PROBLEM 11.4:

(a) $x(t) = u(t) - u(t-4)$ is a shifted pulse

$$= \delta(t-2) * \underbrace{\left[u(t+2) - u(t-2)\right]}$$

time-shift

F.T. $= \dfrac{\sin(2\omega)}{\omega/2}$

$$X(j\omega) = e^{-j2\omega} \frac{\sin(2\omega)}{\omega/2}$$

(b) Each impulse in ω inverts to a complex exponential

$$S(j\omega) = 4\pi\delta(\omega) + 2\pi\delta(\omega - 10\pi) + 2\pi\delta(\omega + 10\pi)$$

$$s(t) = 2e^{j0} + e^{j10\pi t} + e^{-j10\pi t}$$

$$= 2 + 2\cos(10\pi t)$$

(c) $R(j\omega) = \frac{1}{2} - \frac{2}{4 + j2\omega} = \frac{1}{2} - \frac{1}{2 + j\omega}$

$$r(t) = \tfrac{1}{2}\delta(t) - e^{-2t}u(t)$$

(d) $y(t) = \delta(t+1) + 2\delta(t) + \delta(t-1)$

$$Y(j\omega) = e^{j\omega} + 2 + e^{-j\omega}$$

$$= 2 + 2\cos(\omega)$$

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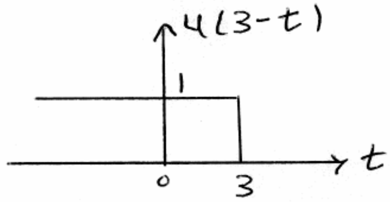
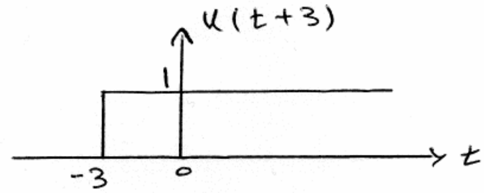
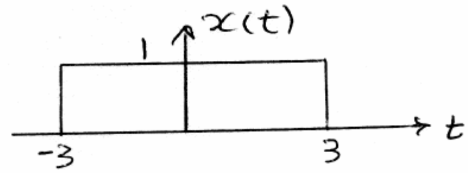
PROBLEM 11.6:

(a)





$$x(t) = u(t+3)\cdot u(3-t) \longrightarrow$$



$$T_0/2 = 3 \rightarrow X(j\omega) = \frac{\sin(3\omega)}{\omega/2}$$

(b) Note

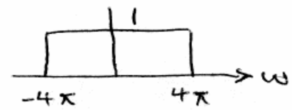
	t-domain		ω-domain
	$\sin 4\pi t$	$\longleftrightarrow$	$\frac{\pi}{j}\delta(\omega-4\pi) - \frac{\pi}{j}\delta(\omega+4\pi)$

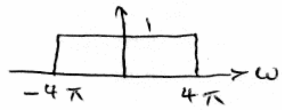
Convolution property of the Fourier Transform:

$$X(j\omega) = \frac{1}{2\pi}\left\{\frac{\pi}{j}\delta(\omega-4\pi) - \frac{\pi}{j}\delta(\omega+4\pi)\right\} * \left\{\frac{\pi}{j}\delta(\omega-50\pi) - \frac{\pi}{j}\delta(\omega+50\pi)\right\}$$

$$X(j\omega) = \frac{\pi}{2}\delta(\omega-46\pi) + \frac{\pi}{2}\delta(\omega+46\pi) - \frac{\pi}{2}\delta(\omega-54\pi) - \frac{\pi}{2}\delta(\omega+54\pi)$$

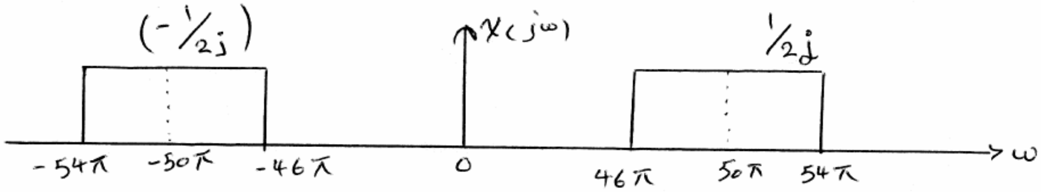
PROBLEM 11.6 (more):

