PROBLEM 7.4:

(a) use filter coeffs: $H(z) = \frac{1}{3} + \frac{1}{3}z^{-1} + \frac{1}{3}z^{-2}$

(b) Use positive powers to extract poles and zeros

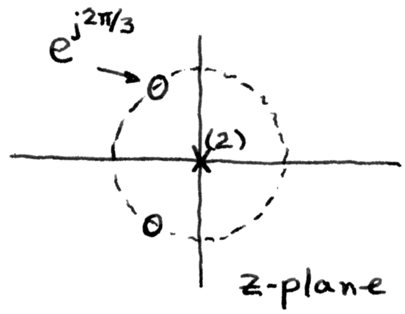
$$H(z) = \frac{1}{z^2}\left(\frac{1}{3}z^2 + \frac{1}{3}z + \frac{1}{3}\right)$$

TWO POLES AT $z=0$

zeros at

$$z = \frac{-1 \pm \sqrt{1-4}}{2} = -\frac{1}{2} \pm j\frac{\sqrt{3}}{2}$$

zeros: $1e^{\pm j2\pi/3}$



(c) $\mathcal{H}(\hat{\omega}) = H(e^{j\hat{\omega}}) = H(z)\big|_{z=e^{j\hat{\omega}}}$

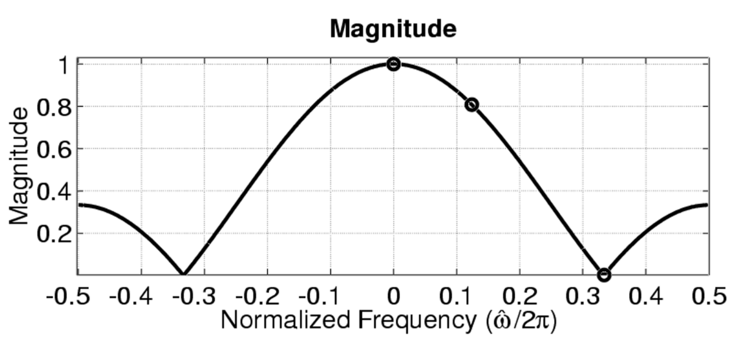
$$= \frac{1}{3} + \frac{1}{3}e^{-j\hat{\omega}} + \frac{1}{3}e^{-j2\hat{\omega}} = \frac{1}{3}e^{-j\hat{\omega}}\left(e^{j\hat{\omega}} + 1 + e^{-j\hat{\omega}}\right)$$

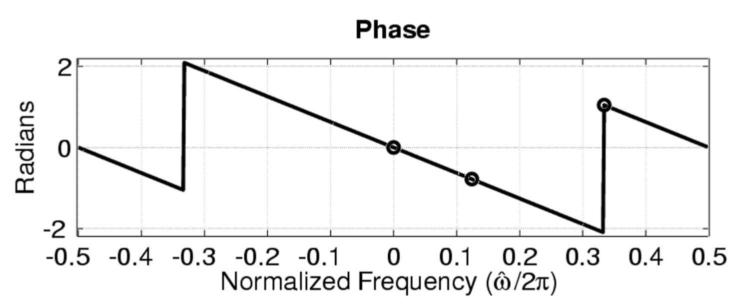
$$= e^{-j\hat{\omega}}\left(\frac{1+2\cos\hat{\omega}}{3}\right)$$

ANOTHER FORMULA:

$$\boxed{\mathcal{H}(\hat{\omega}) = e^{-j\hat{\omega}}\left(\frac{\sin(3\hat{\omega}/2)}{3\sin(\hat{\omega}/2)}\right)}$$

(d) use MATLAB





PROBLEM 7.6:

(a) $Y_1(z) = H_1(z) X(z)$

$$Y(z) = H_2(z) Y_1(z) = H_2(z)\left(H_1(z) X(z)\right)$$

$$= \left(\underbrace{H_2(z) H_1(z)}_{H(z)}\right) X(z) \quad \text{because } H(z) = \frac{Y(z)}{X(z)}$$

