CH4 – Solutions

P_4.2

$$x(t) = 7\sin(11\pi t) = 7\cos(11\pi t - \pi/2) \longrightarrow \boxed{A/D}_{f_s} \longrightarrow x[n] = A\cos(\hat{\omega}_0 n + \varphi).$$

(a) $f_s = 10$ samples/sec.

$$x(t)\Big|_{t=n/f_s} = x\left(\frac{n}{f_s}\right) = 7\cos\left(\frac{11\pi n}{10} - \pi/2\right).$$

$$= 7\cos\left(\frac{11\pi n}{10} - 2\pi n - \pi/2\right).$$

$$= 7\cos\left(-\frac{9\pi n}{10} - \pi/2\right) = 7\cos(.9\pi n + \pi/2).$$

$$\boxed{A = 7,\ \hat{\omega}_0 = 0.9\pi,\ \varphi = \pi/2}$$

(b) $f_s = 5$ samples/sec

$$x(t)\Big|_{t=n/f_s} = x\left(\frac{n}{5}\right) = 7\cos\left(\frac{11\pi n}{5} - \pi/2\right)$$

$$= 7\cos\left(\frac{\pi n}{5} - \frac{\pi}{2}\right)$$

$$\boxed{A = 7,\ \hat{\omega}_0 = \frac{\pi}{5},\ \varphi = -\frac{\pi}{2}}$$

(c) $f_s = 15$ samples/sec

$$x(t)\Big|_{t=n/f_s} = x\left(\frac{n}{15}\right) = 7\cos\left(\frac{11\pi n}{15} - \frac{\pi}{2}\right)$$

$$A = 7,\ \hat{\omega}_0 = \frac{11\pi}{15} = 2\pi\left(\frac{5.5}{15}\right)\ ;\ \varphi = -\pi/2$$

P_ 4.5

(a) Let $x(t) = 10\cos(\omega_0 t + \varphi)$

Sampling at a rate of $f_s \Rightarrow x[n] = x(t)\big|_{t=n/f_s} = x(n/f_s)$

$x[n] = 10\cos(\omega_0 \frac{n}{f_s} + \varphi)$ $\quad$ $\boxed{f_s = 1000}$

Equate this to $\quad \frac{\omega_0}{f_s} = 0.2\pi \Rightarrow \omega_0 = 0.2\pi \times 1000 = 200\pi$

$x[n] = 10\cos(0.2\pi n - \pi/7)$ $\quad \varphi = -\pi/7$

A second possible signal is the "folded alias" at $(f_s - f_0)$

$$f_s - f_0 = f_s - \frac{\omega_0}{2\pi} = 1000 - \frac{200\pi}{2\pi} = 900\,\text{Hz}$$

In this case, the phase (ψ) changes.

$$\tilde{x}(t) = 10\cos(2\pi(f_s - f_0)t + \psi)$$

$$\tilde{x}[n] = 10\cos(2\pi(f_s - f_0)\frac{n}{f_s} + \psi) = 10\cos(2\pi n - 2\pi f_0 \frac{n}{f_s} + \psi)$$

$$= 10\cos(-2\pi \frac{f_0 n}{f_s} + \psi) = 10\cos(2\pi \frac{f_0}{f_s} n - \psi).$$

$\boxed{f_0 \text{ is still } 100\text{Hz}}$

$\Rightarrow \psi = +\pi/7$

(b) Reconstruction of $x[n]$ with $f_s = 2000$ samples/sec.

The discrete and continuous domains are related by: $\frac{n}{f_s} \leftrightarrow t$ or $n \leftrightarrow f_s t$

So we replace "n" in $x[n]$ with $f_s t$. This is what an <u>ideal</u> D-to-A would do.

$$x[n] = 10\cos(0.2\pi n - \pi/7)$$

$$x(t) = 10\cos(0.2\pi f_s t - \pi/7) \leftarrow f_s = 2000$$

$$= 10\cos(400\pi t - \pi/7)$$

$\omega_0 = 400\pi \Rightarrow f_0 = 200\,\text{Hz}.$

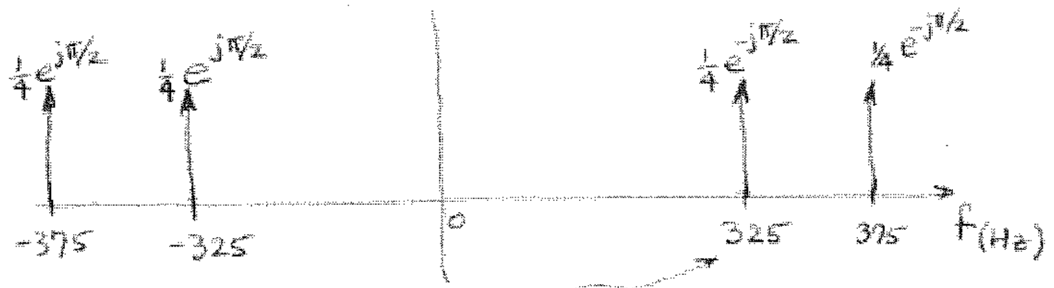
P_6.8

$$x(t) = \cos(50\pi t)\sin(700\pi t)$$

(a) $$x(t) = \left(\tfrac{1}{2}e^{j50\pi t} + \tfrac{1}{2}e^{-j50\pi t}\right)\left(\tfrac{1}{2j}e^{j700\pi t} - \tfrac{1}{2j}e^{-j700\pi t}\right)$$

