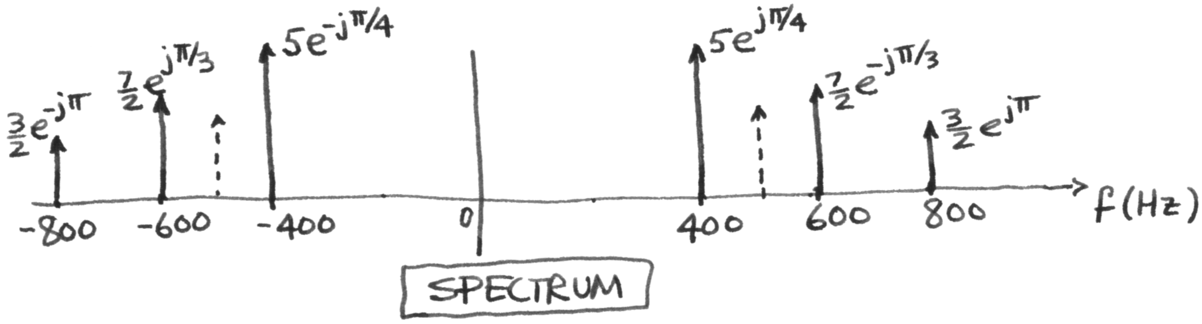
## PROBLEM 3.1:

(a) There are 3 components

$$10\cos(800\pi t + \pi/4) = \Re e\{10e^{j\pi/4}e^{j800\pi t}\} \qquad \text{freq} = 400\text{ Hz}$$

$$7\cos(1200\pi t - \pi/3) = \Re e\{7e^{-j\pi/3}e^{j1200\pi t}\} \qquad \text{freq} = 600\text{ Hz}$$

$$-3\cos(1600\pi t) = \Re e\{3e^{j\pi}e^{j1600\pi t}\} \qquad \text{freq} = 800\text{ Hz}$$




(b) $x(t)$ is periodic because there is a fundamental frequency $f = 200\text{Hz}$ that divides all 3 freqs. The period is the fundamental period $= 1/200$ sec

(c)

$$5\cos(1000\pi t + \pi/2) = \Re e\{5e^{j\pi/2}e^{j1000\pi t}\} \qquad \text{freq} = 500\text{Hz}$$

The spectrum will have two additional lines at $f = -500\text{Hz}$ and $f = 500\text{Hz}$

$\frac{5}{2}e^{-j\pi/2}$

component is $\frac{5}{2}e^{j\pi/2}$

The dotted lines in the sketch above show where these two lines will be.

Yes, $y(t)$ is periodic. The fundamental frequency is now $f_0 = 100\text{Hz}$ because it has to divide into 500Hz as well as 400, 600 and 800.

The period is now $\frac{1}{100}$ sec.

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## PROBLEM 3.8:

x(t) has four components:

$2 = 2e^{j0} \Rightarrow$ freq = 0 , period = anything

$4\cos(40\pi t - \pi/5) = \Re e\{4e^{-j\pi/5}e^{j40\pi t}\}$  freq = 20 Hz, period = 1/20 sec.

$3\sin(60\pi t) = 3\cos(60\pi t - \pi/2)$
$= \Re e\{3e^{-j\pi/2}e^{j60\pi t}\}$  freq = 30 Hz, period = 1/30 sec.

$4\cos(120\pi t - \pi/3) = \Re e\{4e^{-j\pi/3}e^{j120\pi t}\}$  freq = 60 Hz, period = 1/60 sec.

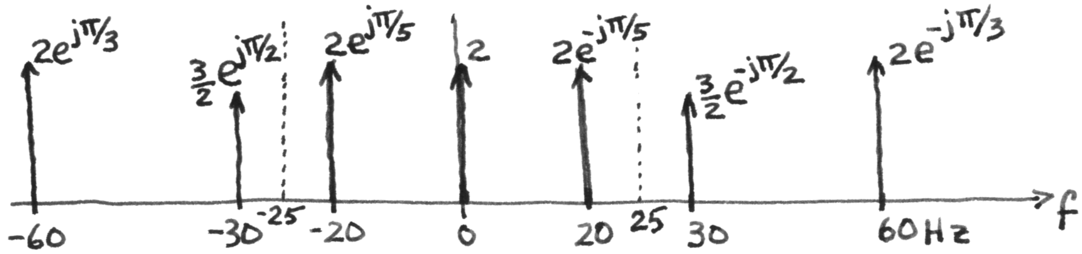
(a) The fundamental period is the smallest time that is exactly divisible by 1/20, 1/30 and 1/60.