(b) Since $H_2(z)H_1(z) = H_1(z)H_2(z)$ because $H_1(z)$ and $H_2(z)$ are scalar functions.

$$\Rightarrow Y(z) = H_1(z) \underbrace{H_2(z) X(z)}$$

means that $H_2(z)$ is applied first

(c) $H_1(z) = \frac{1}{3}(1 + z^{-1} + z^{-2})$ by using the filter coeffs.

$$H(z) = H_2(z) H_1(z)$$

$$= \tfrac{1}{3}(1+z^{-1}+z^{-2}) \cdot \tfrac{1}{3}(1+z^{-1}+z^{-2})$$

$$= \tfrac{1}{9}(1 + 2z^{-1} + 3z^{-2} + 2z^{-3} + z^{-4})$$

(d) Convert to difference equation (i.e., filter coeffs)

$$y[n] = \tfrac{1}{9}\left(x[n] + 2x[n-1] + 3x[n-2] + 2x[n-3] + x[n-4]\right)$$

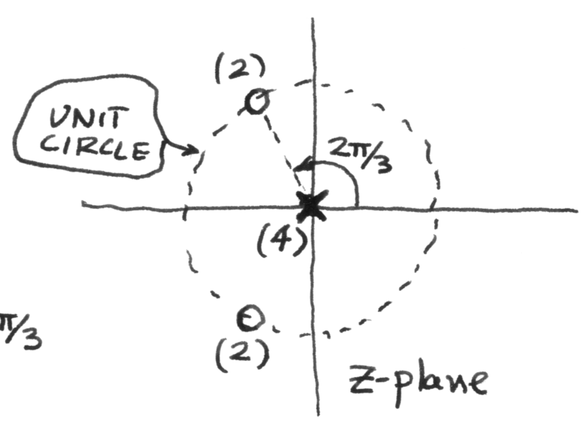
(e) Find the poles & zeros of $H_2(z)$, then "double" them because $H_1(z) = H_2(z)$.

$$H_2(z) = \tfrac{1}{3} z^{-2} \left(\underbrace{z^2 + z + 1}\right)$$

$\frac{1}{z^2}$ contributes two poles at $z=0$

zeros are:

$$\frac{-1 \pm \sqrt{1-4}}{2} = -\tfrac{1}{2} \pm j\tfrac{\sqrt{3}}{2} = e^{\pm j2\pi/3}$$



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PROBLEM 7.8:

(a) $h[n] = \frac{1}{4}\{\delta[n] + \delta[n-1] + \delta[n-2] + \delta[n-3]\}$

(b) $H(z) = \frac{1}{4}(1 + z^{-1} + z^{-2} + z^{-3})$ by using $h[n]$.

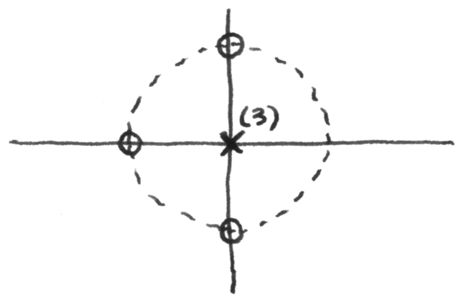
(c) Poles and zeros:

$$H(z) = \frac{1}{4}\,\frac{z^3 + z^2 + z + 1}{z^3}$$

3 Poles at $z = 0$

$$z^3 + z^2 + z + 1 = \frac{z^4 - 1}{z - 1}$$

zeros at $z = \pm j$ & $z = -1$



(d) $H(z) = \frac{1}{4}\,\frac{1 - z^{-4}}{1 - z^{-1}} = \frac{1}{4}(1 + z^{-1} + z^{-2} + z^{-3})$

$$H(e^{j\hat{\omega}}) = \frac{1}{4}\,\frac{1 - e^{-j4\hat{\omega}}}{1 - e^{-j\hat{\omega}}} = \frac{1}{4}\,\frac{e^{-j2\hat{\omega}}(e^{j2\hat{\omega}} - e^{-j2\hat{\omega}})}{e^{-j\hat{\omega}/2}(e^{+j\hat{\omega}/2} - e^{-j\hat{\omega}/2})}$$

$$= \frac{1}{4} e^{-j3\hat{\omega}/2}\,\frac{\sin(2\hat{\omega})}{\sin(\frac{1}{2}\hat{\omega})}$$

