PROBLEM 4.2:

$$x(t) = 7\sin(11\pi t) = 7\cos(11\pi t - \pi/2) \longrightarrow \boxed{A/D} \longrightarrow x[n] = A\cos(\hat{\omega}_0 n + \varphi).$$

$$f_s.$$

(a) $f_s = 10$ samples/sec.

$$x(t)\Big|_{t=n/f_s} = x\left(\frac{n}{f_s}\right) = 7\cos\left(\frac{11\pi n}{10} - \pi/2\right).$$

$$= 7\cos\left(\frac{11\pi n}{10} - 2\pi n - \pi/2\right).$$

$$= 7\cos\left(-\frac{9\pi n}{10} - \pi/2\right) = 7\cos(.9\pi n + \pi/2).$$

$$\boxed{A = 7,\ \hat{\omega}_0 = 0.9\pi,\ \varphi = \pi/2}$$

(b) $f_s = 5$ samples/sec

$$x(t)\Big|_{t=n/f_s} = x\left(\frac{n}{5}\right) = 7\cos\left(\frac{11\pi n}{5} - \pi/2\right)$$

$$= 7\cos\left(\frac{\pi n}{5} - \frac{\pi}{2}\right)$$

$$\boxed{A = 7,\ \hat{\omega}_0 = \frac{\pi}{5},\ \varphi = -\frac{\pi}{2}}$$

(c) $f_s = 15$ samples/sec

$$x(t)\Big|_{t=n/f_s} = x\left(\frac{n}{15}\right) = 7\cos\left(\frac{11\pi n}{15} - \frac{\pi}{2}\right)$$

$$A = 7,\ \hat{\omega}_0 = \frac{11\pi}{15} = 2\pi\left(\frac{5.5}{15}\right)\ \ \&\ \ \varphi = -\pi/2$$

McClellan, Schafer and Yoder, *Signal Processing First*, ISBN 0-13-065562-7.
Pearson Prentice Hall, Inc. Upper Saddle River, NJ 07458.

PROBLEM 4.5:

(a) Let $x(t) = 10\cos(\omega_o t + \varphi)$

Sampling at a rate of $f_s \Rightarrow x[n] = x(t)\big|_{t=n/f_s} = x(n/f_s)$

$$x[n] = 10\cos\left(\omega_o \frac{n}{f_s} + \varphi\right)$$

$f_s = 1000$

$\frac{\omega_o}{f_s} = 0.2\pi \Rightarrow \omega_o = 0.2\pi \times 1000 = 200\pi$

Equate this to

$$x[n] = 10\cos(0.2\pi n - \pi/7)$$

$\varphi = -\pi/7$

A second possible signal is the "folded alias" at $(f_s - f_o)$

$$f_s - f_o = f_s - \frac{\omega_o}{2\pi} = 1000 - \frac{200\pi}{2\pi} = 900\text{ Hz}$$

In this case, the phase (φ) changes.

$$\tilde{x}(t) = 10\cos(2\pi(f_s - f_o)t + \psi)$$

$$\tilde{x}[n] = 10\cos\left(2\pi(f_s - f_o)\frac{n}{f_s} + \psi\right) = 10\cos\left(2\pi n - 2\pi f_o \frac{n}{f_s} + \psi\right)$$

$$= 10\cos\left(-\frac{2\pi f_o n}{f_s} + \psi\right) = 10\cos\left(2\pi\frac{f_o}{f_s} n - \psi\right).$$

f_o is still 100 Hz

$$\Rightarrow \psi = +\pi/7$$

(b) Reconstruction of $x[n]$ with $f_s = 2000$ samples/sec.

The discrete and continuous domains are related by: $\frac{n}{f_s} \leftrightarrow t$ or $n \leftrightarrow f_s t$

So we replace "n" in $x[n]$ with $f_s t$. This is what an ideal D-to-A would do.

$$x[n] = 10\cos(0.2\pi n - \pi/7)$$

$$x(t) = 10\cos(0.2\pi f_s t - \pi/7)$$ $f_s = 2000$

$$= 10\cos(400\pi t - \pi/7)$$

$\omega_o = 400\pi \Rightarrow f_o = 200\text{ Hz}.$

McClellan, Schafer and Yoder, *Signal Processing First*, ISBN 0-13-065562-7.
Pearson Prentice Hall, Inc. Upper Saddle River, NJ 07458. © 2003

PROBLEM 4.8:

$$x(t) = \cos(50\pi t)\sin(700\pi t)$$

(a) $$x(t) = \left(\frac{1}{2}e^{j50\pi t} + \frac{1}{2}e^{-j50\pi t}\right)\left(\frac{1}{2j}e^{j700\pi t} - \frac{1}{2j}e^{-j700\pi t}\right)$$

$$= \frac{1}{4j}e^{j750\pi t} + \frac{1}{4j}e^{j650\pi t} - \frac{1}{4j}e^{-j650\pi t} - \frac{1}{4j}e^{-j750\pi t}$$

SAME AS $\frac{1}{4}e^{-j\pi/2}$

SAME AS $\frac{1}{4}e^{+j\pi/2}$

SPECTRUM.

