Chapter 3 Solutions

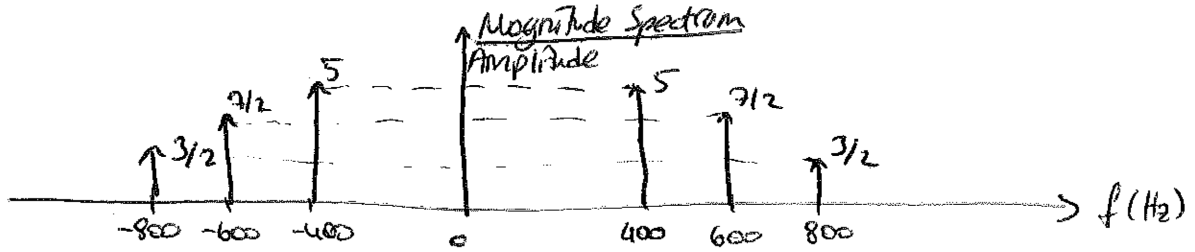
Problems: 3.1 / 3.8 / 3.10 / 3.12 / 3.14 / 3.19

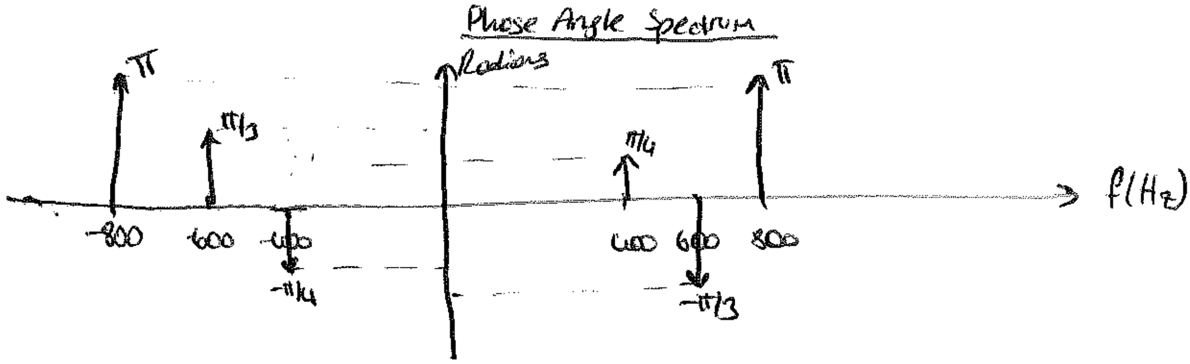
P-3.1

a) $x(t) = 10\cos(800\pi t + \pi/4) + 7\cos(1200\pi t - \pi/3) - 3\cos(1600\pi t)$

$$= 5e^{+j\pi/4} e^{j2\pi(400)t} + 5e^{-j\pi/4} e^{-j2\pi(400)t} + \tfrac{7}{2}e^{-j\pi/3} e^{j2\pi(600)t} +$$

$$\tfrac{7}{2}e^{j\pi/3} e^{-j2\pi(600)t} - \tfrac{3}{2}e^{j2\pi(800)t} - \tfrac{3}{2}e^{-j2\pi(800)t} \qquad (\rightarrow e^{j\pi} = e^{-j\pi} = -1)$$





b) Yes. Frequency of the resulting periodic signal is the largest common divisor of the frequencies of the signals composing it.

$lcd(400, 600, 800) = 200 \qquad f_c = 200\,Hz \quad T_c = \frac{1}{200}\,s = 0.005\,s$

c) $y(t) = x(t) + \frac{5}{2}e^{j\pi/2} e^{j2\pi(500)t} + \frac{5}{2}e^{-j\pi/2} e^{-j2\pi(500)t}$

- We need to add component to spectrum at frequency 500 Hz with amplitude $\frac{5}{2}$ and phase angle $\pi/2$ and at frequency -500 Hz with amplitude $\frac{5}{2}$ and phase angle $-\pi/2$.

