

Calculus Related Problems (Taken from old Exams)

- ① Let $f(x,y) = x^2y - 2xy + y^2 - 15y$. Find the direction of most rapid increase for $f(x,y)$ at the point $(1,1)$ and the rate of change of f in this direction.
- ② The derivative of a differentiable function $f(x,y)$ at a point P in the direction of $\hat{a}_x + \hat{a}_y$ is 2, and in the direction of $3\hat{a}_x - 4\hat{a}_y$ is $-3\sqrt{2}$. Find the derivative of $f(x,y)$ at P in the direction of $7\hat{a}_x - \hat{a}_y$.
- ③ Evaluate $\iiint_V \frac{1}{1+(x^2+y^2+z^2)^{3/2}} dx dy dz$ where $V = \{(x,y,z) : x^2+y^2+z^2 \leq 1\}$
- ④ Evaluate $\iint_S (x^2+y^2) dx dy$ where $S = \{(x,y) : 1 \leq x^2+y^2 \leq 4, x \geq 0 \text{ and } y \geq 0\}$
- ⑤ Let V be the volume bounded by the surfaces $x^2+y^2+z^2=9$ and $x^2+y^2+z^2=1$ and lies above the surface $z^2=3(x^2+y^2)$ with $z > 0$. Write down 3 integrals in cartesian, cylindrical and spherical coordinates that give the value of the volume. (Do not evaluate these integrals)
- ⑥ Evaluate $\int_0^1 \int_0^{\sqrt{y-y^2}} \frac{1}{[x^2+y^2-(x^2+y^2)^2]^{1/2}} dx dy$
- ⑦ Find the value of the line integral $\oint_C [(3x^2y^2+y) dx + 2x^3y dy]$, where C is the cardioid $\rho = 1 + \cos\phi$ parametrized counterclockwise.
- ⑧ Let Q be the volume bounded by the paraboloid $z = 4 - x^2 - y^2$ and the xy -plane. Find the outward flux of the vector field $\vec{F} = \hat{a}_x x^3 + \hat{a}_y y^3 + \hat{a}_z z$ over the boundary of Q .