(c) Note $\frac{\sin 4\pi t}{\pi t} \longleftrightarrow$ 

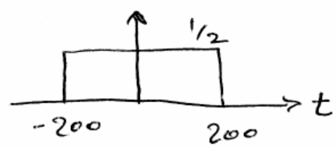
$X(j\omega) = \frac{1}{2\pi}\Big\{$  $\Big\} * \Big\{ \frac{\pi}{j}\delta(\omega - 50\pi) - \frac{\pi}{j}\delta(\omega + 50\pi)\Big\}$

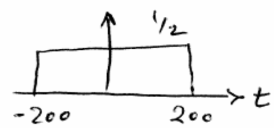
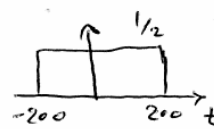
convolution

Thus



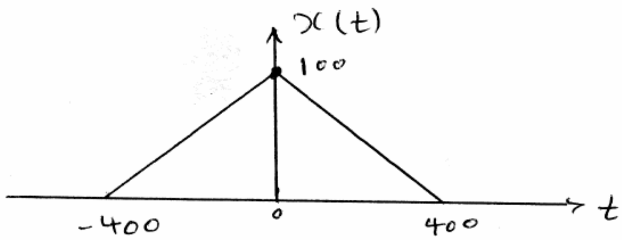
$$X(j\omega) = \begin{cases} \frac{1}{2j} & 46\pi \le \omega \le 54\pi \\ -1/2j & -54\pi \le \omega \le -46\pi \\ 0 & \text{else} \end{cases}$$

(d) Note: $\frac{1}{2}\frac{\sin(200\omega)}{\omega/2} \longleftrightarrow$ 

$\frac{\sin^2(200\omega)}{\omega^2} \longleftrightarrow \Big\{$  $*$  $\Big\}$

convolution

Thus



(e) $\cos\omega \longleftrightarrow \frac{1}{2}\{\delta(t-1) + \delta(t+1)\}$

$\cos^2\omega \longleftrightarrow \frac{1}{2}\{\delta(t-1) + \delta(t+1)\} * \frac{1}{2}\{\delta(t-1) + \delta(t+1)\}$

$x(t) = \frac{1}{4}\{\delta(t-2) + \delta(t+2) + 2\,\delta(t)\}$

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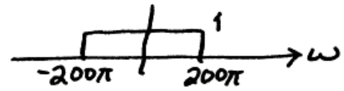
PROBLEM 11.7:

(a) Use derivative property: $\frac{d}{dt}x(t) \longrightarrow j\omega X(j\omega)$

F.T. of $\dfrac{\sin(200\pi t)}{\pi t}$ is a rectangle



Thus

$$X(j\omega) = \begin{cases} j10\omega & \text{if } |\omega| \leq 200\pi \\ 0 & \text{if } |\omega| > 200\pi \end{cases}$$

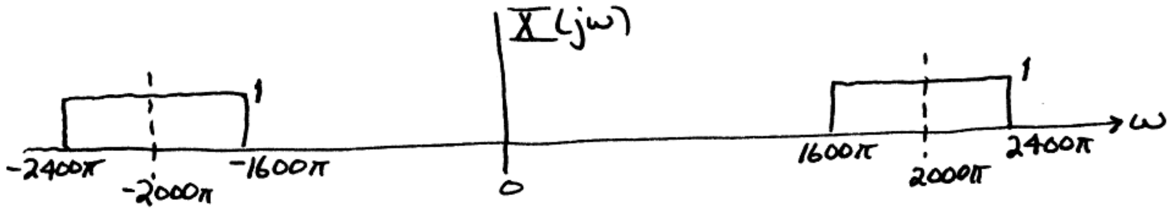
or,

$$X(j\omega) = j10\omega\left[u(\omega+200\pi) - u(\omega-200\pi)\right]$$

(b) Multiply by cosine $\Longrightarrow$ frequency shifting

$$x(t) = 2\,\underbrace{\frac{\sin(400\pi t)}{\pi t}}_{\text{F.T. is a rectangle}}\left\{\tfrac{1}{2}e^{j2000\pi t} + \tfrac{1}{2}e^{-j2000\pi t}\right\}.$$

shift to $\omega = 2000\pi$ shift to $\omega = -2000\pi$



(c) The Fourier Transform of an impulse train in time is a (different) impulse train in frequency.

$T = 10$ secs from the definition of $x(t)$.

$$\Rightarrow \quad X(j\omega) = \frac{2\pi}{10}\sum_{k=-\infty}^{\infty}\delta\left(\omega - \frac{2\pi k}{10}\right)$$

spacing is $\frac{\pi}{5}$ rads.