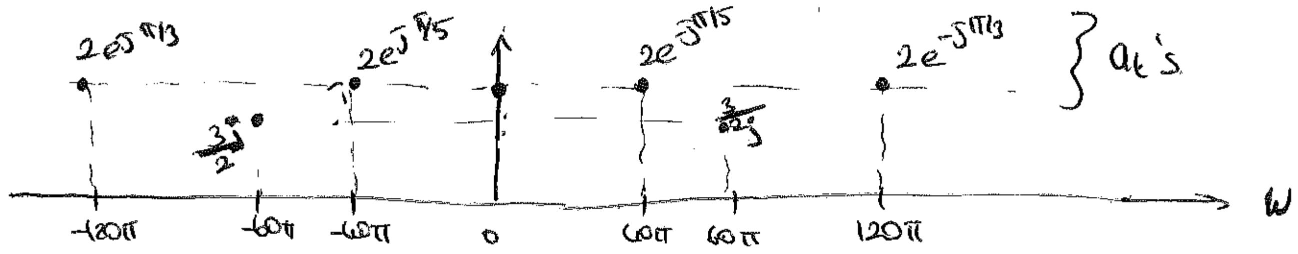
- Yes. $lcd(400, 600, 800, 500) = 100$, new common frequency is $f_c = 100\,Hz$

  and $T_c = \frac{1}{100}\,s = 0.01\,s$.

P-3.8

b-a) $\text{lcd}(40\pi, 60\pi, 120\pi) = 20\pi \qquad \omega_0 = 20\pi \qquad T_0 = \frac{2\pi}{\omega_0} = \frac{1}{10} = 0.1s.$

Maximum frequency of the signals composing $x(t)$ is 120π so we will need N to be $N = \frac{120\pi}{20\pi} = 6$. a_k's can be found from euler's formula directly as in the previous question or from the formula given in 3.26 (Fourier Analysis Equation). We can use Euler's formula because these are sinusoidal signals.

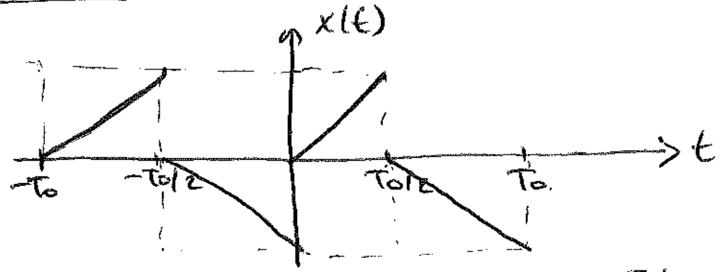


c) $y(t) = x(t) + \underbrace{5e^{-j\pi/6}e^{j50\pi t}}_{\substack{\omega = 50\pi \\ \text{comp.amp.} = 5e^{-j\pi/6}}} + \underbrace{5e^{j\pi/6}e^{-j50\pi t}}_{\substack{\omega = -50\pi \\ \text{comp.amp.} = 5e^{j\pi/6}}}$ } Addition of these components to spectrum.

Yes. $\text{lcd}(40\pi, 60\pi, 120\pi, 50\pi) = 10\pi \qquad \omega_0 = 10\pi \qquad T_0 = 0.5s.$

P-3.10

a)



b) $a_0 = \frac{1}{T_0}\int_0^{T_0} x(t)\,dt = \frac{1}{T_0}\left(\int_0^{T_0/2} x(t)\,dt + \int_{T_0/2}^{T_0} -x(t+\tfrac{T_0}{2})\,dt\right)$ | let $s = t + \frac{T_0}{2}$, $t = s - T_0/2$, $ds = dt$

$= \frac{1}{T_0}\left(\int_0^{T_0/2} x(t)\,dt - \int_{T_0}^{\frac{3T_0}{2}} x(s)\,ds\right) = \frac{1}{T_0}\left(\int_0^{T_0/2} x(t)\,dt - \int_0^{T_0/2} x(t)\,dt\right) = 0.$

c) let $k = 2n$, n is an integer.