At $\hat{\omega} = 0$, $H(e^{j\hat{\omega}}) = \frac{1}{4} e^{j0} \cdot 4 = 1$

At $\hat{\omega} = \pi/2, \pi, -\pi/2$, $H(e^{j\hat{\omega}}) = 0$ because $\sin(2\hat{\omega}) = 0$.

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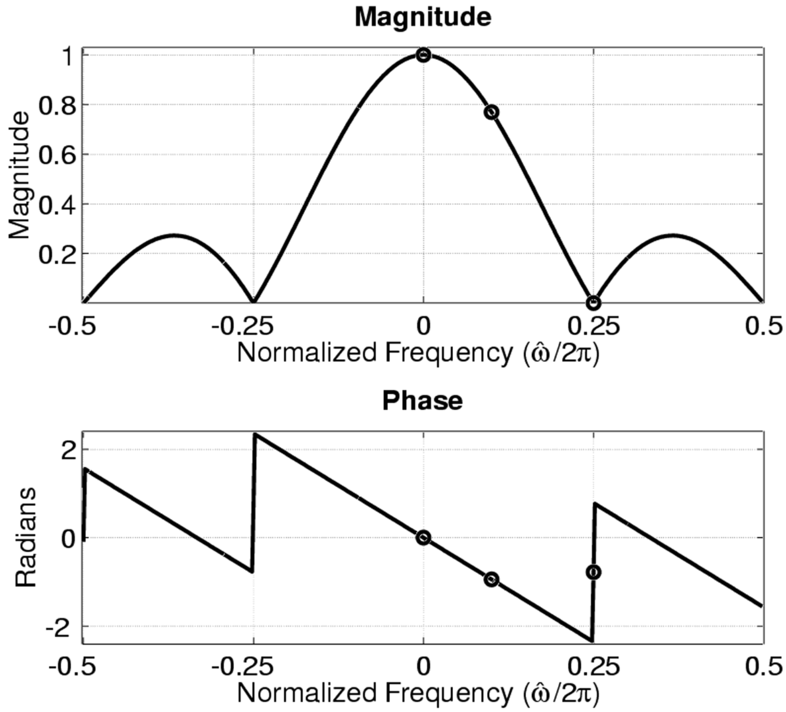
(e) use MATLAB



(f) Evaluate $H(e^{j\hat{\omega}})$ at $\hat{\omega}=0$, $\hat{\omega}=0.2\pi$ and $\hat{\omega}=0.5\pi$.

These are marked on the frequency response plots

$$H(e^{j0})=1 \qquad H(e^{j0.2\pi})=0.771e^{-j0.3\pi} \qquad H(e^{j0.5\pi})=0$$

$$\Rightarrow y[n] = 5 + 4(0.771)\cos(0.2\pi n - 0.3\pi) + 0$$

$$= 5 + 3.084\cos(0.2\pi n - 0.3\pi)$$

ANGLE $= -54^\circ$ or -0.94 rads

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PROBLEM 7.10:

(a) Convert $H(z)$ to a difference equation:

$$y[n] = x[n] - 3x[n-2] + 2x[n-3] + 4x[n-6]$$

The most delay is 6 samples, so the term $4\delta[n-4]$ in $x[n]$ is delayed to $16\delta[n-10]$.

The least amount of delay is $2\delta[n]$ experiencing no delay. Thus the output starts at $n=0$ and ends at $n=10$.

$$\Rightarrow \quad y[n] = 0 \quad \text{for } n<0 \ \& \ n>10$$

$N_1 = 0$ and $N_2 = 10$.

