PROBLEM 2.1:

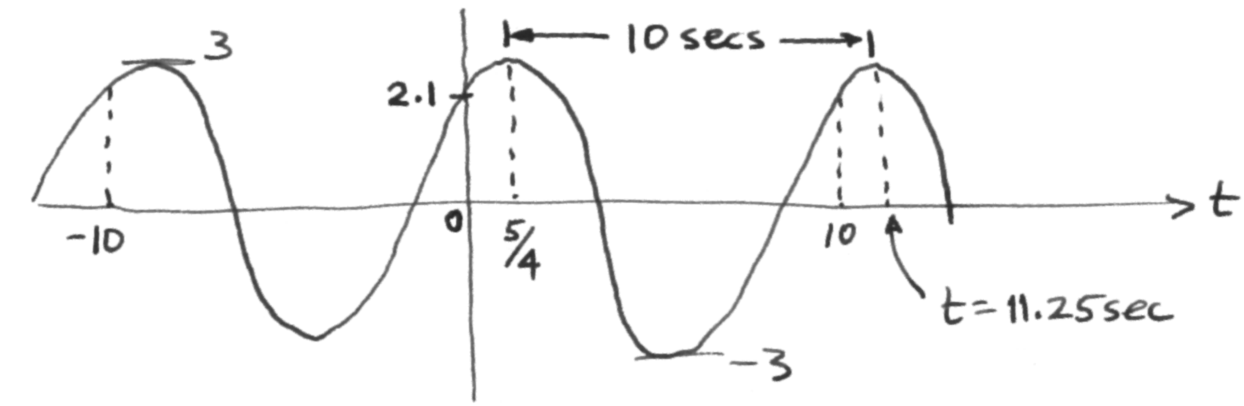
$$x(t) = 3\cos\left(\frac{\pi}{5}t - \frac{\pi}{4}\right)$$

$$2\pi f = \frac{\pi}{5} \implies f = \frac{1}{10} \implies T = 10 \text{ sec. (PERIOD)}$$

$$\varphi = -2\pi\frac{t_1}{T} \implies t_1 = -\frac{\varphi}{2\pi}T = \frac{\pi/4}{2\pi} \times 10 = \frac{5}{4} = 1.25 \text{sec}$$

at $t=0$

$$x(t) = 3\cos(-\pi/4) = \frac{3\sqrt{2}}{2} \approx 2.1$$



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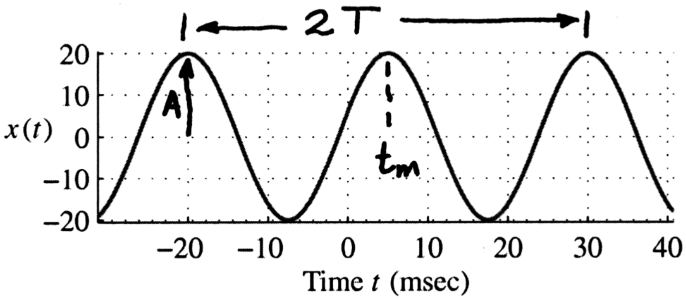
PROBLEM 2.2:



Period: $2T = (30-(-20))\text{ msec} = 50\text{ msec}$

$\Rightarrow T = 25\text{ msec}$

Frequency: $\omega_0 = 2\pi/T = 2\pi\left(\frac{1}{25\times10^{-3}}\right) = 2\pi(40)\text{ rad/s}$

$f = 40\text{ Hz}$

Amplitude: $A = 20$

Time-Shift: $t_m = +5\text{ msec}$

Phase: $\varphi = -\omega_0 t_m = -2\pi(40)\times 5\times10^{-3}$

$\varphi = -2\pi(0.2) = -0.4\pi$

$$x(t) = 20\cos(80\pi t - 0.4\pi)$$

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PROBLEM 2.4:

$$e^{j\theta} = 1 + j\theta + \frac{(j\theta)^2}{2!} + \frac{(j\theta)^3}{3!} + \frac{(j\theta)^4}{4!} + \frac{(j\theta)^5}{5!} + \dots$$

$$= 1 + j\theta - \frac{\theta^2}{2!} - j\frac{\theta^3}{3!} + \frac{\theta^4}{4!} + j\frac{\theta^5}{5!} + \dots$$

Separate the real and imaginary parts:

$$e^{j\theta} = \underbrace{\left(1 - \frac{\theta^2}{2!} + \frac{\theta^4}{4!} - \dots\right)}_{\cos\theta} + j\underbrace{\left(\theta - \frac{\theta^3}{3!} + \frac{\theta^5}{5!} - \dots\right)}_{\sin\theta}$$

$$\therefore e^{j\theta} = \cos\theta + j\sin\theta$$

which proves Euler's formula.

