EEE 391: Basics of Signals and Systems

Fall 2024–2025

Analytical Assignment 2

1. The frequency response of a linear time-invariant filter is given by the formula:

$$H(e^{j\hat{\omega}}) = (1 - e^{-j\hat{\omega}})(1 - e^{j\pi/3}e^{-j\hat{\omega}})(1 - e^{-j\pi/3}e^{-j\hat{\omega}})$$

   (a) Write the difference equation that gives the relation between the input $x[n]$ and the output $y[n]$.
   (b) What is the output if the input is $x[n] = \delta[n]$?
   (c) If the input is of the form $x[n] = Ae^{j\phi}e^{j\hat{\omega}n}$, for what values of $-\pi \leq \hat{\omega} \leq \pi$ will $y[n] = 0$ for all n?

2. Consider an FIR filter with coefficients $b_k = \{2, 0, 5, 0, 2\}$.

   (a) If the input signal is $x[n] = \delta[n] - \delta[n-2]$, make a plot of the output $y[n]$. Include zero values on the plot.
   (b) Determine the frequency response, $H(e^{j\omega})$
   (c) Determine and plot the magnitude and the phase of $H(e^{j\omega})$

3. Suppose that a LTI system has a z-transform system function equal to:

$$H(z) = 7 - 6z^{-1} - 5z^{-3} + 4z^{-4}$$

   (a) Determine the difference equation that relates the output $y[n]$ of the system to the input $x[n]$.
   (b) Determine and plot the impulse response $h[n]$.
   (c) Determine the step response, i.e., the output when the input is:

$$u[n] = \begin{cases} 1, & n \geq 0 \\ 0, & n < 0 \end{cases}$$

4. A linear time-invariant system is described by the difference equation:

$$y[n] = x[n] + x[n-4]$$

   (a) Find its impulse response $h[n]$.
   (b) Find its system function $H(z)$.
   (c) Plot the poles and zeros of $H(z)$ in the z-plane.
   (Recall that $-1 = e^{j\pi k}$ where k is an odd integer.)
   (d) Find the frequency response $H(e^{j\omega})$ and express it in polar form (magnitude and phase).
   (e) Carefully sketch and label a plot of $|H(e^{j\omega})|$ for $-\pi < \omega < \pi$.

5. Answer the following questions about the system whose z-transform system function is:

$$H(z) = \frac{1 + 0.8z^{-1}}{1 - 0.9z^{-1}}$$

   (a) Determine the poles and zeros of $H(z)$.

   (b) Determine the difference equation relating the input and output of this filter.

   (c) Derive a simple expression (purely real) for the magnitude-squared of the frequency response $|H(e^{j\hat{\omega}})|^2$.

   (d) Is this filter a Lowpass or Highpass filter? **Explain** your answer.

6. You are given the following two signals:

$$x_1(t) = \begin{cases} t + 3, & \text{for } t \in [-3, 0], \\ -t + 3, & \text{for } t \in [0, 3], \\ 0, & \text{otherwise,} \end{cases}$$

$$x_2(t) = \begin{cases} 1, & \text{for } t \in [-3, 3], \\ 0, & \text{otherwise.} \end{cases}$$

   (a) Sketch $x_1(t)$ and $x_2(t)$ over the interval $-3 \leq t \leq 3$.

   (b) Calculate $x_1(t) * x_2(t)$, $x_1(t) * x_1(t)$, and $x_2(t) * x_2(t - 3)$.

7. Consider the following continuous-time LTI system:

Diagram:

$$x(t) \longrightarrow \boxed{H_1(j\omega)} \longrightarrow \boxed{H_2(j\omega)} \longrightarrow y(t)$$

where

$$H_1(j\omega) = \frac{1}{j\omega + 1} \quad \text{and} \quad H_2(j\omega) = \frac{1}{j\omega + 2}.$$

   (a) Determine the differential equation describing the overall system.

   (b) Find the impulse response of this system from the frequency response.

   (c) Find the output $y(t)$ when $x(t)$ is a signal with the Fourier Transform $X(j\omega) = j\omega$. Use the frequency response.

8. Consider the following *high-pass filter* whose frequency response is given below:

$$H(j\omega) = 1 - \text{rect}\left(\frac{\omega}{4\pi}\right)$$

Recall that the rect function is defined as:

$$\text{rect}(\theta) = \begin{cases} 1, & \text{if } -\frac{1}{2} < \theta < \frac{1}{2}, \\ 0, & \text{otherwise.} \end{cases}$$

(a) The following signal is the input signal of this system:

$$x(t) = \frac{\sin(4\pi t)}{\pi t}\cos(2\pi t).$$

i. Let $X(j\omega)$ be the Fourier transform of the input signal $x(t)$. Sketch and clearly label $X(j\omega)$.

ii. Let $Y(j\omega)$ be the Fourier transform of the output signal $y(t)$. Sketch and clearly label $Y(j\omega)$.

(b) The output signal $y(t)$ is processed to obtain a new signal:

$$z(t) = \frac{\sin(2\pi t)}{\pi t}y(t).$$

i. Let $Z(j\omega)$ be the Fourier transform of $z(t)$. Sketch and clearly label $Z(j\omega)$.

ii. Find the time-domain expression for $y(t)$.