Bilkent University
Electrical and Electronics Engineering Department
Fall 2025-26

EEE - 321: Signals and Systems

Lab Assignment 3

Important note: Due to a mismatch between 0.5 s digit durations and the 8192-length ω grid, we updated DTMF digit durations to 0.25 s throughout. (If you already implemented 0.5 s digit durations by extending the ω grid accordingly, that is acceptable as long as you clearly state in your report which approach you used.)

Please carefully study this assignment before coming to the laboratory. You may begin working on it or even complete it if you wish, but you do not have to. There will be a short quiz in the lab session to test your understanding of the content of the assignment. Within one week, complete the assignment in the form of a report and upload to Moodle. Some of the exercises will be performed by hand and others by using Matlab. What you should include in your report is indicated within the exercises. Also, after you upload your report, there will be another quiz to test your understanding of what you did in the report, given in the next lab session.

Part 1

In this lab, you are going to see two common daily life applications in which the concepts you learned in the signals and systems course are used. In this part, you will see the first of these applications: transmission and detection of Dual Tone Multi Frequency (DTMF) signals. In Part 2, you will see the second application: cancellation of the echoes from a sound signal.

Dual tone multi frequency (DTMF) is the name of the standard technique used over analog telephone lines to transmit and receive the information about the dialed phone number.

Table 1: DTMF frequencies

	1209 Hz	**1336 Hz**	**1477 Hz**	**1633 Hz**
697 Hz	1	2	3	A
770 Hz	4	5	6	B
852 Hz	7	8	9	C
941 Hz	*	0	#	D

The interpretation of the table is as follows: Suppose you press the button for 5 on your phone for 0.25 seconds. Then, the DTMF transmitter on your phone sends the following signal

$$x(t) = \begin{cases} \cos(2\pi 770t) + \cos(2\pi 1336t) & \text{for } 0 \leq t \leq 0.25, \\ 0 & \text{otherwise} \end{cases} \tag{1}$$

Or if you press 9, the transmitted signal becomes

$$x(t) = \begin{cases} \cos(2\pi 852t) + \cos(2\pi 1477t) & \text{for } 0 \leq t \leq 0.25, \\ 0 & \text{otherwise} \end{cases} \tag{2}$$

At the receiver (that is the telephone exchange - telefon santrali), the DTMF receiver examines the incoming signal, tries to understand which frequencies were transmitted, so tries to decide which number is dialed.

1.1 DTMF Transmitter

In this part, you will write a function that prepares the analog signal to be transmitted when a phone number containing only numerical digits 0,1,...,9 is dialed. Assume that for each button, the duration of the transmitted signal is only 0.25 seconds. Thus, for instance, if the dialed number is 2016, you should prepare the following signal:

$$x(t) = \begin{cases} \cos(2\pi 852t) + \cos(2\pi 1477t) & \text{for } 0 \leq t \leq 0.25, \\ \cos(2\pi 770t) + \cos(2\pi 1209t) & \text{for } 0.25 \leq t \leq 0.5, \\ \cos(2\pi 852t) + \cos(2\pi 1209t) & \text{for } 0.5 \leq t \leq 0.75, \\ \cos(2\pi 697t) + \cos(2\pi 1336t) & \text{for } 0.75 \leq t \leq 1, \\ 0 & \text{otherwise} \end{cases} \tag{3}$$

Note that if N digits are dialed, the duration of the final signal is 0.25N. Of course, in Matlab we can only compute the samples of $x(t)$. Your code should compute the samples within $0 \leq t \leq 0.25N$ using the sampling period $T_s = 1/8192$. Your function should look like `function [x]=DTMFTRA(Number)` where

- `Number` of size $1 \times N$ contains the phone number that is dialed. For instance, if the dialed number is 2019, you should have `Number=[2 0 1 9]`.
- `x` contains the samples of the transmitted signal $x(t)$.

Once your code is ready to run, prepare the signal for your own cellular phone number and listen to it using the Matlab command `soundsc(x,8192)`. Does what you listen to sound familiar to you? Include your code and comments to your report.

1.2 DTMF Receiver

(Note: Before starting this part, clear everything in the workspace issuing the command `clear all`. Place the m-files named `FT.m` and `IFT.m` under the current directory.)

First run `x=DTMFTRA(Number)`, where `Number` includes the last 4 digits of your ID number in reverse order (i.e., for 20191234, choose `Number = [4 3 2 1]`). Now, suppose we are on the receiver side, `x` is the received signal and assume we do not know what `x` includes.