(b) $X(z) = 2 + z^{-1} - 2z^{-2} + 4z^{-4}$

$$\begin{aligned} Y(z) &= H(z)X(z) \\ &= (1 - 3z^{-2} + 2z^{-3} + 4z^{-6})(2 + z^{-1} - 2z^{-2} + 4z^{-4}) \\ &= 2 + z^{-1} - 2z^{-2} + 4z^{-4} - 6z^{-2} - 3z^{-3} + 6z^{-4} - 12z^{-6} \\ &\quad + 4z^{-3} + 2z^{-4} - 4z^{-5} + 8z^{-7} + 8z^{-6} + 4z^{-7} - 8z^{-8} + 16z^{-10} \end{aligned}$$

Combine terms with common exponents

$$\begin{aligned} Y(z) = {} & 2 + z^{-1} - 8z^{-2} + z^{-3} + 12z^{-4} - 4z^{-5} - 4z^{-6} + 12z^{-7} \\ & - 8z^{-8} + 16z^{-10} \end{aligned}$$

Invert:

$$\begin{aligned} y[n] = {} & 2\delta[n] + \delta[n-1] - 8\delta[n-2] + \delta[n-3] + 12\delta[n-4] \\ & - 4\delta[n-5] - 4\delta[n-6] + 12\delta[n-7] - 8\delta[n-8] + 16\delta[n-10] \end{aligned}$$

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PROBLEM 7.14:

$$H(z) = 1 - 2z^{-2} - 4z^{-4}$$

$$h[n] = \delta[n] - 2\delta[n-2] - 4\delta[n-4]$$

$$x[n] = 20e^{j0n} + 20\cos\left(\tfrac{\pi}{2}n + \tfrac{\pi}{4}\right) - 20\delta[n]$$

$$H(e^{j0})\cdot 20 \qquad \text{Need } H(e^{j\pi/2}) \qquad -20h[n]$$

$$H(e^{j\hat{\omega}}) = 1 - 2e^{-j2\hat{\omega}} - 4e^{-j4\hat{\omega}}$$

$$H(e^{j0}) = 1 - 2 - 4 = -5$$

$$H(e^{j\pi/2}) = 1 - 2e^{-j\pi} - 4e^{-j2\pi}$$

$$= 1 + 2 - 4 = -1$$

$$\therefore\ y[n] = -100 - 20\cos\left(\tfrac{\pi}{2}n + \tfrac{\pi}{4}\right) - 20\delta[n] + 40\delta[n-2] + 80\delta[n-4]$$

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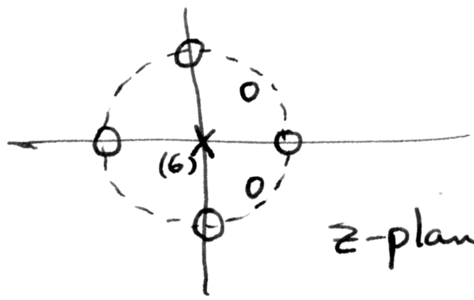
PROBLEM 7.16:

(a) $H(z)$ has 6 zeros & 6 poles at $z=0$

The zeros are: $z = \pm 1, \pm j, 0.8e^{\pm j\pi/4}$



z-plane

(b) $w[n] = x[n] - x[n-4]$

$$\Rightarrow H_1(z) = 1 - z^{-4} = (1+z^{-2})(1-z^{-2}).$$

To get $H_2(z)$ divide:

$$H_2(z) = \frac{H(z)}{H_1(z)} = (1-0.8e^{-j\pi/4}z^{-1})\cdot(1-0.8e^{+j\pi/4}z^{-1}).$$

$$H_2(z) = 1 - 1.6\cos\pi/4\, z^{-1} + .64z^{-2}$$

(c) $$y[n] = x[n] - \underbrace{(0.8\sqrt{2})}_{=1.1314}x[n-1] + 0.64x[n-2].$$

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