EEE 391: Basics of Signals and Systems
Analytical Assignment 1 Solutions

Solutions

1.9. (a) $x_1(t)$ is a periodic complex exponential.

$$x_1(t) = je^{j10t} = e^{j(10t+\frac{\pi}{2})}$$

The fundamental period of $x_1(t)$ is $\frac{2\pi}{10} = \frac{\pi}{5}$.

(b) $x_2(t)$ is a complex exponential multiplied by a decaying exponential. Therefore, $x_2(t)$ is not periodic.

(c) $x_3[n]$ is a periodic signal.

$$x_3[n] = e^{j7\pi n} = e^{j\pi n}$$

$x_3[n]$ is a complex exponential with a fundamental period of $\frac{2\pi}{\pi} = 2$.

(d) $x_4[n]$ is a periodic signal. The fundamental period is given by $N = m(\frac{2\pi}{3\pi/5}) = m(\frac{10}{3})$. By choosing $m = 3$, we obtain the fundamental period to be 10.

(e) $x_5[n]$ is not periodic. $x_5[n]$ is a complex exponential with $\omega_0 = 3/5$. We cannot find any integer m such that $m(\frac{2\pi}{\omega_0})$ is also an integer. Therefore, $x_5[n]$ is not periodic.

Figure 1: Solution for Question 1

3)

- Linear
- Memoryless, linear, casual
- Time invariant, linear, casual
- Linear

Figure 2: Solution for Question 2

3.3. The given signal is

$$\begin{aligned} x(t) &= 2 + \frac{1}{2}e^{j(2\pi/3)t} + \frac{1}{2}e^{-j(2\pi/3)t} - 2je^{j(5\pi/3)t} + 2je^{-j(5\pi/3)t} \\ &= 2 + \frac{1}{2}e^{j2(2\pi/6)t} + \frac{1}{2}e^{-j2(2\pi/6)t} - 2je^{j5(2\pi/6)t} + 2je^{-j5(2\pi/6)t} \end{aligned}$$

From this, we may conclude that the fundamental frequency of $x(t)$ is $2\pi/6 = \pi/3$. The non-zero Fourier series coefficients of $x(t)$ are:

$$a_0 = 2, \qquad a_2 = a_{-2} = \frac{1}{2}, \qquad a_5 = a_{-5}^* = -2j$$

Figure 3: Solution for Question 3

3.1. Using the Fourier series synthesis eq. (3.38),

$$\begin{aligned} x(t) &= a_1 e^{j(2\pi/T)t} + a_{-1}e^{-j(2\pi/T)t} + a_3 e^{j3(2\pi/T)t} + a_{-3}e^{-j3(2\pi/T)t} \\ &= 2e^{j(2\pi/8)t} + 2e^{-j(2\pi/8)t} + 4je^{j3(2\pi/8)t} - 4je^{-j3(2\pi/8)t} \\ &= 4\cos(\frac{\pi}{4}t) - 8\sin(\frac{6\pi}{8}t) \\ &= 4\cos(\frac{\pi}{4}t) + 8\cos(\frac{3\pi}{4}t + \frac{\pi}{2}) \end{aligned}$$

Figure 4: Solution for Question 4

6)

