EEE - 321: Signals and Systems

Lab Assignment 6

Please carefully study this assignment before coming to the laboratory. You may begin working on it or even complete it if you wish, but you do not have to. There will be a short pre-quiz at the beginning, post-quiz will be held on your sections class hours, 22/23 December at 12.00, please note that post-quiz will be held before you submit your reports. You have 1 week to submit your reports. These quizzes may contain conceptual, analytical, and Matlab-based questions. Within one week, complete the assignment in the form of a report and turn it in to the assistant. Some of the exercises will be performed by hand and others by using Matlab. What you should include in your report is indicated within the exercises.

Note: Include all your code to your report.

Part 1

An important class of discrete time (DT) linear time invariant (LTI) systems are those whose input output relations are specified through linear constant coefficient difference equations as follows:

$$y[n] = \sum_{l=1}^{N} a[l]\, y[n-l] + \sum_{k=0}^{M} b[k]\, x[n-k]. \tag{1}$$

Assuming that $x[n] = 0$ and $y[n] = 0$ for $n < 0$, find expressions for $y[0]$ and $y[1]$ in terms of $x[n]$, $a[l]$ and $b[k]$. Include your work to your report.

Taking the Z-transforms of both sides of Eq. (1) and arranging the terms, show that the transfer function of the system whose input output relation is given by Eq. (1) is in the following form:

$$H(z) = \frac{Y(z)}{X(z)} = \frac{\displaystyle\sum_{p=0}^{P} c_n[p] z^{-p}}{\displaystyle\sum_{q=0}^{Q} c_d[q] z^{-q}} \tag{2}$$

In particular, find P, Q, $c_n[p]$ and $c_d[q]$ in terms of M, N, $a[l]$ and $b[k]$. Note that Eq. (2) is basically the ratio of two polynomials in z^{-1}. Include your work to your report.

Write a Matlab function which computes the output of a DTLTI system whose input output relation is given by Eq. 1. Your function should look like

function [y]=DTLTI(a,b,x,Ny) *where*

- **a** of size $1 \times N$ contains the coefficients denoted by a_i in Eq. 1 such that $\mathbf{a(1) = a_1}, \ldots,$ $\mathbf{a(N) = a_N}$.
- **b** of size $1 \times (M+1)$ contains the coefficients denoted by b_k in Eq. 1 such that $\mathbf{b(1) = b_0}$, $\mathbf{b(2) = b_1}, \ldots$, $\mathbf{b(M+1) = b_M}$.
- **x** of size $1 \times N_x$ represents the input signal $x[n]$. We assume that $x[n] = 0$ for $n < 0$ and $n \geq N_x - 1$ and that $\mathbf{x(1) = x[0]}$, $\mathbf{x(2) = x[1]}$, $\ldots$, $\mathbf{x(N_x) = x[N_x - 1]}$.
- **Ny**: You will compute the output signal $y[n]$ for $0 \leq n \leq N_y - 1$, so that $\mathbf{Ny = N_y}$. Always assume that $\mathbf{N_y \geq 1}$. Note that N_y can be greater than N_x.
- **y** of size $1 \times N_y$ represents the output signal $y[n]$ such that $\mathbf{y(1) = y[0]}$,$\mathbf{y(2) = y[1]}, \ldots,$ $\mathbf{y(N_y) = y[N_y - 1]}$.

Part 2

Let D denote your ID number, and let D_5 denote your ID number in modulo 5. That is

$$D \equiv D_5 \pmod 5$$

with $0 \leq D_5 \leq 4$.

Suppose $a[l] = 0$ for $1 \leq l \leq N$. Therefore, the value of N is not important.

Suppose $M = 1 + 2D_5$ and let

$$b[k] = e^{-k+1} \quad \text{for } 0 \leq k \leq M - 1,$$

and 0 otherwise.

(a) Using your **DTLTI** function , compute the impulse response $h[n]$ of the filter (that is, the output of the filter when $x[n] = \delta[n]$) for $0 \leq n \leq 10$, and display it against n.

(b) Compare the nonzero values of impulse response to the coefficients $b[k]$. What pattern do you notice? Include your answer to your report.

(c) Does the impulse response have finite length or infinite length? Is the system FIR (finite impulse response) or IIR (infinite impulse response)? If the system is FIR, what is the length of the impulse response? How does it compare to M ? Include your answers to your report.

(d) Compute the **z-transform** of the impulse response **analytically**. Show your work clearly. Then, by using the z-transform expression, find the **discrete-time Fourier transform** of the impulse response.

(e) Using the expression you found in part (d), plot the magnitude response of the impulse response in Matlab for the frequency interval $[-\pi, \pi)$ and include it in your report. What kind of a filter (lowpass, highpass, bandpass, etc.) is this system? What does 3 dB bandwidth mean? What is the 3 dB bandwidth of this system? What does cut-off frequency mean? What is the cut-off frequency of this system? Include your answers in your report.

