8.5

$$y[n] = \tfrac{1}{2}y[n-1] - \tfrac{1}{3}y[n-2] - x[n]$$

$$Y(z) = \tfrac{1}{2}z^{-1}Y(z) - \tfrac{1}{3}z^{-2}Y(z) - X(z)$$

$$\left(1 - \tfrac{1}{2}z^{-1} + \tfrac{1}{3}z^{-2}\right)Y(z) = -X(z)$$

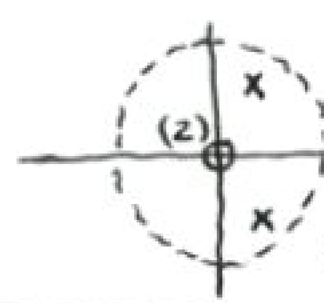
$$H(z) = \frac{Y(z)}{X(z)} = \frac{-1}{1 - \frac{1}{2}z^{-1} + \frac{1}{3}z^{-2}}$$

Change to positive powers of z when finding poles and zeros.

$$H(z) = \frac{-z^2}{z^2 - \frac{1}{2}z + \frac{1}{3}}$$

poles are at
$z = 0.25 \pm j0.52$
$= 0.5774e^{\pm j0.357\pi}$
ANGLE $= \pm 64.34°$
or ± 1.123 rads

Numerator is z^2, so we have two zeros at $z=0$.



z-plane

$$y[n] = \tfrac{1}{2}y[n-1] - \tfrac{1}{3}y[n-2] - x[n-2]$$

$$H(z) = \frac{-z^{-2}}{1 - \frac{1}{2}z^{-1} + \frac{1}{3}z^{-2}} = \frac{-1}{z^2 - \frac{1}{2}z + \frac{1}{3}}$$ ← Same poles

If we take $\lim_{z\to\infty} H(z)$ we get $H(z) \to \frac{1}{z^2}$ so we have 2 zeros at $z = \infty$

$$y[n] = \tfrac{1}{2}y[n-1] - \tfrac{1}{3}y[n-2] - x[n-4]$$

$$H(z) = \frac{-z^{-4}}{1 - \frac{1}{2}z^{-1} + \frac{1}{3}z^{-2}} = \frac{-1}{z^2\left(z^2 - \frac{1}{2}z + \frac{1}{3}\right)}$$

Now $H(z) \to 1/z^4$ as $z \to \infty$, so we have 4 zeros at $z = \infty$

We have 4 poles. The same two as above, plus 2 more poles at $z=0$.

$\boxed{8.7}$

$$y[n] = -0.8y[n-1] + 0.8x[n] + x[n-1]$$

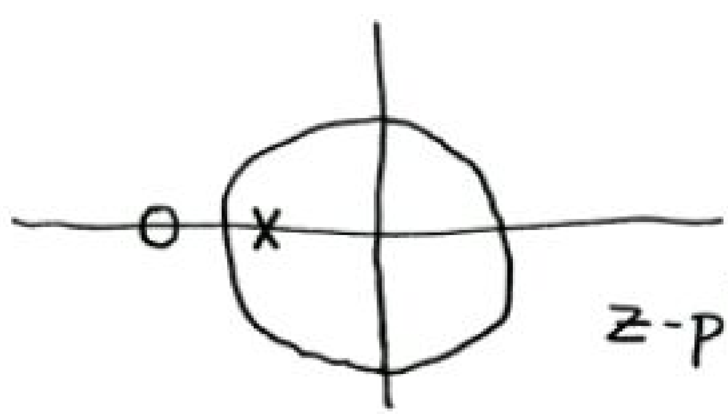
(a) $Y(z) = -0.8z^{-1}Y(z) + 0.8X(z) + z^{-1}X(z)$

$$H(z) = \frac{Y(z)}{X(z)} = \frac{0.8 + z^{-1}}{1 + 0.8z^{-1}}$$

$$= \frac{0.8z + 1}{z + 0.8}$$

(b) Pole at: $z + 0.8 = 0 \Rightarrow z = -0.8$

Zero at: $0.8z + 1 = 0 \Rightarrow z = -1/0.8 = -1.25$



(c) $H(e^{j\hat{\omega}}) = H(z)\big|_{z=e^{j\hat{\omega}}}$

$$= \frac{0.8 + e^{-j\hat{\omega}}}{1 + 0.8e^{-j\hat{\omega}}}$$

(d) $|H(e^{j\hat{\omega}})|^2 = H(e^{j\hat{\omega}})H^*(e^{j\hat{\omega}})$

$$= \frac{0.8 + e^{-j\hat{\omega}}}{1 + 0.8e^{-j\hat{\omega}}} \cdot \frac{0.8 + e^{j\hat{\omega}}}{1 + 0.8e^{j\hat{\omega}}}$$

