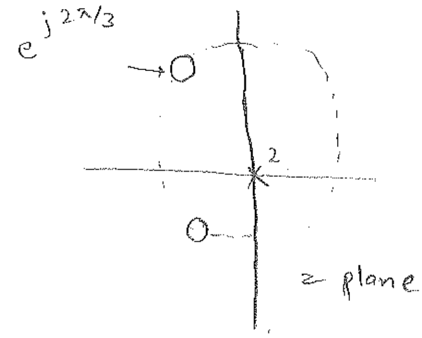
CHAPTER 7

7.4, 7.6, 7.8, 7.10, 7.14, 7.16

7.4: (a) use filter coeffs: $H(z) = \frac{1}{3} + \frac{1}{3}z^{-1} + \frac{1}{3}z^{-2}$.

(b) Use positive powers to extract poles and zeros

$$H(z) = \underbrace{\frac{1}{z^2}}_{\text{Two poles at } z=0}\left(\frac{1}{3}z^2 + \frac{1}{3}z + \frac{1}{3}\right)$$



Zeros at

$$z = \frac{-1 \pm \sqrt{1-4}}{2} = -\frac{1}{2} \pm j\frac{\sqrt{3}}{2}$$

zeros: $1\, e^{\pm j2\pi/3}$

(c) $H(\hat{\omega}) = H(e^{j\hat{\omega}}) = H(z)\big|_{z=e^{j\hat{\omega}}}$

$$= \frac{1}{3} + \frac{1}{3}e^{-j\hat{\omega}} + \frac{1}{3}e^{-j2\hat{\omega}} = \frac{1}{3}e^{-j\hat{\omega}}\left(e^{j\hat{\omega}} + 1 + e^{-j\hat{\omega}}\right)$$

$$= e^{-j\hat{\omega}}\left(\frac{1+2\cos\hat{\omega}}{3}\right)$$

ANOTHER FORMULA

$$\boxed{H(\hat{\omega}) = e^{-j\hat{\omega}}\left(\frac{\sin(3\hat{\omega}/2)}{3\sin(\hat{\omega}/2)}\right)}$$

(e) Use Linearity & Frequency response at $\hat{\omega}=0$, $\hat{\omega}=\pi/4$ and $\hat{\omega}=2\pi/3$. These are marked on the plots of frequency response.

$$y[n] = 4H(0) + |H(\pi/4)| \cos\left(\frac{\pi}{4}n - \frac{\pi}{4} + \angle H(\pi/4)\right) - 3|H(2\pi/3)| \cos(2\pi/3\, n + \angle H(2\pi/3))$$

$$H(0) = \frac{1}{3} + \frac{1}{3} + \frac{1}{3} = 1.$$

$$H(\pi/4) = e^{-j\pi/4}(1 + 2\sqrt{2}/2)/3 = \frac{1+\sqrt{2}}{3} e^{-j\pi/4} = 0.8047 e^{-j\pi/4}$$

$H(2\pi/3) = 0$ because $H(z) = 0$ at $z = e^{\pm j2\pi/3}$.

$$\therefore\ y[n] = 4 + 0.8047 \cos(\pi/4\, n - \pi/2)$$

7.6 (a) $Y_1(z) = H_1(z)\, X(z)$

$$Y(z) = H_2(z)\, Y_1(z) = H_2(z)\,(H_1(z)\, X(z))$$

$$= \underbrace{(H_2(z)\, H_1(z))}_{H(z)}\, X(z)$$

because $H(z) = \dfrac{Y(z)}{X(z)}$

(b) since $H_2(z)\, H_1(z) = H_1(z)\, H_2(z)$ because $H_1(z)$ and $H_2(z)$ are scalar functions

$$\Rightarrow\ Y(z) = H_1(z)\, \underbrace{H_2(z)\, X(z)}$$

means that $H_2(z)$ is applied first.

(c) $H_1(z) = \frac{1}{3}(1 + z^{-1} + z^{-2})$ by using filter coeffs

$$H(z) = H_2(z)\, H_1(z)$$

$$= \frac{1}{3}(1 + z^{-1} + z^{-2}) \cdot \frac{1}{3}(1 + z^{-1} + z^{-2})$$

$$= \frac{1}{4}(1 + 2z^{-1} + 3z^{-2} + 2z^{-3} + z^{-4})$$

(d) convert to difference equation (i.e filter coeffs)

$$y[n] = \frac{1}{9}\left(x[n] + 2x[n-1] + 3x[n-2] + 2x[n-3] + x[n-4]\right)$$

(e) Find the poles and zeros of $H_2(z)$, then "double" them because $H_1(z) = H_2(z)$