SAME AS 650π rad/sec

(b) Sampling Thm says sample at a rate greater than $\boxed{\text{two}}$ $\boxed{\text{times}}$ the highest freq.

HIGHEST FREQ = 375 Hz

$$\Rightarrow f_s \geq 750\text{ Hz}.$$

McClellan, Schafer and Yoder, *Signal Processing First*, ISBN 0-13-065562-7.
Pearson Prentice Hall, Inc. Upper Saddle River, NJ 07458. © 2003

PROBLEM 4.13:

Assume that the sampling rates of a C-to-D and D-to-C conversion system are equal, and the input to the Ideal C-to-D converter is

$$x(t) = 2\cos(2\pi(50)t + \pi/2) + \cos(2\pi(150)t)$$

(a) If the output of the ideal D-to-C Converter is equal to the input $x(t)$, i.e.,

$$y(t) = 2\cos(2\pi(50)t + \pi/2) + \cos(2\pi(150)t)$$

what general statement can you make about the sampling frequency f_s in this case?

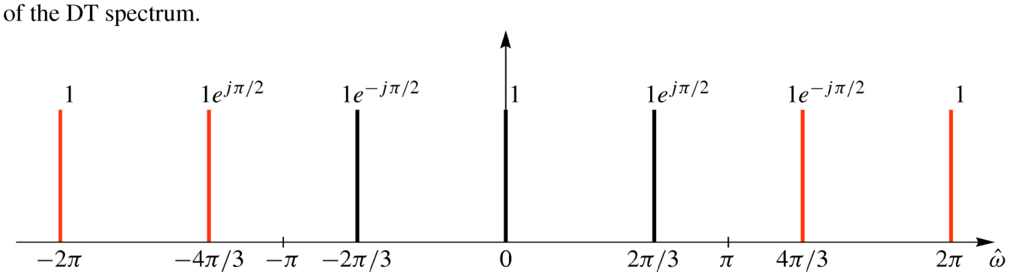
Solution: The sampling frequency must be greater than twice the highest frequency, because there was no aliasing. Thus, we can say that

$$F_s > 2 \times 150 = 300 \text{ Hz}$$

(b) If the sampling rate is $f_s = 250$ samples/sec., determine the discrete-time signal $x[n]$, and give an expression for $x[n]$ as a sum of cosines. *Make sure that all frequencies in your answer are positive and less than π radians.* *Solution:* Replace t with $n/f_s = n/250$ to get

$$\begin{aligned} x[n] = x(n/250) &= 2\cos(2\pi(50)(n/250) + \pi/2) + \cos(2\pi(150)(n/250)) \\ &= 2\cos(2\pi(0.2)n + \pi/2) + \cos(2\pi(0.6)n) \\ &= 2\cos(2\pi(0.2)n + \pi/2) + \cos(2\pi(0.4)n) \end{aligned}$$

(c) Plot the spectrum of the signal in part (b) over the range of frequencies $-\pi \le \hat{\omega} \le \pi$. The plot below shows the periodicity of the DT spectrum.



(d) If the output of the Ideal D-to-C Converter is

$$y(t) = 2\cos(2\pi(50)t + \pi/2) + 1$$

determine the value of the sampling frequency f_s. (Remember that the input signal is $x(t)$ defined above.)