$$= \frac{0.64 + 0.8e^{-j\hat{\omega}} + 0.8e^{j\hat{\omega}} + 1}{1 + 0.8e^{-j\hat{\omega}} + 0.8e^{j\hat{\omega}} + 0.64}$$

$$= \frac{1.64 + 1.6\cos\hat{\omega}}{1.64 + 1.6\cos\hat{\omega}}$$

$$= 1$$

$\boxed{8.10}$

$$y[n] = -\tfrac{1}{2}y[n-1] + x[n]$$

(a) $Y(z) = -\frac{1}{2}z^{-1}Y(z) + X(z)$

$(1 + \frac{1}{2}z^{-1})Y(z) = X(z) \implies H(z) = \dfrac{Y(z)}{X(z)} = \dfrac{1}{1+\frac{1}{2}z^{-1}}$

To find poles & zeros change to positive powers of z.

$H(z) = \dfrac{z}{z + 1/2} \implies$ 1 zero at $z=0$, one pole at $z = -1/2$.

(b) The impulse response of the system is the inverse transform of $H(z)$:

$$H(z) = \frac{1}{1+\frac{1}{2}z^{-1}} \longrightarrow h[n] = \left(-\tfrac{1}{2}\right)^n u[n]$$

To get the output when $x[n] = \delta[n] + \delta[n-1] + \delta[n-2]$ use superposition.

$$y[n] = h[n] + h[n-1] + h[n-2]$$
$$= \left(-\tfrac{1}{2}\right)^n u[n] + \left(-\tfrac{1}{2}\right)^{n-1} u[n-1] + \left(-\tfrac{1}{2}\right)^{n-2} u[n-2]$$

For $n=0$, $y[0] = 1 + 0 + 0 = 1$

For $n=1$, $y[1] = -\frac{1}{2} + 1 + 0 = 1/2$

For $n \geq 2$, $y[n] = \left(-\frac{1}{2}\right)^n + \left(-\frac{1}{2}\right)^{n-1} + \left(-\frac{1}{2}\right)^{n-2}$

$$= \left(-\tfrac{1}{2}\right)^n (1 - 2 + 4) = 3\left(-\tfrac{1}{2}\right)^n$$

Formula for $y[n]$:

$$y[n] = \delta[n] + \tfrac{1}{2}\delta[n-1] + 3\left(-\tfrac{1}{2}\right)^n u[n-2]$$

$\boxed{8.13}$

Characterize each system ($S_1 \to S_7$)

S_1: $H_1(z) = \dfrac{\frac{1}{2} + \frac{1}{2}z^{-1}}{1 - 0.9z^{-1}}$ $\Rightarrow$ pole at $z = 0.9$
zero at $z = -1$

$H_1(e^{j\hat{\omega}})$ is a LPF with a null at $\hat{\omega} = \pi$.

S_2: $H_2(z) = \dfrac{9 + 10z^{-1}}{1 + 0.9z^{-1}}$ $\Rightarrow$ pole at $z = -0.9$
zero at $z = -10/9$

$H_2(e^{j\hat{\omega}})$ is an all-pass filter

S_3: $H_3(z) = \dfrac{\frac{1}{2}(1 - z^{-1})}{1 + 0.9z^{-1}}$ $\Rightarrow$ pole at $z = -0.9$
zero at $z = 1$

$H_3(e^{j\hat{\omega}})$ is a HPF with a null at $\hat{\omega} = 0$.

S_4: $H_4(z) = \frac{1}{4}(1 + 4z^{-1} + 6z^{-2} + 4z^{-3} + z^{-4})$

$= \frac{1}{4}(1 + z^{-1})^4$ $\Rightarrow$ 4 zeros at $z = -1$

$H_4(e^{j\hat{\omega}})$ is a LPF with null at $\hat{\omega} = \pi$.

DC value: $H_4(e^{j0}) = 4$.

S_5: $H_5(z) = 1 - z^{-1} + z^{-2} - z^{-3} + z^{-4} = \dfrac{1 + z^{-5}}{1 + z^{-1}}$

has 4 zeros around the unit circle.
No zero at $z = -1$; others at $e^{j(2\pi k/5 - \pi/5)}$

$H_5(e^{j\hat{\omega}})$ is a HPF with nulls at $\hat{\omega} = \pm\frac{\pi}{5}, \pm\frac{3\pi}{5}$

S_6: $H_6(z) = 1 + z^{-1} + z^{-2} + z^{-3} = \dfrac{1 - z^{-4}}{1 - z^{-1}}$

has 3 zeros around the unit circle at $z = \pm j, -1$

$H_6(e^{j\hat{\omega}})$ is a LPF with nulls at $\hat{\omega} = \pm\frac{\pi}{2}, \pi$