(f) In this step, we suppose that we do not know what our system's frequency behaviour is. In order to examine the frequency response, we will introduce a linear chirp signal to the system. As you remember from Lab 1, the instantaneous frequency of a signal of the form

$$x(t) = \cos\big(2\pi\phi(t)\big) \tag{3}$$

is defined as

$$f_{\text{ins}}(t) = \frac{d\phi(t)}{dt}. \tag{4}$$

For the linear chirp signal, $f_{\text{ins}}(t) = f_0 + kt$ where

- k is the rate of frequency change or "chirpyness",
- f_0 is the initial frequency (at $t = 0$),
- the signal sweeps from f_0 to $f_{\text{ins}}(t_{\text{final}})$.

In our experiment, we want our chirp signal to sweep from 0 Hz to 1000 Hz. Sampling frequency will be $f_s = 2000$ Hz. Take the samples of this linear chirp signal from the interval $0 \leq t \leq 1$ (calculate the k accordingly). Using your function **DTLTI**, compute and plot the output. In this experiment, each time sample of the output corresponds to the system's frequency response for a certain frequency. So, in order to understand the frequency response of the system, the x-axis of the plot should take values from 0 to π.

Include the plot in your report. Compare this plot with the one you produced in part e. How is the general trend in the output signal? Are there any sudden jumps? If yes, comment on them. Include your answer in your report. Repeat the same experiment by taking the samples from the interval $0 \leq t \leq 10$ and $0 \leq t \leq 1000$ (calculate the corresponding k 's).
Do not use the built-in Matlab chirp function. Your plots should be produced using subplot. They should have appropriate titles, xlabels and ylabels. Also, you should use xlim function to trim the plot.

Part 3 (Optional, non-grading)

Repeat Part 2 by taking $b[k]$ as $b[k] = \delta[k] - e^{-k}$ for $0 \leq k \leq M-1$, and 0 otherwise.
Repeat Part 2 by taking $b[k]$ as ideal-bandpass interpolating function (as in previous lab).

Part 4

In this part you will design a filter. The design specifications are as follows:

- The filter should have causal impulse response.
- The filter should have one zero, z_1, and two poles p_1, p_2. The values of these will be determined as follows: Let n_i be the $(2i + i \bmod 7) \bmod 5$ where i should take the value of the i^{th} digit of your ID number. For example, if your ID is xxxxxx9x, you should calculate n_7 by writing $i = 9$ and so on. Then,

$$z_1 = \frac{n_1 + jn_8}{\sqrt{n_1^2 + n_8^2}}, \qquad p_1 = \frac{n_3 + jn_4}{\sqrt{1 + n_3^2 + n_4^2}}, \qquad p_2 = \frac{n_2 + jn_7}{\sqrt{1 + n_2^2 + n_7^2}}.$$

  If your ID is such that $z_1 = p_1$ or $z_1 = p_2$ then you can ignore that zero and pole. In this case, the filter will contain just one pole.

Now answer the following questions and add to your report:

a) What is the z-transform of this filter?

b) Express the filter in terms of Equation 1.

c) Compute the impulse response of the filter.

d) Draw the pole-zero plot of this filter and region of convergence. By looking at the pole-zero diagram, roughly draw the magnitude response of this system.

e) Is this system stable?

f) Is this filter FIR or IIR?

g) By using the z-transform of the filter, analytically find the discrete-time Fourier transform of your filter and plot its magnitude in Matlab for the frequency interval of $[-\pi, \pi)$. What kind of filter is this (low-pass, high-pass, band-pass, band-stop)?

h) In this step, we will introduce a linear chirp signal to the system again. This time the signal will be in the form

$$x(t) = \exp\bigl(j2\pi\phi(t)\bigr). \tag{5}$$

This time, we want our chirp signal to sweep from $-1000\,\mathrm{Hz}$ to $1000\,\mathrm{Hz}$ (we want to see both negative and positive side). Sampling frequency will be $f_s = 2000\,\mathrm{Hz}$, again. Take the samples of this linear chirp signal from the interval $0 \le t \le 1$ (calculate the k accordingly). Using your function **DTLTI**, compute the output. Plot the magnitude and phase of the output. In order to understand the frequency response of the system, the x-axis of the plot should take values from $-\pi$ to π.

Include the plot in your report. Compare the magnitude plot with the one you produced in the previous item. Is this magnitude response symmetric w.r.t. the origin? Explain why. If the chirp is sweeping from -800 to 1200, what should be f_s in order to understand the full behaviour of the system? Include your answer in your report. Repeat the same experiment by taking the samples from the intervals $0 \le t \le 10$ and $0 \le t \le 1000$ (calculate the corresponding k's accordingly).

Do not use the built-in Matlab `chirp` function. Your plots should be produced using `subplot`. They should have appropriate titles, `xlabels` and `ylabels`. Also, you should use the `xlim` function to trim the plot.