Solution: Since the frequency of 50 Hz is preserved, the other frequency of 150 Hz must have been aliased to 0 Hz. This can happen if the sampling frequency is $f_s = 150$ Hz, in which case the discrete-time signal is

$$\begin{aligned} x[n] = x(n/150) &= 2\cos(2\pi(50)(n/150) + \pi/2) + \cos(2\pi(150)(n/150)) \\ &= 2\cos(2\pi n/3 + \pi/2) + \cos(2\pi n) \\ &= 2\cos(2\pi n/3 + \pi/2) + 1 \end{aligned}$$

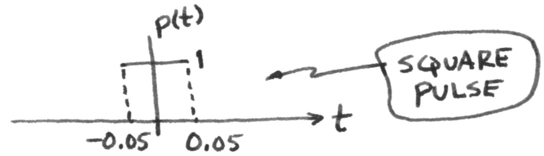
When $x[n]$ is reconstructed by the D/A converter running at $f_s = 150$ Hz, the final output will be

$$y(t) = x[n]\Big|_{n \to f_s t} = 2\cos(2\pi(150t)/3 + \pi/2) + 1 = 2\cos(2\pi(50)t + \pi/2) + 1$$

McClellan, Schafer and Yoder, *Signal Processing First*, ISBN 0-13-065562-7.
Pearson Prentice Hall, Inc. Upper Saddle River, NJ 07458. © 2003

PROBLEM 4.6:

(a) $p(t) = \begin{cases} 1 & -0.05 \le t \le 0.05 \\ 0 & \text{otherwise} \end{cases}$

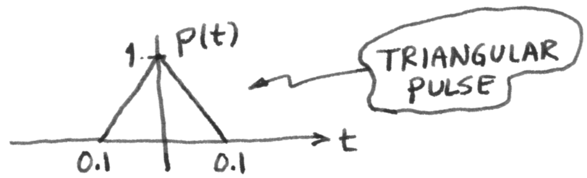


In the formula for $y(t)$

$$y(t) = \ldots + y[0]\,p(t) + y[1]\,p(t-T_s) + y[2]\,p(t-2T_s) + \ldots\ldots$$

The square pulses will not overlap, so the values of $y[n]$ will be extended over an interval of T_s.

(b) $p(t) = \begin{cases} 1 - 10|t| & -0.1 \le t \le 0.1 \\ 0 & \text{otherwise} \end{cases}$

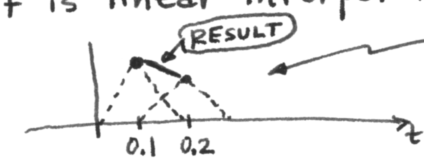


In this case, the neighboring terms do overlap

$$y(t) = \ldots + y[0]\,p(t) + y[1]\,p(t-T_s) + y[2]\,p(t-2T_s) + \ldots$$

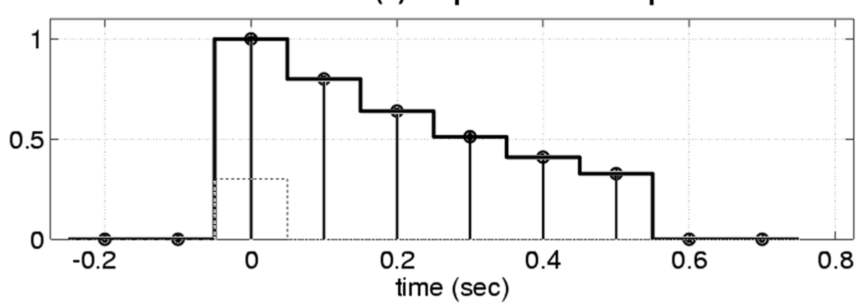
The result is linear interpolation.

Example:

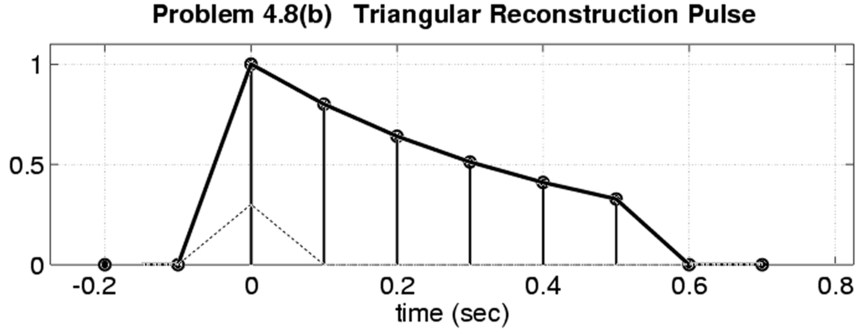


When we add these two triangles, the result between $t=0.1$ and $t=0.2$ is a straight line.