S_7: $H_7(z) = 1 + z^{-1} + z^{-2} + z^{-3} + z^{-4} + z^{-5} = \dfrac{1 - z^{-6}}{1 - z^{-1}}$

has 5 zeros around the unit circle at $z = e^{j\pi k/3}$

$H_7(e^{j\hat{\omega}})$ is a LPF with nulls at $\hat{\omega} = \pm\frac{\pi}{3}, \pm\frac{2\pi}{3}, \pi$

PZ#1: S_7	PZ#3: S_2	PZ#5: S_5
PZ#2: S_1	PZ#4: S_6	PZ#6: S_3

$\boxed{3.14}$

Characterize each system ($S_1 \to S_7$)

S_1: $H_1(z) = \dfrac{\frac{1}{2} + \frac{1}{2} z^{-1}}{1 - 0.9 z^{-1}}$ $\Rightarrow$ pole at $z = 0.9$
zero at $z = -1$

$H_1(e^{j\hat{\omega}})$ is a LPF with a null at $\hat{\omega} = \pi$.

S_2: $H_2(z) = \dfrac{9 + 10 z^{-1}}{1 + 0.9 z^{-1}}$ $\Rightarrow$ pole at $z = -0.9$
zero at $z = -10/9$

$H_2(e^{j\hat{\omega}})$ is an all-pass filter

S_3: $H_3(z) = \dfrac{\frac{1}{2}(1 - z^{-1})}{1 + 0.9 z^{-1}}$ $\Rightarrow$ pole at $z = -0.9$
zero at $z = 1$

$H_3(e^{j\hat{\omega}})$ is a HPF with a null at $\hat{\omega} = 0$.

S_4: $H_4(z) = \frac{1}{4}(1 + 4z^{-1} + 6z^{-2} + 4z^{-3} + z^{-4})$

$= \frac{1}{4}(1 + z^{-1})^4$ $\Rightarrow$ 4 zeros at $z = -1$

$H_4(e^{j\hat{\omega}})$ is a LPF with null at $\hat{\omega} = \pi$.

DC value: $H_4(e^{j0}) = 4$.

S_5: $H_5(z) = 1 - z^{-1} + z^{-2} - z^{-3} + z^{-4} = \dfrac{1 + z^{-5}}{1 + z^{-1}}$

has 4 zeros around the unit circle.
No zero at $z = -1$; others at $e^{j(2\pi k/5 - \pi/5)}$

$H_5(e^{j\hat{\omega}})$ is a HPF with nulls at $\hat{\omega} = \pm\frac{\pi}{5}, \pm\frac{3\pi}{5}$

S_6: $H_6(z) = 1 + z^{-1} + z^{-2} + z^{-3} = \dfrac{1 - z^{-4}}{1 - z^{-1}}$

has 3 zeros around the unit circle at $z = \pm j, -1$

$H_6(e^{j\hat{\omega}})$ is a LPF with nulls at $\hat{\omega} = \pm\frac{\pi}{2}, \pi$

S_7: $H_7(z) = 1 + z^{-1} + z^{-2} + z^{-3} + z^{-4} + z^{-5} = \dfrac{1 - z^{-6}}{1 - z^{-1}}$

has 5 zeros around the unit circle at $z = e^{j\pi k/3}$

$H_7(e^{j\hat{\omega}})$ is a LPF with nulls at $\hat{\omega} = \pm\frac{\pi}{3}, \pm\frac{2\pi}{3}, \pi$

(A) S_1	(C) S_6	(E) S_5
(B) S_3	(D) S_2	(F) S_4

$\boxed{8.16}$

PZ#1: zero at $z=1 \Rightarrow$ zero at $\hat{\omega}=0$
only (D) has a zero at DC

PZ#2: pole on real axis but far from $z=1$.
$\Rightarrow$ LPF with very wide passband. (B)

PZ#3: pole very close to $z=1 \Rightarrow$ narrow LPF
also, zero at $z=-1 \Rightarrow$ zero at $\hat{\omega}=\pi$ (A)

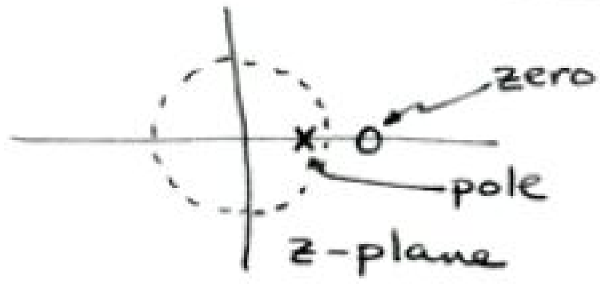
PZ#4: pole angles are approximately $\pm\pi/6$
$\Rightarrow$ peaks near $\hat{\omega}=\pm\pi/6$ (E)

$\boxed{8.18}$

(a) $H(z) = \dfrac{-0.8 + z^{-1}}{1 - 0.8z^{-1}}$ ← BY PICKING THE COEFFS FROM THE DIFF. EQN.