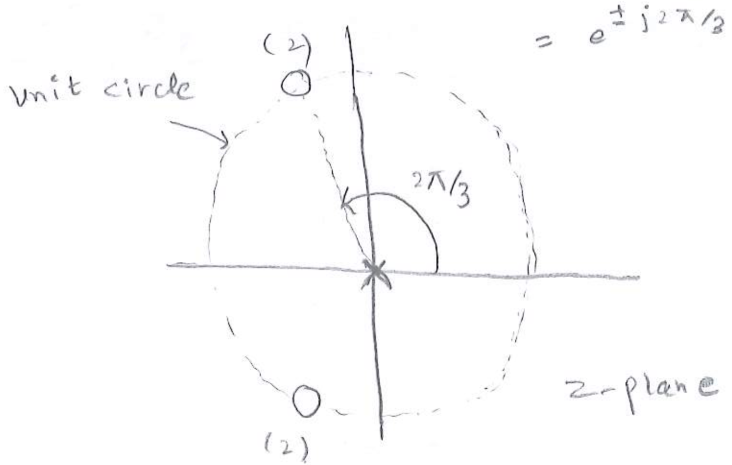
$$H_2(z) = \frac{1}{3} z^{-2} \underbrace{(z^2 + z + 1)}$$

$\frac{1}{z^2}$ contributes two poles at $z=0$

zeros are:

$$\frac{-1 \pm \sqrt{1-4}}{2} = \frac{-1}{2} \pm j\frac{\sqrt{3}}{2}$$

$$= e^{\pm j 2\pi/3}$$



(f)
$$H(e^{j\hat{\omega}}) = H_1(e^{j\hat{\omega}})\, H_2(e^{j\hat{\omega}})$$

$$= \frac{1}{9}\left(1 + e^{-j\hat{\omega}} + e^{-j2\hat{\omega}}\right)^2$$

$$= \frac{1}{9} e^{-j2\hat{\omega}} \left(e^{j\hat{\omega}} + 1 + e^{-j\hat{\omega}}\right)^2$$

$$= \frac{1}{9} e^{-j2\hat{\omega}} \left(1 + 2\cos(\hat{\omega})\right)^2$$

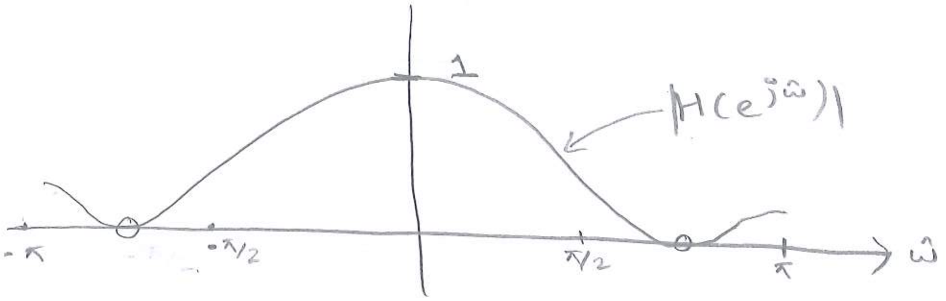
$$|H(e^{j\hat{\omega}})| = \frac{1}{9}\left(1 + 2\cos(\hat{\omega})\right)^2$$

At $\hat{\omega} = 0$, $|H| = \frac{1}{9}(3)^2 = 1$

At $\hat{\omega} = \pi/2$, $|H| = \frac{1}{9}(1)^2 = 1/9$

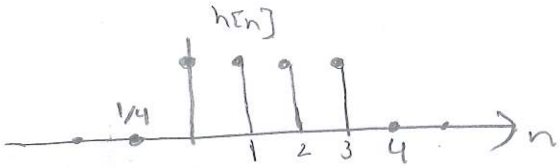
At $\hat{\omega} = 2\pi/3$, $|H| = 0$ because there is a zero on the unit circle

At $\hat{\omega} = \pi$, $|H| = \frac{1}{9}(1-2)^2 = 1/9$



Problem 7.8:

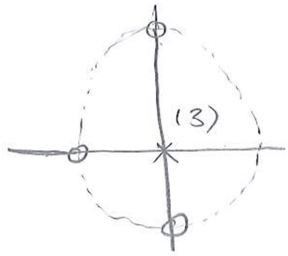
(a) $h[n] = \frac{1}{4}\{\delta[n] + \delta[n-1] + \delta[n-2] + \delta[n-3]\}$.



(b) $H(z) = \frac{1}{4}(1 + z^{-1} + z^{-2} + z^{-3})$ by using $h[n]$.

(c) Poles and zeros:

$$H(z) = \frac{1}{4}\,\frac{z^3 + z^2 + z + 1}{z^3} \rightarrow \boxed{\text{3 poles at } z = 0}$$



$$z^3 + z^2 + z + 1 = \frac{z^4 - 1}{z - 1}$$ ← zeros at z

(d) $$H(z) = \frac{1}{4}\,\frac{1 - z^{-4}}{1 - z^{-1}} = \frac{1}{4}\,\frac{e^{-j2\hat{\omega}}(e^{j2\hat{\omega}} - e^{-j2\hat{\omega}})}{e^{-jw/2}(e^{jw/2} - e^{-j\hat{\omega}/2})}$$

$$= \frac{1}{4} e^{-j3\hat{\omega}/2}\,\frac{\sin(2\hat{\omega})}{\sin(\frac{1}{2}\hat{\omega})}$$

At $\hat{\omega} = 0$, $H(e^{j\hat{\omega}}) = \frac{1}{4}e^{j0} \cdot 4 = 1$

At $\hat{\omega} = \pi/2, \pi, -\pi/2$, $H(e^{j\hat{\omega}}) = 0$ because $\sin(2\hat{\omega}) = 0$.

