum_{k=-\infty}^{\infty} x[k]\, h[n-k] = x[-1]h[n+1] + x[0]h[n] + x(1)h[n-1]$

$k = -1$
$k = 0$
$k = 1$

$\alpha \cdot \frac{1}{n+1}$ if $n \geq 0$

$\beta \cdot \frac{1}{n}$ if $n \geq 1$

$\gamma \cdot \frac{1}{n-1}$ if $n \geq 2$

$$y = \begin{cases} 0 & \text{if } n < 0 \\ \alpha/(n+1) & \text{if } 1 \geq n \geq 0 \\ (\alpha/(n+1)) + (\beta/n) & 2 \geq n \geq 1 \\ (\alpha/(n+1)) + (\beta/n) + (\gamma/(n-1)) & n \geq 2 \end{cases}$$ ✓

c) $x[n] = \alpha$

$$y[n] = \sum_{k=-\infty}^{\infty} x[k]\, h[n-k] = \alpha \sum_{k=-\infty}^{\infty} h[n-k] = \alpha \cdot \underbrace{\sum_{m=1}^{\infty} \frac{1}{m}}_{\text{Diverges to } \infty}$$

$h[m] \neq 0 \quad m \geq 1$

Q7. Consider the cascade (series) connection of two LTI systems, whose impulse responses are $h_1(t) = e^{-\alpha t}u(t)$ and

$$h_2(t) = \delta(t) - \delta(t - \alpha),$$

where α is a real constant, $0 < \alpha < 1$.

(a) Check whether the overall system is *stable* or not. Check its *causality*. Briefly explain your reasonings.

(b) A continuous-time signal is given as

$$x(t) = \cos(\beta t) + e^{jm\beta t},$$

where $m > 1$ is an integer and $\beta > 0$ is a real constant. In terms of β and m, find the fundamental periods T_1 and T_2 of $\cos(\beta t)$ and $e^{jm\beta t}$, respectively.

(c) Find the continuous-time Fourier series coefficients of $x(t)$.

(d) When the system with the impulse response $h_1(t)$ is excited by the input signal e^{st} (where $s \in \mathbb{C}$), the output signal is observed to be

$$\frac{1}{s + \alpha} e^{st}. \quad H(j\omega)$$

solve later

Evaluate the output signal $y(t)$ for the overall cascaded system corresponding to the input signal $x(t)$ in part (b).

$h(t) = h_1(t) * h_2(t)$ ✓ $\rightarrow$ $\int_{-\infty}^{\infty} |h(t)|\,dt = \int_{-\infty}^{\infty} e^{-\alpha t} u(t)\,dt - \int_{-\infty}^{\infty} e^{-\alpha(t-\alpha)} \cdot u(t-\alpha)\,dt$

$h(t) = h_1(t) - h_1(t-\alpha)$

$= e^{-\alpha t} u(t) - e^{-\alpha(t-\alpha)} u(t-\alpha)$

$\int_{0}^{\infty} e^{-\alpha t} \cdot dt - \int_{\alpha}^{\infty} e^{-\alpha t} \cdot e^{\alpha^2} dt$

$h(t) = 0 \qquad t<0$

$\left. \frac{e^{-\alpha t}}{-\alpha} \right|_{0}^{\infty} - \left[e^{\alpha^2} \left. \frac{e^{-\alpha t}}{-\alpha} \right|_{\alpha}^{\infty} \right]$

$y(t) = x(t) * h(t)$

$\int_{-\infty}^{\infty} x(\tau) h(t-\tau)\,dt$

they will bring future dependence

$0 + \frac{1}{\alpha} - \left[e^{\alpha^2} \left[0 + \frac{1}{\alpha} \right] \right]$

$\frac{1}{\alpha} - \frac{e^{\alpha^2}}{\alpha} \rightarrow \underbrace{\frac{1}{\alpha}\left[1 - e^{\alpha^2}\right]}_{\text{constant}}$

it is absolutely summable $\rightarrow$ stable

$h(t) = \underbrace{e^{-\alpha t} u(t)}_{0 \checkmark\ t<0} - e^{-\alpha} \ldots \underbrace{u(t-\alpha)}_{t<\alpha} \rightarrow$ causal

0 ✓

b) $\cos(\beta t) \rightarrow \frac{2\pi}{\beta} = T_1$

T_1 is the overall T of the expression

$e^{jm\beta t} \rightarrow m \cdot \frac{2\pi}{m\beta} = T_2$

c) $\cos(\beta t) = \frac{e^{j\beta t} + e^{-j\beta t}}{2}$ $\quad \omega_0 \quad \omega_0$

