PROBLEM 9.2:

(a) An *exponentiation system* is defined by the input/output relation $y(t) = \exp\{x(t+2)\} = e^{x(t+2)}$

(i) *Linear:* The system is ***not*** linear because the sum of two inputs will give an output that is the product of the corresponding outputs:

$$\begin{aligned} x_1(t) &\rightarrow y_1(t) = e^{x_1(t+2)} \\ x_2(t) &\rightarrow y_2(t) = e^{x_2(t+2)} \\ x_1(t) + x_2(t) &\rightarrow e^{x_1(t+2)+x_2(t+2)} = e^{x_1(t+2)}e^{x_2(t+2)} = y_1(t)y_2(t) \end{aligned}$$

(ii) *Time-invariant:* The system is time-invariant because the system definition is a point-wise operator:

$$\begin{aligned} x_1(t) &\rightarrow y_1(t) = e^{x_1(t+2)} \\ x_1(t-t_1) &\rightarrow y_2(t) = e^{x_1(t+2-t_1)} = e^{x_1((t-t_1)+2)} = y_1(t-t_1) \end{aligned}$$

(iii) *Stable:* The system is stable because the system definition is a point-wise operator. If the input signal is bounded by M_x, i.e., $\max\{|x[n]|\} < M_x$, then the output signal is bounded by $M_y = e^{M_x}$.

(iv) *Causal:* The system is ***not*** causal because the system definition involves a time-shift of $(t+2)$ which is a shift by -2. Here is a counter-example:

$$x_1(t) = u(t) \rightarrow y_1(t) = e^{u(t+2)} = e^1 u(t+2)$$

In other words, the input "starts" at $t = 0$, while the output "starts earlier" at $t = -2$.

(b) A *phase modulator* is a system whose input and output satisfy a relation of the form $y(t) = \cos[\omega_c t + x(t)]$

(i) *Linear:* The system is ***not*** linear because the sum of two inputs will give an output that is the not the sum of the corresponding outputs. Let one of the input signals be the zero signal to get a counterexample:

$$\begin{aligned} x_1(t) &\rightarrow y_1(t) = \cos[\omega_c t + x_1(t)] \\ x_2(t) = 0 &\rightarrow y_2(t) = \cos[\omega_c t + x_2(t)] = \cos[\omega_c t] \\ x_1(t) + x_2(t) &\rightarrow \cos[\omega_c t + x_1(t) + x_2(t)] = \cos[\omega_c t + x_1(t)] = y_1(t) \neq y_1(t) + y_2(t) \end{aligned}$$

(ii) *Time-invariant:* The system is ***not*** time-invariant because the system definition contains a component that does not depend on $x(t)$. Here is a counterexample with a unit-step signal:

$$\begin{aligned} x_1(t) = \pi u(t) &\rightarrow y_1(t) = \cos[\omega_c t + \pi u(t)] = \cos[\omega_c t]u(-t) - \cos[\omega_c t]u(t) \\ x_1(t-1) = \pi u(t-1) &\rightarrow y_2(t) = \cos[\omega_c t + \pi u(t-1)] = \cos[\omega_c t]u(1-t) - \cos[\omega_c t]u(t-1) \\ \text{but, } y_1(t-1) &= \cos[\omega_c(t-1)]u(1-t) - \cos[\omega_c(t-1)]u(t-1) \end{aligned}$$

Thus, $y_2(t) \neq y_1(t-1)$ which means that $y_2(t)$ is not a shifted version of $y_1(t)$.

PROBLEM 9.2 (more):

(iii) *Stable:* The system is stable because the output will always be bounded by one, independent of the values of $x(t)$.

(iv) *Causal:* The system is causal because the output $y(t)$ depends only on the value of the input $x(t)$ **at the same time**. No values of $x(t)$ from the future (or the past) are used.