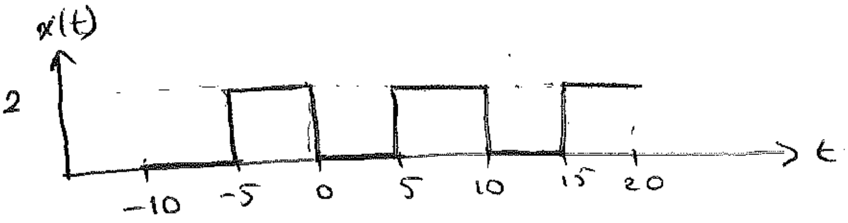
$a_k = a_{2n} = \frac{1}{T_0}\int_0^{T_0} x(t)e^{-j2\pi(2n)f_0 t}\,dt = \frac{1}{T_0}\left(\int_0^{T_0/2} x(t)e^{-j2\pi(2n)f_0 t}\,dt + \int_{T_0/2}^{T_0} -x(t+\tfrac{T_0}{2})e^{-j2\pi(2n)f_0 t}\,dt\right)$

$= \frac{1}{T_0}\left(\int_0^{T_0/2} x(t)e^{-j2\pi(2n)f_0 t}\,dt - \int_{T_0}^{\frac{3T_0}{2}} x(s)e^{-j2\pi(2n)f_0(s - \frac{T_0}{2})}\,ds\right)$

$= \frac{1}{T_0}\left(\int_0^{T_0/2} x(t)e^{-j2\pi(2n)f_0 t}\,dt - \int_0^{T_0/2} x(t)e^{-j2\pi(2n)f_0 t}\cdot\underbrace{e^{j2\pi(\cancel{2}n)f_0\frac{T_0}{\cancel{2}}}}_{1}\,dt\right) = 0$

P-3.12

a)



b) $$a_0 = \frac{1}{T_0}\int_0^{T_0} x(t)\,dt = 1$$

c) $$a_1 = \frac{1}{10}\int_0^{10} x(t)\, e^{-j\left(\frac{2\pi}{10}\right)t}\,dt = \frac{1}{10}\int_5^{10} 2\, e^{-j\frac{2\pi}{10}t}\,dt$$

$$= \frac{2}{10}\left(-j\frac{2\pi}{10}\right) e^{-j\frac{2\pi}{10}t}\Big|_5^{10} = \frac{2}{10}\left(-j\frac{2\pi}{10}\right)\Big[\underbrace{e^{-j2\pi} - e^{-j\pi}}_{\frac{1-(-1)}{2}}\Big] = -j\frac{8\pi}{100}$$

d) $y(t) = 1 + x(t)$

is also DC → so $b_0 = a_0 + 1$

$b_1 = a_1$ → no change withe the parts with frequencies different than 0.

P-3.14

a) $$x(t) = \sum_{k=-\infty}^{\infty} a_k\, e^{j(2\pi/T_0)kt}$$

* Summation, integration, derivation are linear functions, which means scaling property holds. Eg. $a+b=c$
$ka+kb=kc$

$$a_k = \frac{1}{T_0}\int_0^{T_0} x(t)\, e^{-j(2\pi/T_0)kt}\,dt$$

$$Aa_k = \frac{1}{T_0}\int_0^{T_0} Ax(t)\, e^{-j(2\pi/T_0)kt}\,dt$$

$$b_k = \frac{1}{T_0}\int_0^{T_0} y(t)\, e^{-j(2\pi/T_0)kt}\,dt$$

b) $y(t) = x(t-t_d)$

$$a_k = \frac{1}{T_0}\int_0^{T_0} x(t)\, e^{-j(2\pi/T_0)kt}\,dt$$

$$b_k = \frac{1}{T_0}\int_0^{T_0} x(t-t_d)\, e^{-j(2\pi/T_0)kt}\,dt$$

let $s = t - t_d \rightarrow t = s + t_d \rightarrow dt = ds$

$$b_k = \frac{1}{T_0}\int_{-t_d}^{T_0 - t_d} x(s)\, e^{-j(2\pi/T_0)k(s+t_d)}\,ds$$