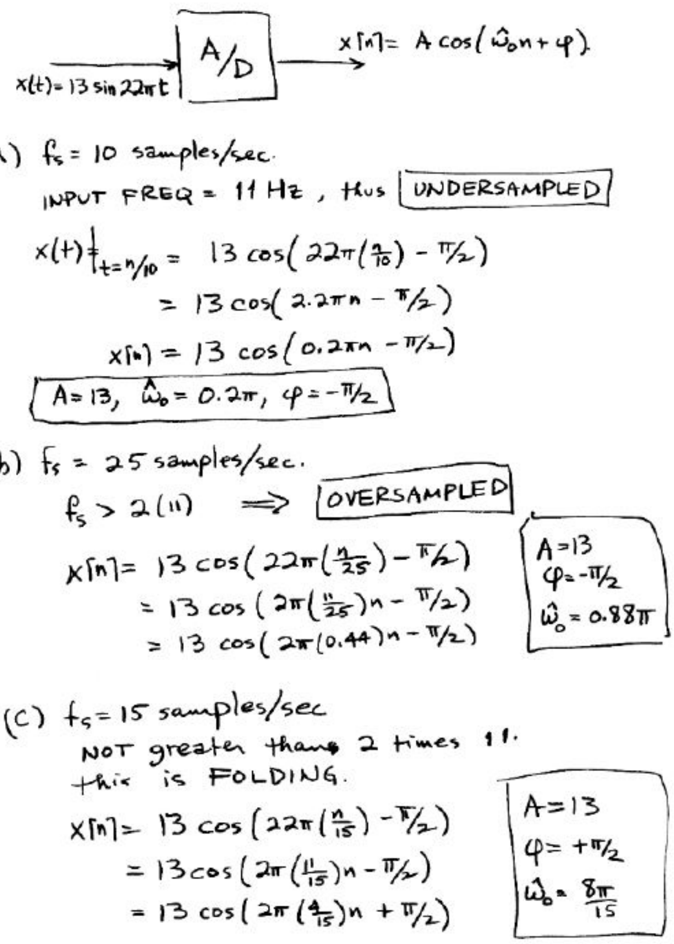

Figure 5: Solution for Question 5

2.7. (a) Given that

$$x[n] = \delta[n-1],$$

we see that

$$y[n] = \sum_{k=-\infty}^{\infty} x[k]g[n-2k] = g[n-2] = u[n-2] - u[n-6]$$

(b) Given that

$$x[n] = \delta[n-2],$$

we see that

$$y[n] = \sum_{k=-\infty}^{\infty} x[k]g[n-2k] = g[n-4] = u[n-4] - u[n-8]$$

(c) The input to the system in part (b) is the same as the input in part (a) shifted by 1 to the right. If S is time invariant then the system output obtained in part (b) has to the be the same as the system output obtained in part (a) shifted by 1 to the right. Clealry, this is not the case. Therefore, the system is **not** LTI.

(d) If $x[n] = u[n]$, then

$$\begin{aligned} y[n] &= \sum_{k=-\infty}^{\infty} x[k]g[n-2k] \\ &= \sum_{k=0}^{\infty} g[n-2k] \end{aligned}$$

The signal $g[n-2k]$ is plotted for $k = 0, 1, 2$ in Figure S2.7. From this figure it is clear that

$$y[n] = \begin{cases} 1, & n = 0, 1 \\ 2, & n > 1 \\ 0, & \text{otherwise} \end{cases} = 2u[n] - \delta[n] - \delta[n-1]$$

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Therefore,

$$A = t - 5, \quad B = t - 4.$$

Figure 6: Solution for Question 6

2.1. (a) We know that

$$y_1[n] = x[n] * h[n] = \sum_{k=-\infty}^{\infty} h[k]x[n-k] \qquad \text{(S2.1-1)}$$

The signals $x[n]$ and $h[n]$ are as shown in Figure S2.1.

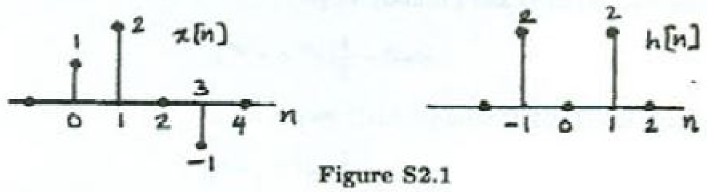

Figure S2.1

From this figure, we can easily see that the above convolution sum reduces to

$$\begin{aligned} y_1[n] &= h[-1]x[n+1] + h[1]x[n-1] \\ &= 2x[n+1] + 2x[n-1] \end{aligned}$$

This gives

$$y_1[n] = 2\delta[n+1] + 4\delta[n] + 2\delta[n-1] + 2\delta[n-2] - 2\delta[n-4]$$