(c) An *amplitude modulator* is a system whose input and output satisfy a relation of the form $y(t) = [A + x(t)]\cos(\omega_c t)$

(i) *Linear:* The system is ***not*** linear because the sum of two inputs will give an output that is the not the sum of the corresponding outputs. Let one of the input signals be the zero signal to get a counterexample:

$$\begin{aligned}
x_1(t) &\to y_1(t) = [A + x_1(t)]\cos(\omega_c t) \\
x_2(t) = 0 &\to y_2(t) = [A + x_2(t)]\cos(\omega_c t) = A\cos[\omega_c t] \\
x_1(t) + x_2(t) &\to [A + x_1(t) + x_2(t)]\cos(\omega_c t) = [A + x_1(t)]\cos(\omega_c t) = y_1(t) \neq y_1(t) + y_2(t)
\end{aligned}$$

(ii) *Time-invariant:* The system is ***not*** time-invariant because the system definition contains a component that does not depend on $x(t)$. Here is a counterexample with a unit-step signal:

$$\begin{aligned}
x_1(t) = -Au(t) &\to y_1(t) = [A - Au(t)]\cos(\omega_c t) = A\cos[\omega_c t]u(-t) \\
x_1(t-1) = Au(t-1) &\to y_2(t) = [A - Au(t-1)]\cos(\omega_c t) = A\cos[\omega_c t]u(1-t) \\
\text{but, } y_1(t-1) &= A\cos[\omega_c(t-1)]u(1-t)
\end{aligned}$$

Thus, $y_2(t) \neq y_1(t-1)$ which means that $y_2(t)$ is not a shifted version of $y_1(t)$.

(iii) *Stable:* The system is stable because the output will always be bounded by $|A + \max\{|x[n]|\}|$.

(iv) *Causal:* The system is causal because the output $y(t)$ depends only on the value of the input $x(t)$ **at the same time**. No values of $x(t)$ from the future (or the past) are used.

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PROBLEM 9.2 (more):

(d) A system that takes the even part of an input signal is defined by a relation of the form $y(t) = \mathcal{E}v\{x(t)\} = \dfrac{x(t) + x(-t)}{2}$

(i) *Linear:* The system is linear, so we have to prove both the scaling property and the superposition property:

$$\begin{aligned}
x_1(t) &\rightarrow y_1(t) = \tfrac{1}{2}x_1(t) + \tfrac{1}{2}x_1(-t) \\
x_2(t) &\rightarrow y_2(t) = \tfrac{1}{2}x_2(t) + \tfrac{1}{2}x_2(-t) \\
x_1(t) + x_2(t) &\rightarrow \tfrac{1}{2}(x_1(t) + x_2(t)) + \tfrac{1}{2}(x_1(-t) + x_2(-t)) \\
&\qquad = \tfrac{1}{2}(x_1(t) + x_1(-t)) + \tfrac{1}{2}(x_2(t) + x_2(-t)) = y_1(t) + y_2(t) \\
\beta x_1(t) &\rightarrow \tfrac{1}{2}(\beta x_1(t)) + \tfrac{1}{2}(\beta x_1(-t)) = \beta\left(\tfrac{1}{2}x_1(t) + \tfrac{1}{2}x_1(-t)\right) = \beta y_1(t)
\end{aligned}$$

(ii) *Time-invariant:* The system is ***not*** time-invariant because the system definition contains a flip. Here is a counterexample with a unit-impulse signal:

$$x_1(t) = \delta(t) \rightarrow y_1(t) = \tfrac{1}{2}\{\delta(t) + \delta(-t)\} = \delta(t)$$

$$x_1(t-1) = \delta(t-1) \rightarrow y_1(t) = \tfrac{1}{2}\{\delta(t-1) + \delta(-t-1))\} = \tfrac{1}{2}$$

$$delta(t-1) + \tfrac{1}{2}\delta(t+1)\}$$

$$\text{but, } y_1(t-1) = \delta(t-1)$$

Thus, $y_2(t) \neq y_1(t-1)$ which means that $y_2(t)$ is not a shifted version of $y_1(t)$.