$\Rightarrow T_0 = 1/10$ sec because $T_0 = 2(1/20) = 3(1/30) = 6(1/60)$

$\Rightarrow f_0 = 10$ Hz $\Rightarrow \omega_0 = 2\pi f_0 = 20\pi$ rad/sec

$X_0 = 2$ , $X_2 = 4e^{-j\pi/5}$ , $X_3 = 3e^{-j\pi/2}$ , $X_6 = 4e^{-j\pi/3}$

(b)




NOTE: since $\Re e\{X_i\} = \frac{1}{2}X_i + \frac{1}{2}X_i^*$ we use $\frac{1}{2}X_i$ in the plot.

(c) $y(t) = x(t) + 10\cos(50\pi t - \pi/6)$

equals $\Re e\{10e^{-j\pi/6}e^{j50\pi t}\}$  freq = 25 Hz, period = 1/25 sec

The new period must be divisible by 1/10 & 1/25

$\Rightarrow T_0 = 1/5$ because $T_0 = 5(1/25)$ and $2(1/10)$

$\Rightarrow f_0 = 5$ Hz $\Rightarrow \omega_0 = 2\pi f_0 = 10\pi$ rad/sec

We must re-index the $X_i$ because $\omega_0$ is lower.

$Y_0 = 2$ , $Y_4 = X_2$ , $Y_5 = 10e^{-j\pi/6}$ , $Y_6 = X_3$ , $Y_{12} = X_6$

There will be one new pair of lines at $f = \pm 25$ Hz

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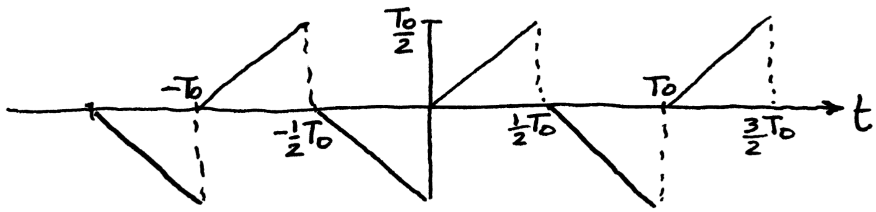
## PROBLEM 3.10:

Half-wave symmetry: $x(t+T_0/2) = -x(t)$

(a) $x(t) = t$ for $0 \le t < T_0/2$




(b) $$a_0 = \frac{1}{T_0}\int_0^{T_0} x(t)\, e^{-j0}\, dt$$

Change of variables $u = t - T_0/2$

$$= \frac{1}{T_0}\int_0^{T_0/2} x(t)\, dt + \frac{1}{T_0}\int_{T_0/2}^{T_0} x(t)\, dt$$

$$= \frac{1}{T_0}\int_0^{T_0/2} x(t)\, dt + \frac{1}{T_0}\int_0^{T_0/2} \underbrace{x(u+T_0/2)}_{=-x(u)}\, du$$

because of half-wave symmetry

$$= \frac{1}{T_0}\int_0^{T_0/2} x(t)\, dt - \frac{1}{T_0}\int_0^{T_0/2} x(u)\, du = \underline{\underline{0}}$$

(c) If $k$ is even, then $k = 2\ell$

$$a_k = \frac{1}{T_0}\int_0^{T_0/2} x(t)\, e^{-j(2\pi/T_0)2\ell t}\, dt + \frac{1}{T_0}\int_{T_0/2}^{T_0} x(t)\, e^{-j(2\pi/T_0)2\ell t}\, dt$$

Make the same change of variables: $u = t - T_0/2$.

$$a_k = \frac{1}{T_0}\int_0^{T_0/2} x(t)\, e^{-j(2\pi/T_0)2\ell t}\, dt + \frac{1}{T_0}\int_0^{T_0/2} \underbrace{x(u+T_0/2)}_{=-x(u)}\, e^{-j(2\pi/T_0)2\ell(u+T_0/2)}\, du$$

$$e^{-j(2\pi/T_0)2\ell u}\, e^{-j(2\pi/T_0)2\ell T_0/2} = e^{-j(2\pi/T_0)2\ell u}\, \underbrace{e^{-j2\pi\ell}}_{=1}$$

$$\Rightarrow a_k = \frac{1}{T_0}\int_0^{T_0/2} x(t)\, e^{-j(2\pi/T_0)2\ell t}\, dt - \frac{1}{T_0}\int_0^{T_0/2} x(u)\, e^{-j(2\pi/T_0)2\ell u}\, du$$

$\Rightarrow a_k = 0$ for $k$ even.