$x(t) = \underbrace{\frac{1}{2}}_{a_1} e^{j\omega_0 t} + \underbrace{\frac{1}{2}}_{a_{-1}} e^{-j\omega_0 t} + \underbrace{1}_{a_m} . e^{jm\omega_0 t}$

$$\boxed{\begin{array}{l} a_1 = 1/2 = a_{-1} \\ a_m = 1 \\ a_k = 0, \quad k \neq 1, -1, m \end{array}}$$

d) $x(t) = \frac{1}{2}e^{j\omega_0 t} + \frac{1}{2}e^{-j\omega_0 t} + e^{jm\omega_0 t}$

$y_1(t) = H(j\omega_0) \cdot \frac{1}{2}e^{j\omega_0 t} + H(j\omega_0) \cdot \frac{1}{2}e^{-j\omega_0 t} + H(jm\omega_0) \cdot e^{jm\omega_0 t}$

$H(j\omega) \xleftarrow{\text{tedious}} h(t)$ no!

$e^{st} \to \boxed{} \to \underbrace{H(s)}_{\frac{1}{s+\alpha}} e^{st}$ $\quad H(j\omega) = \frac{1}{j\omega + \alpha}$

$y_1(t) = \frac{1}{\underbrace{j\omega_0}_{\beta} + \alpha} \cdot \frac{1}{2}e^{j\omega_0 t} + \frac{1}{\underbrace{-j\omega_0}_{\beta} + \alpha} \frac{1}{2}e^{-j\omega_0 t} + \frac{1}{\underbrace{jm\omega_0}_{\beta} + \alpha} e^{jm\omega_0 t}$

$y(t) = \underbrace{h_2(t)}_{\delta(t) - \delta(t-\alpha)} * y_1(t) = y_1(t) - y_1(t-2)$

$= \frac{1}{j\beta + \alpha} \frac{1}{2}\underbrace{\left[e^{j\beta t} - e^{j\beta(t-\alpha)}\right]}_{e^{j\beta t} - e^{j\beta t} \cdot e^{-j\beta\alpha}} = e^{j\beta t}(1 - e^{-j\beta 2})$

$= \frac{1}{2} \frac{e^{j\beta t}}{j\beta + \alpha}\left[1 - e^{-j\beta\alpha}\right] + \frac{1}{2} \frac{e^{-j\beta t}}{-j\beta + \alpha}\left[1 - e^{j\beta\alpha}\right] + \frac{e^{jm\beta t}}{jm\beta + \alpha}(1 - e^{-j\beta\alpha})$

Q8) Consider the continuous-time LTI system described by the differential equation

$$\frac{dy(t)}{dt} + 2y(t) = 2x(t), \qquad -\infty < t < \infty,$$

with zero initial condition $y(0^-) = 0$. $h(0^-) = 0$

(a) Find the impulse response $h(t)$ of the system.

(b) Let $x(t) = u(t)$. Find the output $y(t)$. Compute

$$E_x = \int_{-\infty}^{\infty} |x(t)|^2 \, dt, \qquad E_y = \int_{-\infty}^{\infty} |y(t)|^2 \, dt,$$

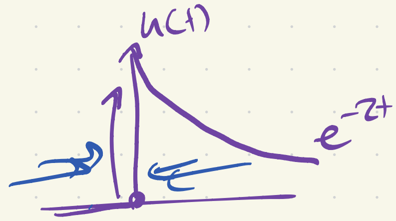
and the average powers P_x and P_y. Classify $x(t)$ and $y(t)$ as *energy* or *power* signals.

(c) Is this system invertible? If yes, derive a differential equation that the inverse system must satisfy (i.e., an equation that recovers $x(t)$ from $y(t)$). State whether this inverse system is ~~causal and BIBO stable, and briefly justify your answers.~~

$$\underbrace{\frac{dh(t)}{dt}}_{} + \underbrace{2h(t)}_{} = 2\delta(t)$$

$t \neq 0 \quad h(t) = e^{-rt} \qquad -re^{-rt} + 2e^{-rt} = 0$

$h(t) = C\,e^{-2t} \qquad r = 2$



$h(t) = \delta(t)$

$$\lim_{\epsilon \to 0} \int_{-\epsilon}^{\epsilon} \frac{dh(t)}{\cancel{dt}}\,\cancel{dt} + \int_{-\epsilon}^{\epsilon} 2h(t) = \underbrace{\int_{-\epsilon}^{\epsilon} 2\delta(t)\,dt}_{2}$$

$h(t) = 2e^{-2t}u(t)$

$$\underbrace{h(0^+)}_{2} - h(0^-) = 2$$

b) $y(t) = x(t) * h(t)$

$= h(t) * u(t) = \int_{-\infty}^{\infty} h(\tau)\,\underbrace{u(t-\tau)}_{\tau < t}\,d\tau = \int_0^t 2e^{-2\tau}\,d\tau$

