6.2, 6.5, 6.7a,b, 6.9, 6.13, 6.18

Problem 6.2 :-

$$y[n] = (x[n])^2$$

(a) $x[n] = A e^{j\phi} e^{j\hat{\omega} n}$

$y[n] = (A e^{j\phi} e^{j\hat{\omega} n})^2 = A^2 e^{j2\phi} e^{j2\hat{\omega} n}$

(b) No

The output cannot be written as

$$y[n] = H(\hat{\omega}) A e^{j\phi} e^{j\hat{\omega} n}$$

because the frequency has changes.

The new frequency is $2\hat{\omega}$.

Problem 6.5 :

$$y[n] = x[n] + 2x[n-1] + x[n-2]$$

(a) use filter coeffs : $\{b_k\} = \{1, 2, 1\}$

$H(\hat{\omega}) = 1 + 2e^{-j\hat{\omega}} + e^{-j2\hat{\omega}}$.

(b) $H(\hat{\omega}) = e^{-j\hat{\omega}} (e^{j\hat{\omega}} + 2 + e^{-j\hat{\omega}})$

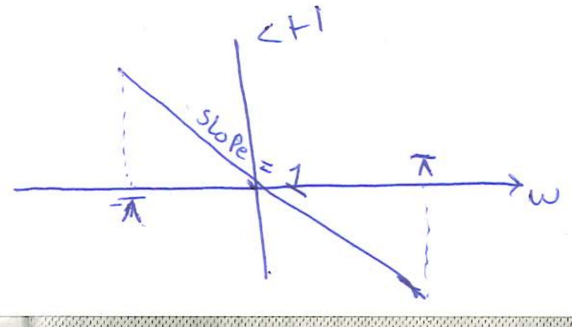
$= e^{-j\hat{\omega}} (2 + 2\cos\hat{\omega})$

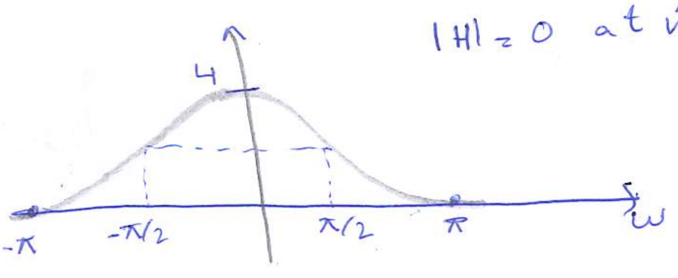
Phase $= -\hat{\omega}$ MAG: $2 + 2\cos\hat{\omega}$.

$|H| = 4$ at $\hat{\omega} = 0$

$|H| = 2$ at $\hat{\omega} = \pi/2$

$|H| = 0$ at $\hat{\omega} = \pi$





(c) $x[n] = 10 + 4\cos(\pi/2\, n + \pi/4)$

$= 10 + 2e^{j\pi/4} e^{j\pi/2 n} + 2e^{-j\pi/4} \cdot e^{-j\pi/2 n}$

$y[n] = 10H(0) + H(\pi/2)\, 2e^{j\pi/4} e^{j\pi/2 n} + 2H(-\pi/2) e^{-j\pi/4} \cdot e^{-j\pi/2}$

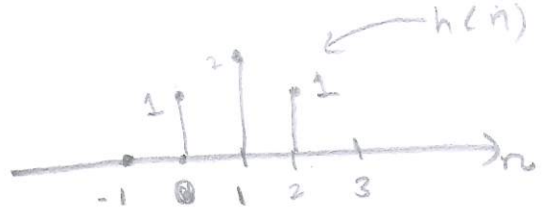
$H(0) = 4e^{j0}$ $\quad H(\pi/2) = e^{-j\pi/2}(2)$

$H(-\pi/2) = 2e^{j\pi/2}$

$\Rightarrow y[n] = 40 + 4e^{-j\pi/2} \cdot e^{j\pi/4} \cdot e^{j\pi/2 n} + 4e^{j\pi/2} \cdot e^{-j\pi/4} \cdot e^{-j\pi/2}$

$= 40 + 8\cos\left(\frac{\pi}{2} n - \frac{\pi}{4}\right)$

(d) $x[n] = \delta[n] \Rightarrow y[n] = \delta[n] + 2\delta[n-1] + \delta[n-2]$



(e) $x[n] = u[n]$

$y[n] = u[n] + 2u[n-1] + u[n-2]$

$y[n] = 0$ for $n < 0$

$y[0] = u[0] + 2u[-1] + u[-2] = 1 + 0 + 0 = 1$

$y[1] = u[1] + 2u[0] + u[-1] = 1 + 2 + 0 = 3$

$y[2] = u[2] + 2u[1] + u[0] = 1 + 2 + 1 = 4$

$y[n] = 4$ for $n \geq 2$

Problem 6.7 :-

(a) $H(e^{j\hat{\omega}}) = 1 + 2e^{-j3\hat{\omega}}$

Solution: Use the fact that the frequency response for $\delta[n-n_0]$ is $H(e^{j\hat{\omega}}) = e^{-j3\hat{\omega}}$

$$h[n] = \delta[n] + 2\delta[n-3].$$

(b) $H(e^{j\hat{\omega}}) = 2e^{-j3\hat{\omega}} \cos(\hat{\omega})$

Use the inverse Euler formula to write the frequency response in terms of complex exponential

$$H(e^{j\hat{\omega}}) = 2e^{-j3\hat{\omega}} \cos(\hat{\omega}) = e^{-j3\hat{\omega}} (e^{j\hat{\omega}} + e^{-j\hat{\omega}})$$

$$H(e^{j\hat{\omega}}) = e^{-j2\hat{\omega}} + e^{-j4\hat{\omega}}$$

$$\Rightarrow \quad h[n] = \delta[n-2] + \delta[n-4]$$

· Problem 6.9:

(a) $H(\hat{\omega}) = (1 - e^{-j\hat{\omega}})\,(1 - 2(0.5)\cos\pi/6\, e^{-j\hat{\omega}} + (0.5)^2 e^{-j2\hat{\omega}})$

$= (1 - e^{-j\hat{\omega}})\,(1 - \frac{\sqrt{3}}{2} e^{-j\hat{\omega}} + \frac{1}{4} e^{-j2\hat{\omega}})$

$= 1 \underbrace{- \frac{1}{2}(\sqrt{3} + 2)}_{-1.866} e^{-j\hat{\omega}} + \underbrace{\left(\frac{1}{4} + \frac{\sqrt{3}}{2}\right)}_{1.116} e^{-j2\hat{\omega}} - \frac{1}{4} e^{-j3\hat{\omega}}$

Difference Equation:

$y[n] = x[n] - 1.866\, x[n-1] + 1.116\, x[n-2] - \frac{1}{4} x[n-3]$

(b) when $x[n] = \delta[n]$, $y[n] = h[n]$ impulse response

$h[n] = \delta[n] - 1.866\,\delta[n-1] + 1.116\,\delta[n-2] - \frac{1}{4}\delta[n-3]$.

(C) Find $\hat{\omega}$ where $H(\hat{\omega}) = 0$

The only frequency is $\hat{\omega} = 0$, because then the factor $(1 - e^{-j\hat{\omega}}) = 0$. The other two factors in $H(\hat{\omega})$ are never zero for $-\pi \le \hat{\omega} \le \pi$.

Problem 6.13:

(a) $y[n] = y_3[n] = x_3[n-1] + x_3[n-2]$.

$= y_2[n-1] + y_2[n-2]$

$= (x_2[n-1] + x_2[n-3]) + (x_2[n-2] + x_2[n-4])$

Now replace $x_2[n]$ with $y_1[n]$.

$y[n] = y_1[n-1] + y_1[n-2] + y_1[n-3] + y_1[n-4]$

$= (x_1[n-1] + x_1[n-2]) + (x_1[n-2] - x_1[n-3]) +$

$(x_1[n-3] - x_1[n-4]) + (x_1[n-4] - x_1[n-5])$

cancel

$y[n] = x_1[n-1] - x_1[n-5]$

$y[n] = x[n-1] - x[n-5]$ $\quad x_1[n] = x[n]$

(b) Same thing as part (a) but use $H_i(\hat{\omega})$

$$\left.\begin{aligned} H_1(\hat{\omega}) &= 1 - e^{-j\hat{\omega}} \\ H_2(\hat{\omega}) &= 1 + e^{-j2\hat{\omega}} \\ H_3(\hat{\omega}) &= e^{-j\hat{\omega}} + e^{-j2\hat{\omega}} \end{aligned}\right\} \text{Multiply these together.}$$

$H_0(\hat{\omega}) = H_1(\hat{\omega})\, H_2(\hat{\omega})\, H_3(\hat{\omega})$

$= (1 - e^{-j\hat{\omega}})(1 + e^{-j2\hat{\omega}})(e^{-j\hat{\omega}} + e^{-j2\hat{\omega}})$

$= (1 - e^{-j\hat{\omega}} + e^{-j2\hat{\omega}} - e^{-j3\hat{\omega}})(e^{-j\hat{\omega}} + e^{-j2\hat{\omega}})$

$= e^{-j\hat{\omega}} - e^{-j2\hat{\omega}} + e^{-j3\hat{\omega}} - e^{-j4\hat{\omega}} + e^{-j2\hat{\omega}} - e^{-j3\hat{\omega}} + e^{-j4\hat{\omega}} - e^{j5\hat{\omega}}$

$H_0(\hat{\omega}) = e^{-j\hat{\omega}} - e^{-j5\hat{\omega}}$

$y[n] = x[n-1] - x[n-5]$

Problem 6.18.

$$x[n] = 5 + 20\cos(\pi/2\, n + \pi/4) + 10\,\delta[n-3]$$

Need $H(0)$ Depends on $H(\pi/2)$ Need impulse respone $h[n]$

$$H(0) = (1-j)\,(1-(-j))\,(1+1)$$
$$= (1-j)\,(1+j)\,2 = 2*2 = 4$$

$$H(\pi/2) = (1-je^{-j\pi/2})\,(1+je^{-j\pi/2})\,(1+e^{-j\pi/2})$$
$$= (1-j(-j))\,(1+j(-j))\,(1-j)$$
$$= (1-1)\,(1+1)\,(1-j) = 0$$

To find $h[n]$, multiply out $H(\hat{\omega})$

$$H(\hat{\omega}) = (1 - je^{-j\hat{\omega}} + je^{-j\hat{\omega}} + e^{-j2\hat{\omega}})\,(1+e^{-j\hat{\omega}})$$
$$= (1 - e^{-j2\hat{\omega}})\,(1+e^{-j\hat{\omega}})$$
$$= 1 + e^{-j\hat{\omega}} + e^{-j2\hat{\omega}} + e^{-j3\hat{\omega}}$$

$$\Rightarrow \quad h[n] = \delta[n] + \delta[n-1] + \delta[n-2] + \delta[n-3]$$

Finally,

$$y[n] = 5(4) + 0 + 10\,h[n-3]$$
$$= 20 + 10\,\delta[n-3] + 10\,\delta[n-4] + 10\,\delta[n-5] + 10\,\delta[n-6]$$