$$= \tfrac{1}{4j}e^{j750\pi t} + \tfrac{1}{4j}e^{j650\pi t} - \tfrac{1}{4j}e^{-j650\pi t} - \tfrac{1}{4j}e^{-j750\pi t}$$

SAME AS $\frac{1}{4}e^{-j\pi/2}$ SAME AS $\frac{1}{4}e^{+j\pi/2}$



SPECTRUM. SAME AS 650π rad/sec

(b) Sampling Thm says sample at a rate greater than $\boxed{\text{two}}$ times the highest freq.

HIGHEST FREQ = 375 Hz

$$\Rightarrow f_s \geq 750\text{ Hz}.$$

P-4.13

a) If $y(t) = x(t)$ $\rightarrow$ Nyquist criterion was ensured.

$$F_s > 2 \cdot f_{max} = 2 \cdot 150 = 300 Hz$$

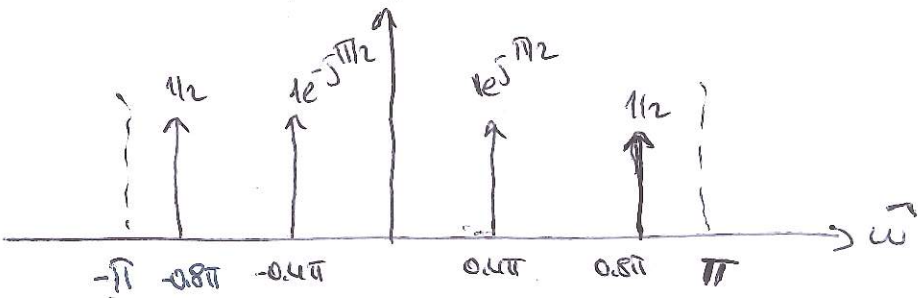
b) $x[n] = x(n.T_s)$, $T_s = 1/250$

$$x[n] = x(n/250) = 2\cos(2\pi(50)(n/250) + \pi/2) + \cos(2\pi(150)(n/250))$$

$$= 2\cos(2\pi(0.2)n + \pi/2) + \cos(2\pi(0.6)n) \quad \Big\} \; \forall f < \pi$$

$$= 2\cos(2\pi(0.2)n + \pi/2) + \cos(2\pi(0.4)n)$$

c) Complex Amplitude



d) If frequency content 50Hz is preserved and 150Hz is aliased to 0Hz then $300 > f_s > 100$ and should be multiple of 150Hz. Thus $f_s = 150 Hz$.

$$x[n] = x(n/150) = 2\cos(2\pi(50)(n/150) + \pi/2) + \cos(2\pi(150)(n/150))$$

$$= 2\cos(2\pi n/3 + \pi/2) + \cos(2\pi n)$$

$$= 2\cos(\tfrac{2}{3}\pi n + \pi/2) + 1.$$

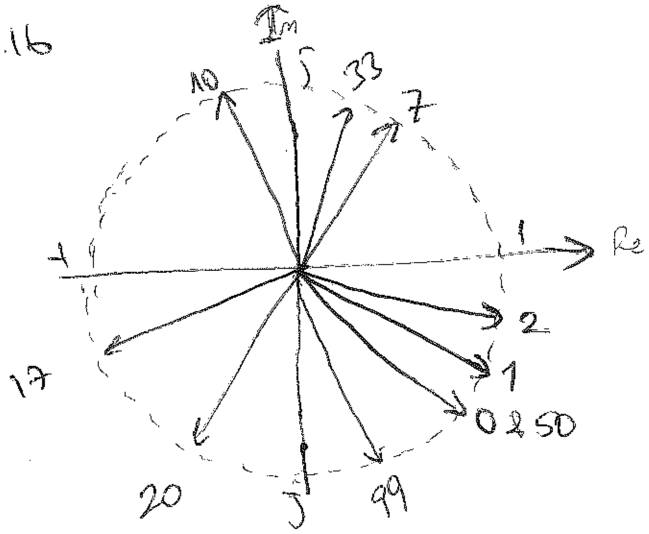
D-C

$$y(t) = x[n]\Big|_{n \to f_s t} = 2\cos\left(\frac{2\pi}{3} . 150 . t + \pi/2\right) + 1 = 2\cos(2\pi(50)t + \pi/2) + 1$$