You can listen to it typing `soundsc(x,8192)`. We know that the form of the signal that we receive is as follows:

$$x(t) = \begin{cases} \cos(2\pi f_{r1}t) + \cos(2\pi f_{c1}t) & \text{for } 0 \leq t \leq 0.25, \\ \cos(2\pi f_{r2}t) + \cos(2\pi f_{c2}t) & \text{for } 0.25 \leq t \leq 0.5, \\ \cos(2\pi f_{r3}t) + \cos(2\pi f_{c3}t) & \text{for } 0.5 \leq t \leq 0.75, \\ \cos(2\pi f_{r4}t) + \cos(2\pi f_{c4}t) & \text{for } 0.75 \leq t \leq 1, \\ 0 & \text{otherwise} \end{cases}$$

where (f_{r1}, f_{c1}) determine the first digit, ..., (f_{r4}, f_{c4}) determine the last digit. To understand the dialed phone number, we need to find $(f_{r1}, f_{c1}), ..., (f_{r4}, f_{c4})$.

The Fourier transform operation is a powerful tool to analyze the frequency content of signals, and we will make use of it to understand the frequency content of the received signal. The Fourier transform of $x(t)$, denoted by $X(\omega)$ is defined as:

$$X(\omega) = \int_{-\infty}^{\infty} x(t)e^{-j\omega t}dt \tag{4}$$

For a particular frequency ω, $X(\omega)$ denotes the contribution of the complex exponential $e^{j\omega t}$ to the signal $x(t)$. As an analogy, $X(\omega)$ shows how many grams of $e^{j\omega t}$ we need to use to form $x(t)$.

Answer the following questions and include the answers to your report. You can directly use the result given in the book for part a. For the other parts show your work clearly. In these parts, you can directly use the definition of the Fourier transform. However, if it is possible, it is recommended to use the properties of the Fourier transform. Note that the rectangular function is defined as

$$\text{rect}(t) = \begin{cases} 0 & \text{if } |t| > 1, \\ \frac{1}{2} & \text{if } |t| = 1, \\ 1 & \text{if } |t| < 1. \end{cases}$$

and the triangular function is defined as $\text{tri}(t) = \max(1 - |t|, 0)$.

a) Let $x(t) = \exp(j2\pi f_0 t)$. What is $X(\omega)$?

b) Let $x(t) = \sin(2\pi f_0 t)$. What is $X(\omega)$?

c) Let $x(t) = \text{rect}\left(\frac{t}{T_0}\right)$. What is $X(\omega)$?

d) Let $x(t) = \text{tri}\left(\frac{t}{T_0}\right)$. What is $X(\omega)$?

e) Let $x(t) = \exp(j2\pi f_0 t)\text{rect}\left(\frac{t}{T_0}\right)$. What is $X(\omega)$?

f) Let $x(t) = \sin(2\pi f_0 t)\text{rect}\left(\frac{t}{T_0}\right)$. What is $X(\omega)$?

g) Let $x(t) = \text{rect}\left(\frac{t+t_0}{T_0}\right)$. What is $X(\omega)$?

h) Let $x(t) = \exp(j2\pi f_0 t)\text{rect}\left(\frac{t+t_0}{T_0}\right)$. What is $X(\omega)$?

i) Let $x(t) = \sin(2\pi f_0 t)\text{rect}\left(\frac{t+t_0}{T_0}\right)$. What is $X(\omega)$?

Then, type `X=FT(x)`, and create a frequency array using the following code:

```
omega=linspace(-8192*pi,8192*pi,8193);
omega=omega(1:8192);
```

This piece of code will create a frequency array in angular frequency. You will learn the details of this code when you learn the details of sampling. Then type `plot(omega,abs(X))`. You will obtain the plot of the magnitude of the Fourier transform of $x(t)$ computed over the grid specified by `omega`. Include the plot to the report. Examine the figure, in particular, determine the frequencies where you see the peaks. Are the frequencies where the peaks occur the ones used by DTMF transceivers? (Here you should consider the conversion of cyclic frequency (with units of Hertz) to angular frequency (with units of rad/sec)). If yes, can you understand ONLY from this figure what the dialed number is? Include your answers to the report.

Now, define a new signal as

$$x_1(t) = \begin{cases} x(t) & \text{for } 0 \le t \le 0.25, \\ 0 & \text{for } 0.25 \le t \le 1 \\ 0 & \text{otherwise.} \end{cases}$$

This operation can be seen as a multiplication of $x(t)$ by a rectangular signal. First write the analytical expression of this rectangular signal. Then, in Matlab, generate this rectangular signal and by multiplying by $x(t)$, create $x_1(t)$. Make sure that the size of the array for `x1(t)` is the same with the size of the array for $x(t)$. Include your code to the report.