(f) Evaluate $H(e^{j\hat{\omega}})$ at $\hat{\omega} = 0$, $\hat{\omega} = 0.2\pi$ and $\hat{\omega} = 0.5\pi$.

These are marked on the frequency response plots.

$H(e^{j0}) = 1$ $\quad H(e^{j0.2\pi}) = 0.771\, e^{-j0.3\pi}$.

$H(e^{j0.5\pi}) = 0$.

$\Rightarrow \quad y[n] = 5 + 4(0.771)\cos(0.2\pi n - 0.3\pi) + 0$.

$\qquad \doteq 5 + 3.084 \cos(0.2\pi n - 0.3\pi)$

$\rightarrow$ ANGLE $= -54°$ or -0.94 rads.

Problem 7.10:

(a) Convert $H(z)$ to a difference equation

$$y[n] = x[n] - 3x[n-2] + 2x[n-3] + 4x[n-6].$$

The most delay is 6 samples, so the term $4\delta[n-4]$ in $x[n]$ is delayed to $16\delta[n-10]$.

The least amount of delay is $2\delta[n]$ experiencing no delay. Thus the output starts at $n=0$ and ends at $n=10$

$\Rightarrow \quad y[n] = 0$ for $n<0$ & $n>10$

$N_1 = 0$ and $N_2 = 10$.

(b) $X(z) = 2 + z^{-1} - 2z^{-2} + 4z^{-4}$.

$$Y(z) = H(z)X(z)$$

$$= (1 - 3z^{-2} + 2z^{-3} + 4z^{-6})(2 + z^{-1} - 2z^{-2} + 4z^{-4})$$

$$= 2 + z^{-1} - 2z^{-2} + 4z^{-4} - 6z^{-2} - 3z^{-3} + 6z^{-4} - 12z^{-6} +$$
$$4z^{-3} + 2z^{-4} - 4z^{-5} + 8z^{-7} + 8z^{-6} +$$
$$4z^{-7} - 8z^{-8} + 16z^{-10}$$

Combine terms with common exponents

$$Y(z) = 2 + z^{-1} - 8z^{-2} + z^{-3} + 12z^{-4} - 4z^{-5} - 4z^{-6}$$
$$+ 12z^{-7} - 8z^{-8} + 16z^{-10}.$$

Invert :

$$y[n] = 2\delta[n] + \delta[n-1] - 8\delta[n-2] + \delta[n-3] +$$
$$12\delta[n-4] - 4\delta[n-5] - 4\delta[n-6] + 12\delta[n-7]$$
$$-8\delta[n-8] + 16\delta[n-10]$$

Problem 7.14 :

$$H(z) = 1 - 2z^{-2} - 4z^{-4}$$

$$h[n] = \delta[n] - 2\delta[n-2] - 4\delta[n-4]$$

$$x[n] = 20e^{j0n} + 20\cos(\pi/2 n + \pi/4) - 20\delta[n]$$

$$\downarrow \qquad \downarrow \qquad \downarrow$$

$H(e^{j0}).20$ Need $H(e^{j\pi/2})$ $-20h[n]$

$$H(e^{j\hat{\omega}}) = 1 - 2e^{-j2\hat{\omega}} - 4e^{-j4\hat{\omega}}$$

$$H(e^{j0}) = 1 - 2 - 4 = -5.$$

$$H(e^{j\pi/2}) = 1 - 2e^{-j\pi} - 4e^{-j2\pi}.$$

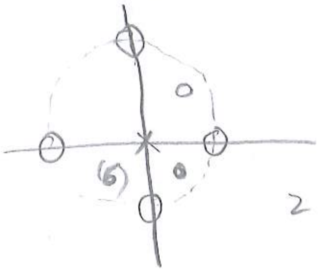
$$= 1 + 2 - 4 = -1.$$

$$\therefore y[n] = -100 - 20\cos(\pi/2\, n + \pi/4) - 20\delta[n] +$$
$$40\delta[n-2] + 80\delta[n-4].$$

Problem 7.16:

(a) $H(z)$ has 6 zeros & 6 poles at $z=0$

The zeros are: $z = \pm 1, \pm j, 0.8\, e^{\pm j\pi/4}$



z-plane.

(b) $w[n] = x[n] - x[n-4]$

$$\Rightarrow H_1(z) = 1 - z^{-4} = (1+z^{-2})(1-z^{-2})$$

To get $H_2(z)$ divide:

$$H_2(z) = \frac{H(z)}{H_1(z)} = (1 - 0.8e^{-j\pi/4}z^{-1}) \cdot (1 - 0.8e^{+j\pi/4}z^{-1})$$

$$H_2(z) = 1 - 1.6\cos \pi/4\, z^{-1} + 0.64\, z^{-2}.$$

(c) $y[n] = x[n] - (0.8\sqrt{2})\, x[n-1] + 0.64\, x[n-2]$

$= 1.1314$