BILKENT UNIVERSITY
Electrical and Electronics Engineering Department

EEE–321: Signals and Systems

Lab Assignment 2

Instructions

Please carefully study this assignment before coming to the laboratory. You may begin working on it or even complete it if you wish, but you do not have to. There will be a short quiz in the lab session to test your understanding of the content of the assignment. Within one week, complete the assignment in the form of a report and upload to Moodle. Some of the exercises will be performed by hand and others by using Matlab. What you should include in your report is indicated within the exercises. Also, after you upload your report, there will be another quiz to test your understanding of what you did in the report, given in the next lab session.

1 Part 1

Consider a signal defined as

$$x_s(t) = \sum_{i=1}^{M} A_i \, \mathrm{e}^{\mathrm{j}\omega_i t}, \tag{1}$$

where $A_i \in \mathbb{C}$ are complex amplitudes (phase is included in A_i) and $\omega_i \in \mathbb{R}$ are angular frequencies.

In this part, you will write a Matlab function (in the form of a Matlab function file) that computes $x_s(t)$ on a vector of time instants using a *single loop* over the index i and vectorized operations over t. Your function should look like

Listing 1: Function header for Part 1 (revised)

```
function xs = SUMCS(t, A, omega)
```

Inputs/Outputs

- `t`: $1 \times N$ or $N \times 1$ vector of time instants over which $x_s(t)$ is computed (your code must work for either row or column input).
- `A`: $1 \times M$ complex-valued vector; the ith element is A_i.
- `omega`: $1 \times M$ real vector; the ith element is ω_i.
- `xs`: complex vector of the same shape as `t`, containing $x_s(t)$ samples.

Important constraints. Use MATLAB's `exp` function and only a single `for`-loop over the index i of (1). **Do not** loop over time points. Treat `t` as a vector and rely on broadcasting/vectorization over `t`. Assume valid, consistent input sizes and reserve the letter `j` for $\sqrt{-1}$.

Hint. You can detect whether `t` is row or column and ensure `xs` matches that shape (use `isrow`, `reshape`).

After writing your function, define the time grid and sizes using your ID number digits:

$$D_7 = \mathrm{mod}(\text{your ID}, 7), \qquad M = 9 + D_7.$$

Take `t = [-0.6:0.0008:1.4]`. Create a random `1xM` complex amplitude vector `A`, where the real and imaginary parts are in the range $[-1.2,\ 2.4]$. Create a real frequency vector `omega` with entries in $[\frac{\pi}{6},\ \frac{5\pi}{4}]$ (radians/s). Use `rand` and simple affine mapping.

Using your function, compute `xs`. Then:

1. Plot the real and imaginary parts of `xs` versus `t` (in separate subplots or separate figures).
2. Plot the magnitude and *unwrapped* phase of `xs` versus `t` (use `unwrap(angle(xs))`).
3. Compute and report the sample energy $\sum_n |x_s(t_n)|^2 \Delta t$ with Δt equal to the step of `t`.

Include your code and plots in the report. Label all axes and add informative titles/captions.

2 Part 2

Consider a periodic signal $x(t)$ with period T, i.e., $x(t) = x(t+T)$ for all t. Its Fourier series expansion is

$$x(t) = \sum_{k=-\infty}^{\infty} X_k\, e^{j\frac{2\pi k}{T}t}, \qquad X_k = \frac{1}{T}\int_{-T/2}^{T/2} x(t)\, e^{-j\frac{2\pi k}{T}t}\, dt. \tag{2}$$

Within one period $t \in [-T/2, T/2)$, let

$$x(t) = \begin{cases} 1 - 3\left(\dfrac{2t}{W}\right)^2 + \left(\dfrac{2t}{W}\right)^4, & \text{if } |t| < \dfrac{W}{2}, \\ 0, & \text{otherwise}, \end{cases} \tag{3}$$

with the constraint $0 < W < T$. Note that (3) is an even, smooth quartic pulse on $(-W/2, W/2)$ and zero elsewhere in the period.

1. Take $T = 2.4$ and $W = 0.9$ and sketch (by hand) $x(t)$ over $-1.5T < t < 1.5T$. Clearly indicate amplitudes and key points ($\pm W/2$, $\pm T/2$). Include the sketch in your report.
2. Determine the Fourier series coefficients X_k for *generic* T and W by evaluating (2). Simplify as much as possible (you may use integration by parts and the even symmetry to reduce the integral to $[0, W/2]$). Include your derivation.