✓

P-4.16

a)



$n = [0,1,2,\cancel{7},10,17,20,33,50,99]$

$\theta = (0.08\pi n - 0.25\pi)$

```
for t = 1 : length(n)
    while θ(t) > π
        θ(t) = θ(t) - 2π
    end
end
```

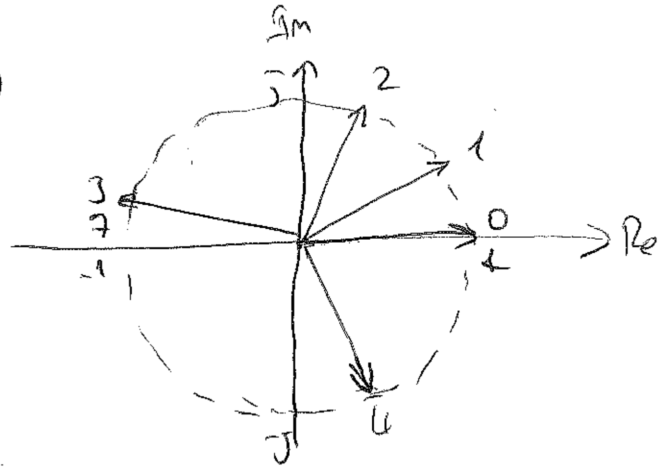
* θ lies between $-\pi$ and π

$\theta/\pi = -0.25,\ -0.17,\ -0.09,\ 0.31,\ 0.55,\ -0.89,\ -0.65,\ 0.39,\ -0.25,\ -0.33$

b) $T = \dfrac{2\pi}{0.08\pi} = 25$

$\hookrightarrow z[50] = z[0]$

c)



$n = [0,1,2,3,4,7]$

$\theta = (0.1\pi n^2)$

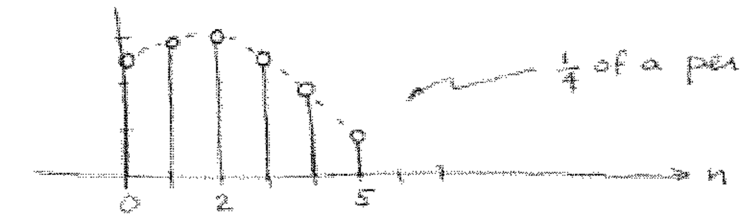
```
for t = 1 : length(n)
    while θ(t) > π
        θ(t) = θ(t) - 2π
    end
end
```

$\theta/\pi = 0,\ 0.1,\ 0.4,\ 0.9,\ -0.4,\ 0.9$

$T = \dfrac{2\pi}{0.1\pi n^2} \rightarrow$ not a integer $\Rightarrow$ NOT PERIODIC

P-6.19

You could estimate the values from a plot.



Looks like $A \approx 3$ $\quad \omega_0 \approx 2\pi\left(\frac{1}{\text{period}}\right) = 2\pi\frac{1}{16} = \frac{\pi}{8}$

$$\varphi = -2\pi\left(\frac{t_1}{T}\right) \cong -2\pi\left(\frac{2}{16}\right) = -\pi/4$$

EXACT:

Write 3 consecutive values of $x[n]$.

$$x[n-1] = \frac{A}{2}e^{j\varphi}e^{j\omega_0 n}e^{-j\omega_0} + \frac{A}{2}e^{-j\varphi}e^{-j\omega_0 n}e^{j\omega_0}$$

$$x[n] = \frac{A}{2}e^{j\varphi}e^{j\omega_0 n} + \frac{A}{2}e^{-j\varphi}e^{-j\omega_0 n}$$

$$x[n+1] = \frac{A}{2}e^{j\varphi}e^{j\omega_0 n}e^{j\omega_0} + \frac{A}{2}e^{-j\varphi}e^{-j\omega_0 n}e^{-j\omega_0}$$

$$\Rightarrow x[n-1]+x[n+1] = \frac{A}{2}e^{j\varphi}e^{j\omega_0 n}(2\cos\omega_0) + \frac{A}{2}e^{-j\varphi}e^{-j\omega_0 n}(2\cos\omega_0)$$

$$= (2\cos\omega_0)\,x[n].$$

$$\Rightarrow \cos\omega_0 = \frac{x[n-1]+x[n+1]}{2x[n]} = \frac{2.4271+2.9816}{2(2.9002)} = 0.9325$$

$$\Rightarrow \boxed{\omega_0 = 2\pi/17}$$

Let $z = \frac{A}{2}e^{j\varphi}$

$x[0] = z + z^* = 2.4271$

$x[1] = e^{j2\pi/17}z + e^{-j2\pi/17}z^* = 2.9002$

2 EQNS in 2 UNKNOWNS

$z = 1.5e^{-j\pi/5}$

$\boxed{A = 3 \;\; ; \;\; \varphi = -\pi/5}$