Problem 4.8(a) Square Pulse Shape

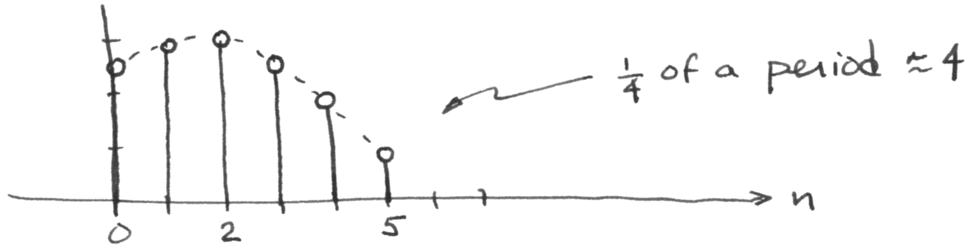


Problem 4.8(b) Triangular Reconstruction Pulse



PROBLEM 4.19:

You could estimate the values from a plot.



Looks like $A \approx 3 \qquad \omega_0 \approx 2\pi\left(\frac{1}{\text{period}}\right) = 2\pi \frac{1}{16} = \frac{\pi}{8}$

$$\varphi = -2\pi\left(\frac{t_1}{T}\right) \cong -2\pi\left(\frac{2}{16}\right) = -\pi/4$$

EXACT:

Write 3 consecutive values of $x[n]$.

$$x[n-1] = \frac{A}{2}e^{j\varphi}e^{j\omega_0 n}e^{-j\omega_0} + \frac{A}{2}e^{-j\varphi}e^{-j\omega_0 n}e^{j\omega_0}$$

$$x[n] = \frac{A}{2}e^{j\varphi}e^{j\omega_0 n} + \frac{A}{2}e^{-j\varphi}e^{-j\omega_0 n}$$

$$x[n+1] = \frac{A}{2}e^{j\varphi}e^{j\omega_0 n}e^{j\omega_0} + \frac{A}{2}e^{-j\varphi}e^{-j\omega_0 n}e^{-j\omega_0}$$

$$\Rightarrow x[n-1] + x[n+1] = \frac{A}{2}e^{j\varphi}e^{j\omega_0 n}(2\cos\omega_0) + \frac{A}{2}e^{-j\varphi}e^{-j\omega_0 n}(2\cos\omega_0)$$

$$= (2\cos\omega_0)\, x[n].$$

$$\Rightarrow \cos\omega_0 = \frac{x[n-1] + x[n+1]}{2x[n]} = \frac{2.4271 + 2.9816}{2(2.9002)} = 0.9325$$

$$\Rightarrow \boxed{\omega_0 = 2\pi/17}$$

Let $Z = \frac{A}{2}e^{j\varphi}$

$$x[0] = Z + Z^* = 2.4271$$

$$x[1] = e^{j2\pi/17}Z + e^{-j2\pi/17}Z^* = 2.9002$$

2 EQNS in 2 UNKNOWNS

$$Z = 1.5e^{-j\pi/5}$$

$$\boxed{A = 3 \quad \& \quad \varphi = -\pi/5}$$

McClellan, Schafer and Yoder, *Signal Processing First*, ISBN 0-13-065562-7.
Pearson Prentice Hall, Inc. Upper Saddle River, NJ 07458. © 2003

PROBLEM 4.19 (more):

SOLVE SIMULTANEOUS EQNS FOR A & φ

Let $Z = \frac{1}{2}Ae^{j\varphi}$

$$\begin{bmatrix} 1 & 1 \\ e^{j2\pi/17} & e^{-j2\pi/17} \end{bmatrix} \begin{bmatrix} Z \\ Z^* \end{bmatrix} = \begin{bmatrix} x[0] \\ x[1] \end{bmatrix}$$

Invert the 2x2 matrix: $\begin{bmatrix} a & b \\ c & d \end{bmatrix}^{-1} = \dfrac{1}{ad-bc}\begin{bmatrix} d & -b \\ -c & a \end{bmatrix}.$

$$\Rightarrow \begin{bmatrix} Z \\ Z^* \end{bmatrix} = \frac{1}{e^{-j2\pi/17} - e^{j2\pi/17}} \begin{bmatrix} e^{-j2\pi/17} & -1 \\ -e^{+j2\pi/17} & 1 \end{bmatrix} \begin{bmatrix} x[0] \\ x[1] \end{bmatrix}$$

$$Z = \frac{x[0]e^{-j2\pi/17} - x[1]}{-2j\sin 2\pi/17} = \frac{1}{2}Ae^{j\varphi}$$

PLUG IN $\left\{ \begin{array}{l} x[0] = 2.4271 \\ x[1] = 2.9002 \end{array} \right\}$ AND SIMPLIFY

$$\Rightarrow Z = 1.2135 - j0.8817 = 1.5e^{-j\pi/5}$$

$$\Rightarrow A = 3 \qquad \varphi = -\pi/5$$