3 Part 3

In this part, you will write a MATLAB function that synthesizes the truncated Fourier series

$$\tilde{x}(t) = \sum_{k=-K}^{K} X_k\, e^{j\frac{2\pi k}{T}t}, \tag{4}$$

for the signal in (3). Your function should look like

Listing 2: Function header for Part 3 (revised)

```
function xt = FSWave(t, K, T, W)
```

Where

- `t` is the time grid over which $\tilde{x}(t)$ is computed,
- `xt` is the vector of values of $\tilde{x}(t)$ computed over `t`,
- `K`, `T`, `W` correspond to (4) and (3).

Use your `SUMCS` from Part 1 to accumulate complex exponentials, and only a single `for`-loop over k in (4). (You will need to compute X_k inside the code, either using the closed-form you derived or a numerically stable approximation.)

Let $D_{13} = \mathrm{mod}(\text{your ID}, 13)$ and $D_9 = \mathrm{mod}(\text{your ID}, 9)$. Use

$$T = 2.4, \qquad W = 0.9, \qquad K = 32 + D_{13}, \qquad t = [-5 : 0.001 : 5].$$

Compute `xt`. Then:

1. Plot $\Re\{\texttt{xt}\}$ and $\Im\{\texttt{xt}\}$ versus `t` (separately). Report the maximum absolute value of the imaginary part and comment on why it is not exactly zero for a real signal (numerical reasons).
2. Overlay one period of the *ideal* $x(t)$ from (3) on top of $\Re\{\tilde{x}(t)\}$ over the interval $[-T/2, T/2]$ to visually assess approximation quality.

Resolution study. Again take $T = 2.4$, $W = 0.9$, $t = [-5 : 0.001 : 5]$ and produce five plots for

$$K = 3 + D_9, \quad K = 10 + D_9, \quad K = 20 + D_9, \quad K = 60 + D_9, \quad K = 120 + D_9.$$

Display only $\Re\{\texttt{xt}\}$. What changes do you observe as K grows? Comment on approximation accuracy and the behavior near the edges of the nonzero support (e.g., ringing/overshoot).

Numerical note. Repeat the $K = 60 + D_9$ case with a finer grid `t = [-5:0.0005:5]` and compare $\|\Im\{\texttt{xt}\}\|_\infty$ with the previous run. Comment briefly on floating-point/phase-accumulation effects.

4 Part 4

In this part, you will make small changes in the function `FSWave` that you wrote in Part 3, and you will observe the resulting changes. Consider the following equation:

$$y(t) = \sum_{k=-K}^{K} Y_k \, e^{j\frac{2\pi k}{T}t}. \tag{5}$$

(a) Suppose $Y_k = X_{-k}$. What is the required change in your `FSWave` function to compute $y(t)$ from the X_ks? Take `T=2`, `W=1`, `K=25 + D11` and `t = [-5:0.001:5]`. Run your code, and compute and plot $y(t)$ versus t (only the real part). Include the plot in your report. What is the effect of the operation that we performed? Include the answer in your report.

(b) Suppose $Y_k = X_k\, \mathrm{e}^{-\mathrm{j}\frac{2\pi k}{T} t_0}$ where t_0 is a new parameter. What is the required change in your `FSWave` function to compute $y(t)$ from the X_ks? Take `T=2`, `W=1`, `K=25 + D11`, `t = [-5:0.001:5]` and `t0 = 0.8`. Run your code, and compute and plot $y(t)$ versus t (only the real part). Include the plot in your report. What is the effect of the operation that we performed? Include the answer in your report.

(c) Suppose $Y_k = \mathrm{j}\,\dfrac{k\,2\pi}{T}\,X_k$. What is the required change in your `FSWave` function to compute $y(t)$ from the X_ks? Take `T=2`, `W=1`, `K=25 + D11` and `t = -5:0.001:5`. Run your code, and compute and plot $y(t)$ versus t (only the real part). Include the plot in your report. What is the effect of the operation that we performed? Include the answer in your report.

(d) Suppose

$$Y_k = \begin{cases} X_{K+1-k}, & k = 1, 2, \ldots, K, \\ X_0, & k = 0, \\ X_{-(K+1+k)}, & k = -K, -K+1, -K+2, \ldots, -1. \end{cases} \tag{6}$$

What is the required change in your `FSWave` function to compute $y(t)$ from the X_ks? Take `T=2`, `W=1`, `K=25 + D11` and `t = [-5:0.001:5]`. Run your code, and compute and plot $y(t)$ versus t (only the real part). Include the plot in your report. What is the effect of the operation that we performed? Include the answer in your report.