Now compute $X_1(\omega)$ using the `FT.m` function and plot its magnitude against ω. Include the plot to the report. Look again to the frequencies where the peaks occur. This time, can you understand what is the first digit that is dialed? Include the answer to your report.

Continue in this manner and find the remaining three digits. Explain these steps clearly. Why do you think that the first method (looking at $X(\omega)$ at once) does not work but the second method (looking at $X_1(\omega), ..., X_4(\omega)$) separately) works? Write your answer to the report.

Part 2

(Note: Before starting this part, clear everything in the workspace issuing the command `clear all`. Place the m-files named `FT.m` and `IFT.m` under the current directory.)

This echo issue may cause problems in some applications, for example, ultrasound imaging, communication systems etc. Our purpose in this part is to eliminate these echoes from the recording.

Instead of recording your own voice, you will use a built-in MATLAB audio file containing a segment from Handel's "Hallelujah" chorus. You can load this data into your workspace with the following commands. The sampling rate of this signal is 8192 Hz, as required.

```
load handel.mat; % Loads signal into 'y' and sampling rate into 'Fs'
x = y'; % Assign the signal to 'x' and take the transpose
CONST = 8192;
% Ensure the signal is exactly 10 seconds long, padding with zeros
x(fs*10) = 0;
```

When you load the signal, listen to it in Matlab in order to make sure that your loading is successful. Include the code used to load and prepare the signal in your report.

Now you will artificially create the signal y which suffers from echo. This signal will represent the one which is described in the first paragraph of this part. It can be represented as

$$y(t) = x(t) + \sum_{i=1}^{M} A_i x(t - t_i), \tag{5}$$

where the summation simulates the environment which causes the echo. `M` represent the number of the echo, `Ai` denotes the amplitude of the i-th echo and `ti` denotes the time delay for the i-th echo with $A_i > 0$ and $t_i > 0$. (If there is no amplifier in the environment, we expect $A_i < 1$ due to the power loss.)

First, answer the following questions and include your answers to your report together with a clear derivation.

a) Find $h(t)$ such that $y(t) = x(t) * h(t)$ where * denotes the convolution operation. Note that in this way, we describe the process that relates $x(t)$ to $y(t)$ as a linear time invariant (LTI) system. The impulse response of this LTI system is $h(t)$.

b) What is the frequency response of this system? That is, what is $H(\omega)$, i.e., the Fourier transform of $h(t)$?

c) What is the relation between $X(\omega)$, $Y(\omega)$ and $H(\omega)$?

d) Given $Y(\omega)$ and $H(\omega)$, how can you determine $X(\omega)$?

e) What are the advantages of performing the recovery operation in the frequency domain instead of time domain?

Now, generate the time variable `t` by issuing the command `t=0:1/8192:10-1/8192;`. This variable indicates the sampling points in time of your voice. Then generate `y` from `x` by assuming `M = 5`, `Ai = [0.8, 0.55, 0.4, 0.3, 0.15]` and `ti = [0.5, 1.75, 2.25, 3, 4.25]` seconds for $i \in \{1, 2, 3, 4, 5\}$. Crop the delayed signals so that each of them is 10 seconds long. Plot $x(t)$ vs. `t` and $y(t)$ vs. `t` in separate figures. Clearly indicate the titles and labels. Also listen to $y(t)$ and describe the sound that you listened.

Now you will extract the original signal from the disturbed signal using Fourier domain relations. In order to do this, compute the Fourier transform of $y(t)$ using the command `Y=FT(y)`. Then, compute $H(\omega)$ over the grid specified by `omega`, which will be generated typing

```
omega=linspace(-8192*pi,8192*pi,81921);
omega=omega(1:81920);
```

Next, compute $h(t)$ typing `h=IFT(H)`. Plot $h(t)$ vs. t and $|H(\omega)|$ vs. ω in separate figures. (Restrict the x-axis limits of $|H(\omega)|$ plot to $[-4\pi, 4\pi]$.) Include your plots to your report together with the appropriate labels and titles. Then, using your result in item d, compute $X_e(\omega)$, where e indicates estimated X. Finally, compute $x_e(t)$ from $X_e(\omega)$ typing `xe=IFT(Xe)`. Then listen to $x_e(t)$. Is the estimated audio different than original audio? Plot $x_e(t)$ and include the plot to your report. Also include your comments and observations to the report.