EEE 391: Basics of Signals and Systems
Analytical Assignment 1
Fall 2024-2025
Deadline: November 04, 2024 by 23.59 on Moodle

Questions

1. Determine whether or not each of the following signals is periodic. If a signal is periodic, specify its fundamental period.

   (a) $x_1(t) = je^{j10t}$

   (b) $x_2(t) = e^{(-1+j)t}$

   (c) $x_3[n] = e^{j7\pi n}$

   (d) $x_4[n] = 3e^{j3\pi(n+1/2)/5}$

   (e) $x_5[n] = 3e^{j3/5(n+1/2)}$

2. We introduced a number of general properties of systems. In particular, a system may or may not be:

   - Memoryless
   - Time invariant
   - Linear
   - Causal

   Determine which of these properties hold and which do not hold for each of the following continuous-time systems. In each example, $y(t)$ denotes the system output and $x(t)$ is the system input.

   (a) $y(t) = x(t-2) + x(2-t)$

   (b) $y(t) = \cos(3t)x(t)$

   (c) $y(t) = \frac{d}{dt}x(t)$

   (d) $y(t) = x\left(\frac{t}{3}\right)$

3. For the continuous-time periodic signal:

$$x(t) = 2 + \cos\left(\frac{2\pi}{3}t\right) + 4\sin\left(\frac{5\pi}{3}t\right)$$

   determine the fundamental frequency ω_0 and the Fourier series coefficients a_k such that:

$$x(t) = \sum_{k=-\infty}^{\infty} a_k e^{jk\omega_0 t}$$

4. A continuous-time periodic signal $x(t)$ is real valued and has a fundamental period $T = 8$. The nonzero Fourier series coefficients for $x(t)$ are:

$$a_1 = a_{-1} = 2, \quad a_3 = a_{-3}^* = 4j$$

Express $x(t)$ in the form:

$$x(t) = \sum_{k=0}^{\infty} A_k \cos(\omega_k t + \phi_k)$$

5. Let $x(t) = 3\sin(22\pi t)$. In each of the following, the discrete-time signal $x[n]$ is obtained by sampling $x(t)$ at a rate f_s, and the resultant $x[n]$ can be written as: $x[n] = A\cos(\hat{\omega}_0 n + \phi)$. For each part below, determine the values of A, ϕ, and $\hat{\omega}_0$. In addition, state whether or not the signal has been oversampled or undersampled.

   (a) Let the sampling frequency be $f_s = 10$ samples/sec.

   (b) Let the sampling frequency be $f_s = 25$ samples/sec.

   (c) Let the sampling frequency be $f_s = 15$ samples/sec.

6. A linear system S has the relationship:

$$y[n] = \sum_{k=-\infty}^{\infty} x[k]g[n-2k]$$

   between its input $x[n]$ and its output $y[n]$, where $g[n] = u[n] - u[n-4]$.

   (a) Determine $y[n]$ when $x[n] = \delta[n-1]$.

   (b) Determine $y[n]$ when $x[n] = \delta[n-2]$.

   (c) Is S LTI (Linear Time-Invariant)?

   (d) Determine $y[n]$ when $x[n] = u[n]$.

7. Let:

$$x[n] = \delta[n] + 2\delta[n-1] - \delta[n-3], \quad h[n] = 2\delta[n+1] + 2\delta[n-1]$$

   Compute and plot each of the following convolutions:

   (a) $y_1[n] = x[n] * h[n]$

   (b) $y_2[n] = x[n+2] * h[n]$

   (c) $y_3[n] = x[n] * h[n+2]$

8. The frequency response of a linear time-invariant filter is given by the formula:

$$H(\hat{\omega}) = (1 - e^{-j\hat{\omega}})(1 - e^{j\pi/3}e^{-j\hat{\omega}})(1 - e^{-j\pi/3}e^{-j\hat{\omega}})$$

   (a) Write the difference equation that gives the relation between the input $x[n]$ and the output $y[n]$.

   (b) What is the output if the input is $x[n] = \delta[n]$?

   (c) If the input is of the form $x[n] = Ae^{j\phi}e^{j\hat{\omega}n}$, for what values of $-\pi \leq \hat{\omega} \leq \pi$ will $y[n] = 0$ for all n?