(iii) *Stable:* The system is stable because the output will always be bounded by $\max\{|x[n]|\}|$.

$$\max\{|y[n]|\} = \max\{|\tfrac{1}{2}x(t) + \tfrac{1}{2}x(-t)|\} \leq \tfrac{1}{2}\max\{|x[n]|\}| + \tfrac{1}{2}\max\{|x[n]|\}|$$

(iv) *Causal:* The system is ***not*** causal because the flip component of the system definition creates a component in negative time. The signal $\delta(t-1)$ provides a counterexample. From above, the input "starts" at $t = 1$, while the output "starts earlier" at $t = -1$.

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PROBLEM 9.3:

(a) $\delta(t-10) * \left[\delta(t+10) + 2e^{-t}u(t) + \cos(100\pi t)\right]$

$$= \delta(t-10) * \delta(t+10) + \delta(t-10) * 2e^{-t}u(t) + \delta(t-10) * \cos(100\pi t)$$

$$= \delta(t) + 2e^{-(t-10)}u(t-10) + \cos\left[100\pi(t-10)\right]$$

(b) $\cos(100\pi t)\left[\delta(t) + \delta(t-.002)\right]$

$$= \cos(100\pi \cdot 0)\,\delta(t) + \cos(100\pi(.002))\,\delta(t-.002)$$

$$= 1\,\delta(t) + \cos(0.2\pi)\,\delta(t-.002) = \delta(t) + 0.809\,\delta(t-.002)$$

(c) $\dfrac{d}{dt}\left[e^{-2(t-2)}u(t-2)\right]$ Use formula for derivative of a product.

$$= \frac{d}{dt}\left[e^{-2t}e^{4}\,u(t-2)\right] = e^{4}(-2)e^{-2t}u(t-2) + e^{-2t}e^{4}\delta(t-2)$$

$$= -2e^{4}e^{-2t}u(t-2) + e^{-2(2)}e^{4}\delta(t-2)$$

$$= -2e^{4}e^{-2t}u(t-2) + \delta(t-2)$$

(d) $\displaystyle\int_{-\infty}^{t} \cos(100\pi\tau)\left[\delta(\tau) + \delta(\tau-.002)\right]d\tau$

$$= \int_{-\infty}^{t} \cos(100\pi \cdot 0)\,\delta(\tau)\,d\tau + \int_{-\infty}^{t} \cos(100\pi(.002))\,\delta(\tau-.002)\,d\tau$$

$$= 1\,u(t) + \cos(0.2\pi)\,u(t-.002) = u(t) + 0.809\,u(t-.002)$$

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PROBLEM 9.5:

Solve for $h(t)$ in

$$\left[e^{-(t-4)}u(t-4)\right] * h(t) = 2e^{-t}u(t)$$

In order to find $h(t)$, use the shifting property of the impulse:

$$x(t) * \delta(t-t_1) = x(t-t_1)$$

Thus we can write the first term above as

$$e^{-t}u(t) * \delta(t-4) = e^{-(t-4)}u(t-4)$$

Then we must solve:

$$e^{-t}u(t) * \left(\delta(t-4) * h(t)\right) = 2e^{-t}u(t)$$

which requires that

$$\delta(t-4) * h(t) = 2\delta(t)$$

Since $\delta(t-a) * \delta(t-b) = \delta(t-a-b)$ we conclude that $\boxed{h(t) = 2\delta(t+4)}$

i.e., $\delta(t+4) * 2\delta(t+4) = 2\delta(t)$

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PROBLEM 9.6:

(a) $x(t)[\delta(t+1)+\delta(t-1)]$

$= x(t)\delta(t+1) + x(t)\delta(t-1)$ impulse at $t=1$

$= x(-1)\delta(t+1) + x(1)\delta(t-1)$

(b) $\int_{-\infty}^{\infty} x(\tau)\delta(\tau-1)\,d\tau$ impulse at $\tau=1$

$= \int_{-\infty}^{\infty} x(1)\delta(\tau-1)\,d\tau = x(1)\int_{-\infty}^{\infty}\delta(\tau-1)\,d\tau = x(1)$

(c) $\int_{-\infty}^{\infty} x(\tau)\delta(t-\tau)\,d\tau$ impulse at $\tau=t$

$\therefore$ replace τ with t in $x(\tau)$

$= \int_{-\infty}^{\infty} x(t)\delta(t-\tau)\,d\tau$

AREA of $\delta(\,)$ is one

$= x(t)\int_{-\infty}^{\infty}\delta(t-\tau)\,d\tau = x(t)$

(d) $\delta^{(1)}(t) * x(t-1) = x^{(1)}(t-1)$ Using eq. (9.47)

Recall that $x^{(1)}(t)$ is the first derivative.