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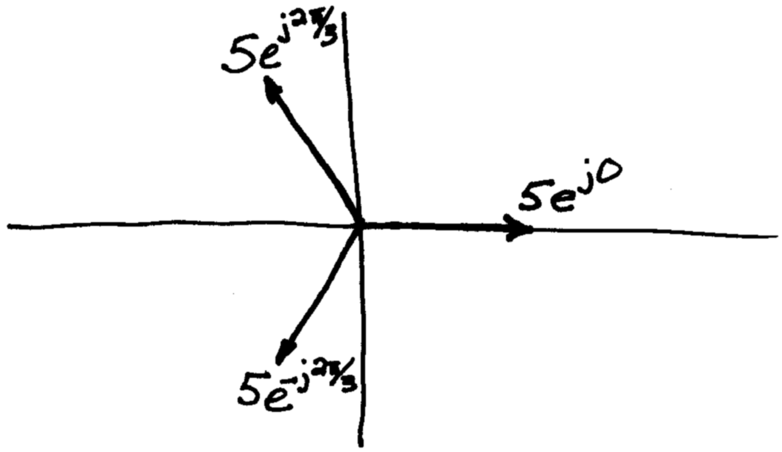
PROBLEM 2.10:

Use Phasors:

$$5\cos(\omega t) \longrightarrow 5e^{j0} = 5 + j0$$

$$5\cos(\omega t + 120°) \longrightarrow 5e^{j2\pi/3} = -\frac{5}{2} + j5\frac{\sqrt{3}}{2}$$

$$5\cos(\omega t - 120°) \longrightarrow 5e^{-j2\pi/3} = -5/2 - j5\frac{\sqrt{3}}{2}$$



Vector Sum:

$$5 + \left(-\frac{5}{2} + j5\frac{\sqrt{3}}{2}\right) + \left(-\frac{5}{2} - j5\frac{\sqrt{3}}{2}\right)$$

$$= \left(5 - \frac{5}{2} - \frac{5}{2}\right) + j\left(5\frac{\sqrt{3}}{2} - 5\frac{\sqrt{3}}{2}\right) = 0$$

Thus, $x(t) = 0$

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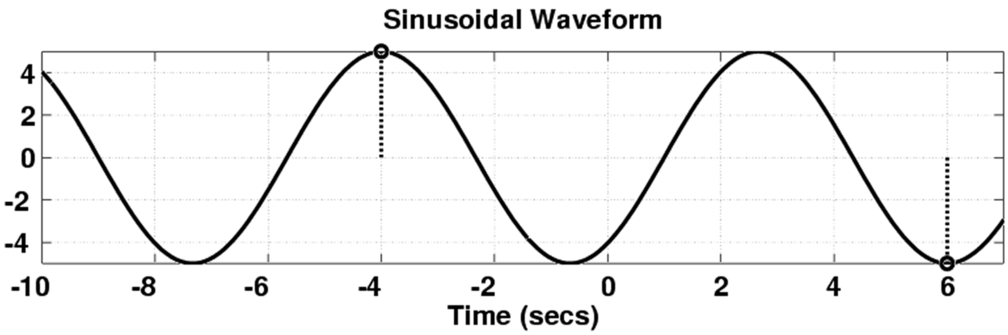
PROBLEM 2.14:

From the graph we can get the following information:

positive peak at $t = -4$ msec, value $= 5$

negative peak at $t = 6$ msec



There are $1\frac{1}{2}$ periods from $t=-4$ ms to $t=6$ ms.

$$(1\tfrac{1}{2})T = 10 \text{ msec}$$

$$\Rightarrow T = \frac{20}{3} \text{ msec} = 6\tfrac{2}{3} \text{ msec}$$

$$\omega_0 = 2\pi/T = 2\pi/(20/3000) = 300\pi \text{ rad/sec}$$

Phase: $$\varphi = -2\pi\left(\frac{t_1}{T}\right) = -2\pi\left(\frac{-4}{20/3}\right) = \frac{12\pi}{10} = 1.2\pi$$

$$\therefore\ x(t) = 5\cos(300\pi t + 1.2\pi)$$

For the complex notation $X = \text{Mag}\, e^{j\,\text{phase}}$

$$X = 5e^{j1.2\pi}$$

$$x(t) = \Re e\left\{ 5e^{j1.2\pi} e^{j300\pi t} \right\}$$

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PROBLEM 2.17:

$$x(t) = 5\cos(\omega_0 t + 3\pi/2) + 4\cos(\omega_0 t + 2\pi/3) + 4\cos(\omega_0 t + \pi/3)$$

(a) Express $x(t)$ in the form $x(t) = A\cos(\omega_0 t + \phi)$ by finding the numerical values of A and ϕ.

$$\left.\begin{aligned} z_1 &= 5e^{j3\pi/2} = 0 - 5j \\ z_2 &= 4e^{j2\pi/3} = -2 + j3.46 \\ z_3 &= 4e^{j\pi/3} = 2 + j3.46 \end{aligned}\right\} \quad \begin{aligned} z &= z_1 + z_2 + z_3 \\ &= 0 + j1.928 \\ &= 1.928e^{j\pi/2} \end{aligned}$$

$$\therefore \quad x(t) = 1.928\cos(\omega_0 t + \pi/2)$$

***

Z = X	+ jY	Magnitude	Phase	Ph/pi	Ph(deg)
-9.185e-16	-5	5	-1.571	-0.500	-90.00
-2	3.464	4	2.094	0.667	120.00
2	3.464	4	1.047	0.333	60.00
4.441e-16	1.928	1.928	1.571	0.500	90.00

(b) Plot all the phasors used to solve the problem in part (a) in the complex plane.