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PROBLEM 11.8:

(a) $X(j\omega) = e^{-j3\omega}\left(\dfrac{1}{2+j\omega}\right)$ — FT^{-1} is $e^{-2t}u(t)$

$$x(t) = e^{-2(t-3)}u(t-3)$$

(b) $X(j\omega) = j\omega\left(\dfrac{1}{2+j\omega}\right)$ use derivative property

$$x(t) = \frac{d}{dt}\left\{e^{-2t}u(t)\right\} = \underbrace{e^{-2t}\delta(t)}_{\text{eval @ } t=0} - 2e^{-2t}u(t)$$

$$x(t) = \delta(t) - 2e^{-2t}u(t)$$

(c) $X(j\omega) = e^{-j3\omega}\left(\dfrac{j\omega}{2+j\omega}\right)$ use time-shift on the result of (b)

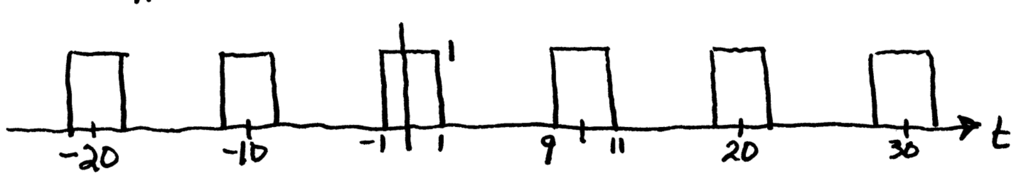
$$x(t) = \delta(t-3) - 2e^{-2(t-3)}u(t-3)$$

(d) $$\frac{2\sin(\omega)}{\omega} = \frac{\sin(\omega)}{\omega/2} \xrightarrow{FT^{-1}} u(t+1) - u(t-1)$$

$$\frac{2\pi}{10}\sum_{k=-\infty}^{\infty}\delta\left(\omega - \frac{2\pi}{10}k\right) \xrightarrow{FT^{-1}} \sum_{n=-\infty}^{\infty}\delta(t-10n)$$

Convolve: $\left[u(t+1) - u(t-1)\right] * \displaystyle\sum_{n=-\infty}^{\infty}\delta(t-10n)$

$$= \sum_{n=-\infty}^{\infty}\left[u(t+1-10n) - u(t-1-10n)\right]$$



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PROBLEM 11.13:

(a) Since the system is LINEAR, the two inputs can be treated separately and then combined.

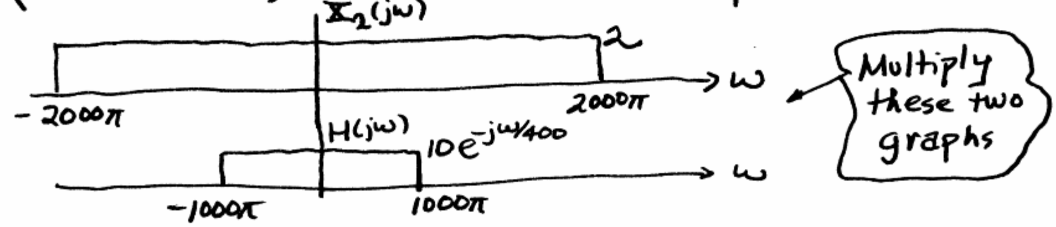
- For the cosine input, the output will be a cosine with a new magnitude and phase. We evaluate $H(j\omega)$ at the input frequency: $\omega = 200\pi$ rad/s.

$$H(j200\pi) = 10e^{-j(200\pi)(0.0025)} = 10e^{-j0.5\pi}$$

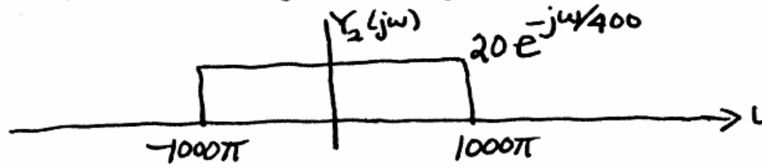
Call this output $y_1(t)$: $\quad y_1(t) = 10\cos(200\pi t - \pi/2)$

- For the "sinc" input, take the F.T. of the input, then multiply by $H(j\omega)$ and then inverse transform.

$$\text{F.T.}\left\{2\frac{\sin(2000\pi t)}{\pi t}\right\} = \text{Rectangular Shape: width of } 4000\pi \text{ rad/s. amp} = 2$$