$= -e^{-2\tau}\Big|_0^t = -e^{-2t} + 1$

$y(t) = (1 - e^{-2t})\,u(t)$

$E_x = \int_{-\infty}^{\infty} |x(t)|^2 dt = \int_0^{\infty} 1^2 dt \to E_x \to \infty$

$E_y = \int_0^{\infty} (1 - e^{-2t})^2 dt$

$E_y \to \infty$

$P_x = \lim_{T \to \infty} \frac{1}{2T} \int_{-T}^{T} \underbrace{|x(t)|^2}_{u(t)} dt = \frac{1}{2T} \underbrace{\int_0^T 1\,dt}_{T}$

$$P_x = \frac{1}{2}$$

$P_y = \lim_{T \to \infty} \int_0^T (1 - e^{-2t})^2 dt$

$\lim_{T \to \infty} \frac{1}{2T} \int_0^T (1 - 2e^{-2t} + e^{-4t})\,dt$

$\frac{1}{2T}\left[\,T + e^{-2t} - \frac{e^{-4t}}{4}\right]_0^T$

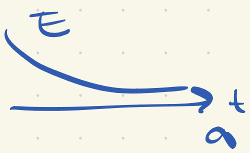
$\lim_{T \to \infty} \frac{1}{2T}\left[T + e^{-2T} - 1 - \frac{e^{-4T}}{4} - 1\right]$

$P_y = \frac{1}{2}$

Energy signals $\quad E = C$

$P = 0 \qquad \frac{E}{T}$

$\frac{E}{\infty}$



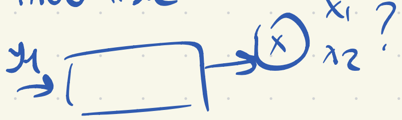
Power signals $\quad P = C \qquad \frac{E}{T}$

$E = \infty$

c) system → invertible (one-to-one

$x_1 \to y_1$

$x_2 \to y_1$



$\tilde{x} = \underbrace{x_1(t)} - \underbrace{x_2(t)}$ $\quad \underbrace{\tilde{y}(t) = 0}$

$\underbrace{\frac{d\tilde{y}(t)}{dt}}_{0} + \underbrace{2\tilde{y}(t)}_{0} = \underbrace{2\tilde{x}(t)}_{0}$ it is one to one (invertible)

$$\frac{1}{2}\frac{dy(t)}{dt} + \frac{2y(t)}{2} = \frac{2x(t)}{2}$$ → Inverse system

$$\int \sum \frac{d^k}{dt} y(t) = \sum \underbrace{\frac{d^m x(t)}{dt}}_{x(t)}$$

Q9) Consider the discrete-time system with input $x[n]$ and output $y[n]$ related by

$$y[n] = x[n] - x[n-1] + x[n-2], \qquad -\infty < n < \infty.$$

For this system, determine whether it is

- linear,
- time invariant,
- causal,
- memoryless,
- BIBO stable,
- ~~invertible (on the class of absolutely summable input~~ sequences).

For each property, answer *yes* or *no* and justify briefly. If you claim that the system is invertible, explicitly give a relation that recovers $x[n]$ from $y[n]$.

$y[n] = x[n] - x[n-1] + x[n-2]$

- linearity $\rightarrow$ linearity holds

$a x_1[n] + b x_2[n] \Rightarrow a y_1 + b y_2$

$$\underbrace{a x_1[n]}_{a y_1} + \underbrace{b x_2[n]}_{b y_2} - a x[n-1] - b x_2[n-1] + a x_1[n-2] + b x_2[n-2]$$

- time invariance

$x[n-n_0] \Rightarrow y[n-n_0]$

$x[n-n_0] - x[n-n_0-1] + x[n-n_0-2]$

- Causality

$y[n] = \checkmark \quad \checkmark \quad \checkmark \rightarrow$ causal

BIBO stability

$y[n] = \underbrace{x[n]}_{BI} * h[n]$

$$\sum_{k=-\infty}^{\infty} h[k] \underbrace{x[n-k]}_{m} < \sum_{k=-\infty}^{\infty} |h[k] \underbrace{x[n-k]}_{m}|$$

∞

$$\sum_{k=-\infty}^{\infty} |h[k]|$$

$$\sum_{k=-\infty}^{\infty} |h[n]| = \sum_{k=-\infty}^{\infty} |\delta[n]| + |\delta[n-1]| + |\delta[n-2]| \leq 3 < \infty$$

1 1 1

BIBO stable