↖ 1 period

$$b_k = \frac{1}{T_0}\int_0^{T_0} x(s)\, e^{-j(2\pi/T_0)ks}\, e^{-j(2\pi/T_0)kt_d}\,ds$$

$$b_k = \frac{1}{T_0}\int_0^{T_0} x(t)\, e^{-j(2\pi/T_0)kt}\,dt\; e^{-j\omega_0 k t_d}$$

$$b_k = a_k\, e^{-jk\omega_0 t_d}$$

P3.19

1) $2 + 3\left(\dfrac{e^{j2\pi 1.2t}e^{j\frac{\pi}{2}} + e^{-j2\pi 1.2t}e^{-j\frac{\pi}{2}}}{2}\right) = 2 + 3\cos\left(2\pi(1.2)t + \frac{\pi}{2}\right)$

at $t=0 \rightarrow 2 + 3\cos\left(\frac{\pi}{2}\right) = 2.$ and as t increases slowly the value decreases

$\Rightarrow$ c

2) $3\left(\dfrac{e^{j2\pi 0.6t}e^{-j\frac{\pi}{4}} + e^{-j2\pi 0.6t}e^{j\frac{\pi}{4}}}{2}\right) + 3\left(\dfrac{e^{j2\pi 1.5t}e^{j\pi} + e^{-j2\pi 1.5t}e^{-j\pi}}{2}\right)$

$= 3\cos\left(2\pi(0.6)t - \frac{\pi}{4}\right) + 3\cos\left(2\pi(1.5)t + \pi\right)$

at $t=0 \rightarrow 3\cos\left(\frac{-\pi}{4}\right) + 3\cos(\pi) = \frac{3\sqrt{2}}{2} - 3 = -0.8787$

at $t=-1 \rightarrow 3\cos\left(-1.2\pi - 0.25\pi\right) + 3\cos(-2\pi) = 2.5307$

$\Rightarrow$ d

3) $2 + 3\left(\dfrac{e^{j2\pi 1.2t}e^{-j\frac{\pi}{4}} + e^{-j2\pi 1.2t}e^{j\frac{\pi}{4}}}{2}\right) = 2 + 3\cos\left(2\pi(1.2)t - \frac{\pi}{4}\right)$

at $t=0 \rightarrow 2 + 3\cos\left(\frac{-\pi}{4}\right) = 4.12$

$\Rightarrow$ a

4) $3\left(\dfrac{e^{j2\pi 1.2t}e^{-j\pi/4} + e^{j2\pi 1.2t}e^{j\pi/4}}{2}\right) + 3\left(\dfrac{e^{j2\pi 2t}e^{j\pi} + e^{j2\pi 2t}e^{-j\pi}}{2}\right)$

$= 3\cos\left(2\pi(1.2)t - \frac{\pi}{4}\right) + 3\cos\left(2\pi(2)t + \pi\right)$

at $t=0 \rightarrow 3\cos\left(\frac{-\pi}{4}\right) + 3\cos(\pi) = -0.8787$

at $t=-1 \rightarrow 3\cos\left(-2.4\pi - 0.25\pi\right) + 3\cos(-3\pi) = -4.3620$

$\Rightarrow$ e

5) $3\left(\dfrac{e^{j2\pi 1.5t}e^{j\pi} + e^{-j2\pi 1.5t}e^{-j\pi}}{2}\right) = 3\cos\left(2\pi(1.5)t + \pi\right)$

at $t=0 \rightarrow -3$

$\Rightarrow$ b