(b) We know that

$$y_2[n] = x[n+2] * h[n] = \sum_{k=-\infty}^{\infty} h[k]x[n+2-k]$$

Comparing with eq. (S2.1-1), we see that

$$y_2[n] = y_1[n+2]$$

(c) We may rewrite eq. (S2.1-1) as

$$y_1[n] = x[n] * h[n] = \sum_{k=-\infty}^{\infty} x[k]h[n-k]$$

Similarly, we may write

$$y_3[n] = x[n] * h[n+2] = \sum_{k=-\infty}^{\infty} x[k]h[n+2-k]$$

Comparing this with eq. (S2.1), we see that

$$y_3[n] = y_1[n+2]$$

Figure 7: Solution for Question 7

The frequency response of a linear time-invariant filter is given by the formula

$$\mathcal{H}(\hat{\omega}) = (1 - e^{-j\hat{\omega}})(1 - e^{j\pi/3}e^{-j\hat{\omega}})(1 - e^{-j\pi/3}e^{-j\hat{\omega}}) \tag{3}$$

(a) Write the difference equation that gives the relation between the input $x[n]$ and the output $y[n]$.

First rearrange the frequency response as follows:

$$\begin{aligned}
\mathcal{H}(\hat{\omega}) &= (1 - e^{-j\hat{\omega}})(1 - e^{j\pi/3}e^{-j\hat{\omega}})(1 - e^{-j\pi/3}e^{-j\hat{\omega}}) \\
&= (1 - e^{-j\hat{\omega}})\left[1 - (e^{j\pi/3} + e^{-j\pi/3})e^{-j\hat{\omega}} + e^{-j2\hat{\omega}}\right] \\
&= (1 - e^{-j\hat{\omega}})\left[1 - (1)e^{-j\hat{\omega}} + e^{-j2\hat{\omega}}\right] \\
&= 1 - 2e^{-j\hat{\omega}} + 2e^{-j2\hat{\omega}} - e^{-j3\hat{\omega}}
\end{aligned}$$

From this equation, we can derive the filter coefficients: $\{b_k\} = \{1, -2, 2, -1\}$. Thus, the output of the filter is given by the following difference equation:

$$y[n] = x[n] - 2x[n-1] + 2x[n-2] - x[n-3]$$

(b) What is the output if the input is $x[n] = \delta[n]$?

When the input to the filter is the unit impulse sequence, the output is unit impulse response:

$$h[n] = \delta[n] - 2\delta[n-1] + 2\delta[n-2] - \delta[n-3]$$

(c) If the input is of the form $x[n] = Ae^{j\phi}e^{j\hat{\omega}n}$, for what values of $-\pi \le \hat{\omega} \le \pi$ will $y[n] = 0$ for all n?

For inputs of this form, the output of the filter is zero for all n when the frequency response of zero, i.e., when $\mathcal{H}(\hat{\omega}) = 0$ at a particular frequency. From Equation (3), the frequency response is zero when one of the factors is zero, i.e., when any ne of the following conditions is true:

$$\begin{aligned}
(1 - e^{-j\hat{\omega}}) &= 0 \\
(1 - e^{j\pi/3}e^{-j\hat{\omega}}) &= 0 \\
(1 - e^{-j\pi/3}e^{-j\hat{\omega}}) &= 0
\end{aligned}$$

These conditions are true when $\hat{\omega} = 0$, $\hat{\omega} = \pi/3$, and $\hat{\omega} = \pi/3$, respectively. For example, we can solve the middle one:

$$\begin{aligned}
(1 - e^{j\pi/3}e^{-j\hat{\omega}}) &= 0 \\
e^{j\hat{\omega}} - e^{j\pi/3} &= 0 \\
e^{j\hat{\omega}} &= e^{j\pi/3} \qquad \Rightarrow \quad \hat{\omega} = \pi/3
\end{aligned}$$

Figure 8: Solution for Question 8