Thus $Y_2(j\omega) = H(j\omega)X_2(j\omega)$ is also a rectangle



$$\boxed{\frac{1}{400} = 0.0025\text{s}}$$

The inverse F.T. of this rectangle is a <u>SHIFTED</u> "sinc"

$$y_2(t) = 20\frac{\sin(1000\pi(t - 1/400))}{\pi(t - 1/400)}$$

Finally, the total output is the sum of $y_1(t)$ & $y_2(t)$

$$y(t) = 10\cos(200\pi t - \pi/2) + 20\frac{\sin(1000\pi(t - 1/400))}{\pi(t - 1/400)}$$

We have used <u>SUPERPOSITION</u> to do this part.

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PROBLEM 11.13 (more):

(b) Use SUPERPOSITION again. Two of the inputs are the same, so we don't have to rework them. We only need to consider the input $x_3(t) = \cos(3000\pi t)$. For a cosine input, we must evaluate $H(j\omega)$ at the input frequency; in this case, at $\omega = 3000\pi$.

$H(j3000\pi) = 0 \Rightarrow$ NO OUTPUT, i.e. $y_3(t) = 0$.

So, the answer is the same as part (a) !

(c) Again, use SUPERPOSITION. We already know the output for $x_1(t) = \cos(200\pi t)$

$$y_1(t) = 10\cos(200\pi t - \pi/2)$$

We need to find the output for $x_4(t) = 2\delta(t)$.

Thus we need the impulse response

But this is just the inverse F.T. of $H(j\omega)$

And we already know that is a shifted "sinc"

$$y_4(t) = 2h(t) = 20\frac{\sin(1000\pi(t - 1/400))}{\pi(t - 1/400)}$$

Finally,

$$y(t) = y_1(t) + y_4(t)$$
$$= 10\cos(200\pi t - \pi/2) + 20\frac{\sin(1000\pi(t - 1/400))}{\pi(t - 1/400)}$$

It is very interesting to see that all three parts have the same answer. Why? Because the filter is an ideal LPF, so Only the part of the input signal between -1000π and $+1000\pi$ matters. For example, in part (c) the F.T. of $2\delta(t)$ is $X_4(j\omega) = 2$ for all ω, but only the part for $|\omega| < 1000\pi$ rad/s matters. Over that range the "sinc" input of part (a) is the same

(d) Superposition simplifies the work.

PROBLEM 11.14:

The periodic input to the above system is defined by the equation:

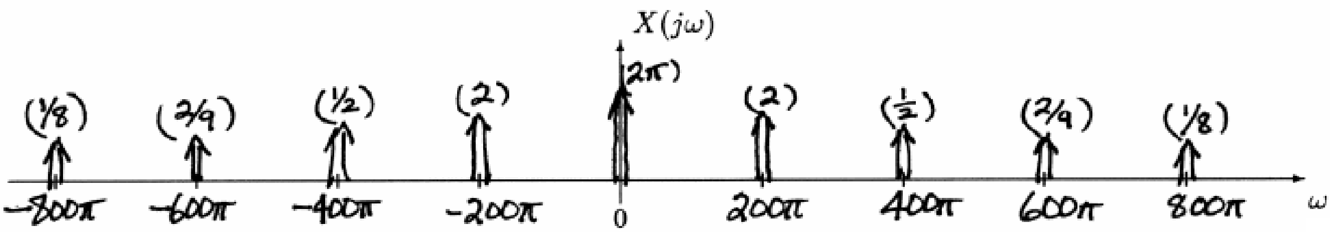
$$x(t) = \sum_{k=-4}^{4} a_k e^{j200\pi kt}, \quad \text{where} \quad a_k = \begin{cases} \dfrac{1}{\pi|k|^2} & k \neq 0 \\ 1 & k = 0 \end{cases}$$

$a_1 = 1/\pi$
$a_2 = 1/4\pi$
$a_3 = 1/9\pi$
$a_4 = 1/16\pi$

(a) Determine the Fourier transform of the periodic signal $x(t)$. Give a formula and then plot it on the graph below.