Thus, $x^{(1)}(t-1) = \frac{d}{dt}x(t-1)$

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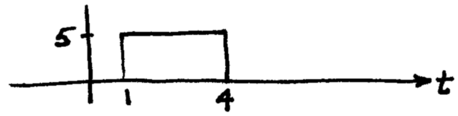
PROBLEM 9.9:

$$h(t) = 5u(t-1) - 5u(t-4)$$



(a)

$$y(t) = u(t) * h(t)$$
$$= u(t) * [5u(t-1) - 5u(t-4)]$$
$$= 5u(t) * u(t-1) - 5u(t) * u(t-4)$$

Use the fact that $u(t) * u(t) = t\,u(t)$ which can be combined with the shift property to write $u(t) * u(t-a) = (t-a)u(t-a)$

Thus, $y(t) = 5(t-1)u(t-1) - 5(t-4)u(t-4)$

(b) The three regions are: $t<1$, $1 \le t \le 4$, and $t>4$.

When $t<1$, both unit-step signals are zero, so $y(t)=0$

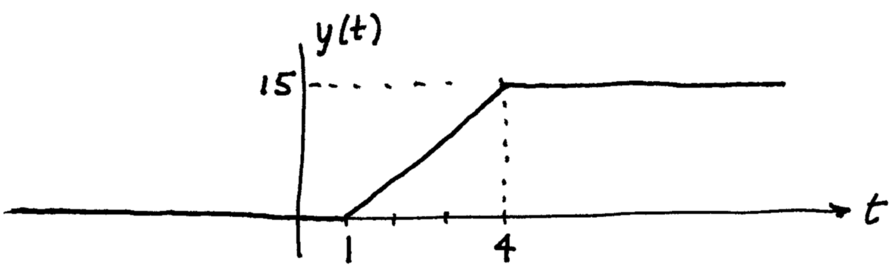
For $1 \le t \le 4$, $u(t-1)=1$ and $u(t-4)=0$, so $y(t) = 5t-5$

For $t>4$, $u(t-1)=1$ and $u(t-4)=1$ so $y(t) = 5t-5-5t+20 = 15$

In summary,

$$y(t) = \begin{cases} 0 & t<1 \\ 5t-5 & 1 \le t \le 4 \\ 15 & 4<t \end{cases}$$

(c)

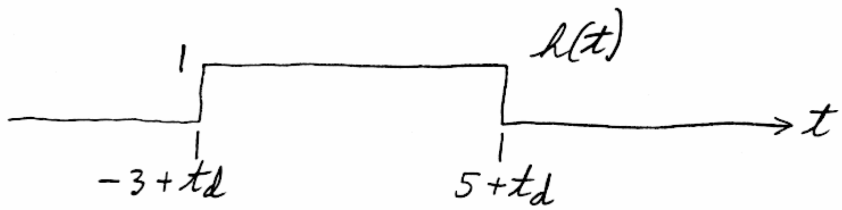


PROBLEM 9.22:

(a) $v(t) = u(t+3) - u(t-5)$

$$h(t) = y(t)\Big|_{x(t)=\delta(t)} = v(t) * \delta(t-t_d) = \delta(t-t_d) * v(t)$$

$$\therefore\ h(t) = u(t+3-t_d) - u(t-5-t_d)$$



(b) $t_d \geq 3$ because $h(t) = 0$ for $t < 0$

(c) #1 and #2 are not <u>stable</u> because

$$\int_{-\infty}^{\infty} |h_1(t)|\,dt \to \infty \quad \text{and} \quad \int_{-\infty}^{\infty} |h_2(t)|\,dt \to \infty$$

#3 is stable

The overall system <u>is</u> stable because

$$\int_{-\infty}^{\infty} |h(t)|\,dt < \infty$$

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