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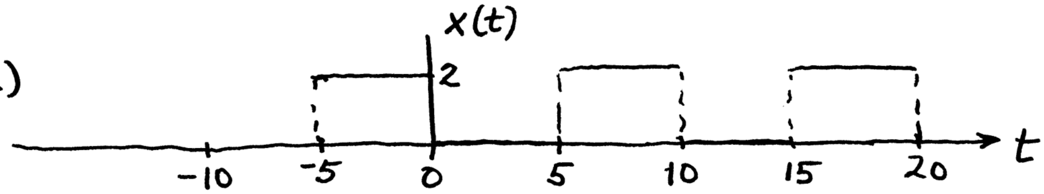
## PROBLEM 3.12:

$$x(t) = \begin{cases} 0 & 0 \le t \le 5 \\ 2 & 5 < t \le 10 \end{cases} \qquad T_0 = 10 \text{ secs.}$$

(a)




(b) $a_0 = \frac{1}{T_0} \times \text{Area} = \frac{1}{10} \times (5 \times 2) = 1$

(c) $$a_1 = \frac{1}{10} \int_5^{10} 2 e^{-j(2\pi/10)t} dt$$

$$= \frac{1}{5} \left. \frac{e^{-j\pi t/5}}{(-j\pi/5)} \right|_5^{10} = \frac{e^{-j2\pi} - e^{-j\pi}}{-j\pi} = \frac{1-(-1)}{-j\pi} = \frac{2j}{\pi}$$

(d) $$y(t) = 1 + x(t) = 1 + \sum_{k=-\infty}^{\infty} a_k e^{j\omega_0 k t}$$

$$= (1 + a_0) + \sum_{k \neq 0} a_k e^{j\omega_0 k t}$$

$$\Longrightarrow \quad b_0 = 1 + a_0 \quad \text{and} \quad b_1 = a_1$$

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## PROBLEM 3.14:

(a) $y(t) = Ax(t) = A \cdot \sum_k a_k e^{jk\omega_0 t}$

$$= \sum_k (Aa_k) e^{jk\omega_0 t} \implies \underline{b_k = Aa_k}$$

(b) $y(t) = x(t - t_d)$

$$= \sum_k a_k e^{j\omega_0 k(t - t_d)}$$

$$= \sum_k \left(a_k e^{-j\omega_0 k t_d}\right) e^{j\omega_0 k t}$$

$$\implies \underline{b_k = a_k e^{-jk\omega_0 t_d}}$$

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## PROBLEM 3.19:

Characterize each time signal:

(a) 6 periods from $t=-2$ to $t=+3$ $\Rightarrow T = \frac{5}{6}$ sec

DC value = 2 $\qquad t_m > 0 \Rightarrow \varphi < 0$

(b) 3 periods from $t=0$ to $t=2$ $\Rightarrow T = \frac{2}{3}$ sec

DC value = 0 $\qquad \varphi = \pi$

(c) 6 periods from $t=-3$ to $t=2$ $\Rightarrow T = \frac{5}{6}$ sec

DC value = 2 $\qquad t_m < 0 \Rightarrow \varphi > 0$

(d) Period $\approx 3\frac{1}{3} = \frac{10}{3}$ secs. Two frequencies

DC value = 0

(e) 2 periods from $t=-2$ to $t=3$ $\Rightarrow T = 2.5$ secs

Two frequencies. DC value = 0

(1) $\omega_0 = 2\pi(1.2) \Rightarrow T = 1/1.2 = 5/6$ sec

$\varphi = 0.5\pi > 0$ $\qquad$ DC = 2

(2) $\omega_0 = 2\pi(0.3)$ because 0.3 divides 0.6 & 1.5

$\Rightarrow T = 1/0.3 = 10/3$ secs. $\qquad$ DC = 0

(3) Like (1). $T = 5/6$ sec. $\qquad$ DC = 2

But $\varphi = -0.25\pi < 0$

(4) $\omega_0 = 2\pi(0.4) \Rightarrow T = 1/0.4 = 2.5$ secs

DC = 0

(5) $\varphi = \pi$ $\qquad T = 1/1.5 = \frac{2}{3}$ sec

Thus, the match is

(a) $\longleftrightarrow$ (3) $\qquad$ (c) $\longleftrightarrow$ (1) $\qquad$ (e) $\longleftrightarrow$ (4)

(b) $\longleftrightarrow$ (5) $\qquad$ (d) $\longleftrightarrow$ (2)

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