$$X(j\omega) = \sum_{k=-4}^{4} 2\pi a_k \delta(\omega - 200\pi k)$$

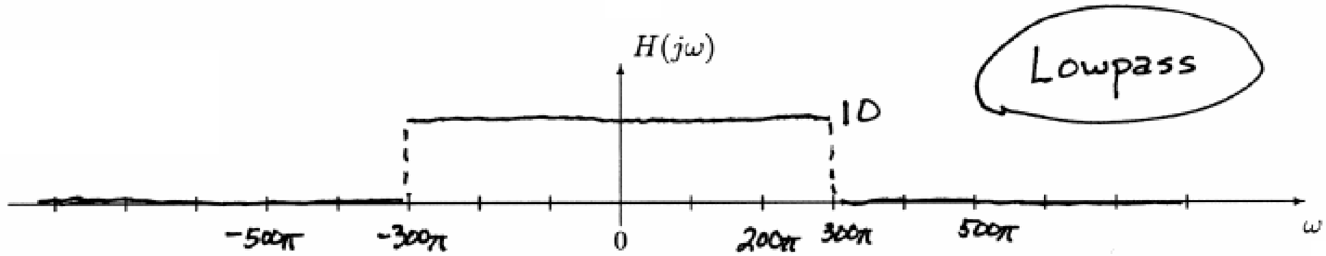
$\omega_0 = 200\pi$



(b) The frequency response of the LTI system is given by the following equation:

$$H(j\omega) = \begin{cases} 10 & |\omega| \leq 300\pi \\ 0 & |\omega| > 300\pi \end{cases}.$$

Plot this function on the graph below using the same frequency scale as the plot in part (a). *Note carefully what type of filter (i.e., lowpass, bandpass, highpass) this is.*



Lowpass

(c) Write an equation for $y(t)$.

$$Y(j\omega) = H(j\omega)X(j\omega) = 20\pi\delta(\omega) + 20\delta(\omega - 200\pi) + 20\delta(\omega + 200\pi)$$

Invert: $y(t) = 10 + \underbrace{\frac{10}{\pi} e^{j200\pi t} + \frac{10}{\pi} e^{-j200\pi t}}_{\text{use Euler's inverse formula}}$

$$y(t) = 10 + \frac{20}{\pi}\cos(200\pi t)$$

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PROBLEM 11.16:

$x(t)$ is real $\Rightarrow$ $x^*(t) = x(t)$

If $x(t) \longrightarrow X(j\omega)$, then $x^*(t) \longrightarrow X^*(-j\omega)$

$\Rightarrow X^*(-j\omega) = X(j\omega)$

Express $X(j\omega)$ in terms of its real and imaginary parts:

$$X(j\omega) = A(\omega) + j\,B(\omega)$$

Then $X^*(-j\omega) = A(-\omega) - j\,B(-\omega)$

$\Rightarrow A(\omega) = A(-\omega)$ and $-B(\omega) = B(-\omega)$

(a) The magnitude is even:

$$|X(-j\omega)| = \sqrt{A^2(-\omega) + B^2(-\omega)}$$
$$= \sqrt{A^2(\omega) + B^2(\omega)} = |X(j\omega)|$$

(b) The phase is odd:

$$\angle X(-j\omega) = \mathrm{Tan}^{-1}\left\{\frac{B(-\omega)}{A(-\omega)}\right\}$$
$$= \mathrm{Tan}^{-1}\left\{\frac{-B(\omega)}{A(\omega)}\right\}$$
$$= -\mathrm{Tan}^{-1}\left\{\frac{B(\omega)}{A(\omega)}\right\} = -\angle X(j\omega)$$

Recall that the tangent function is an <u>ODD</u> function.

PROBLEM 11.17:

Define $s(t) = u(t) - \frac{1}{2}$

$$\Rightarrow \quad S(j\omega) = U(j\omega) - \pi\delta(\omega)$$

Since $s(t)$ is an odd function: $s(-t) = -s(t)$

and $s(-t) \xrightarrow{FT} S(-j\omega)$

$$S(-j\omega) = -S(j\omega)$$

$$\Rightarrow \quad U(-j\omega) \underbrace{- \pi\delta(-\omega)}_{=\pi\delta(\omega)} = -U(j\omega) + \pi\delta(\omega)$$

$$U(-j\omega) = -U(j\omega) + 2\pi\delta(\omega)$$

Thus, if we assume $U(j\omega) = \frac{1}{j\omega} + K\delta(\omega)$

$$\frac{1}{-j\omega} + K\delta(-\omega) = -\frac{1}{j\omega} - K\delta(\omega) + 2\pi\delta(\omega)$$

$$\Rightarrow \quad 2K\delta(\omega) = 2\pi\delta(\omega)$$

$$\Rightarrow \quad K = \pi$$

Note: $\delta(-\omega) = \delta(\omega)$, i.e. $\delta(\,)$ is <u>even</u>

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