(b) POLE @ $z = 0.8$

ZERO @ $z = 1/0.8$
$= 1.25$



(c) $H(e^{j\hat{\omega}}) = H(z)\Big|_{z=e^{j\hat{\omega}}} = \dfrac{-0.8 + e^{-j\hat{\omega}}}{1 - 0.8e^{-j\hat{\omega}}}$

(d) $|H(e^{j\hat{\omega}})|^2 = H(e^{j\hat{\omega}})H^*(e^{j\hat{\omega}})$ ← MULTIPLY BY CONJUGATE

$$= \frac{(-0.8 + e^{-j\hat{\omega}})(-0.8 + e^{+j\hat{\omega}})}{(1 - 0.8e^{-j\hat{\omega}})(1 - 0.8e^{+j\hat{\omega}})}$$

$$= \frac{.64 + 1 - 0.8e^{-j\hat{\omega}} - 0.8e^{+j\hat{\omega}}}{1 + .64 - 0.8e^{-j\hat{\omega}} - 0.8e^{+j\hat{\omega}}}$$

$$= \frac{1.64 - 1.6\cos\hat{\omega}}{1.64 - 1.6\cos\hat{\omega}} \qquad \therefore \quad |H(e^{j\hat{\omega}})|^2 = 1$$

(e) $x[n] = 4 + \cos(\frac{\pi}{4}n) - 3\cos(\frac{2\pi}{3}n)$

Need $H(e^{j0})$ Need $H(e^{j\pi/4})$ Need $H(e^{j2\pi/3})$

Since $|H(e^{j\hat{\omega}})| = 1$ for all freqs, only the phase of the cosine terms will change. Also, the phase at $\hat{\omega} = 0$ is zero, so

$$y[n] = 4 + \cos(\tfrac{\pi}{4}n + \angle H(e^{j\pi/4})) - 3\cos(\tfrac{2\pi}{3}n + \angle H(e^{j2\pi/3}))$$

$\angle H(e^{j\pi/4}) = -149.97° = -2.617 \text{ rads} = -0.833\pi \text{ rads}$

$\angle H(e^{j2\pi/3}) = -172.66° = -3.013 \text{ rads} = -0.959\pi \text{ rads}$

$\boxed{8.19}$

Multiply out $H(z)$

$$H(z) = \frac{(1-z^{-1})(1-jz^{-1})(1+jz^{-1})}{(1-0.9e^{j2\pi/3}z^{-1})(1-0.9e^{-j2\pi/3}z^{-1})}$$

$$= \frac{(1-z^{-1})(1+z^{-2})}{1-2(0.9)\cos(2\pi/3)z^{-1}+(0.9)^2z^{-2}}$$

$$= \frac{1-z^{-1}+z^{-2}-z^{-3}}{1-0.9z^{-1}+0.81z^{-2}}$$

(a) Use the numerator & denominator polynomial coefficients as filter coefficients:

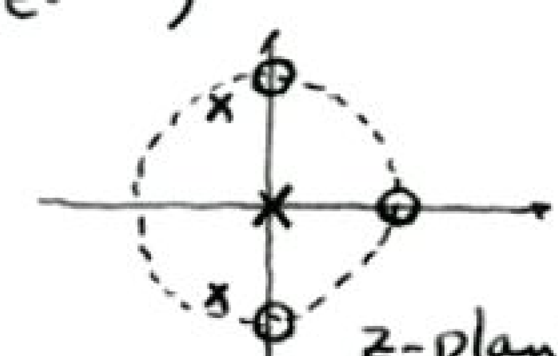
$$y[n] = 0.9y[n-1] - 0.81y[n-2] + x[n] - x[n-1] + x[n-2] - x[n-3]$$

(b) Multiply numerator & denominator by z^3:

$$H(z) = \frac{(z-1)(z-j)(z+j)}{z(z-0.9e^{j2\pi/3})(z-0.9e^{-j2\pi/3})}$$

Zeroes: $z=1$, j and $-j$

Poles: $z=0$, $z=0.9e^{\pm j2\pi/3}$



z-plane

(c) The zeros of the numerator polynomial are on the unit circle at $z=e^{j0}$, $z=e^{j\pi/2}$ and $z=e^{-j\pi/2}$

When $x[n] = Ae^{j\varphi}e^{j\hat{\omega}n}$, the output $y[n]$ is

$$y[n] = H(e^{j\hat{\omega}})\cdot Ae^{j\varphi}e^{j\hat{\omega}n}$$

There the output will be zero when $H(e^{j\hat{\omega}})=0$.

That is, for $\hat{\omega}=0$, $\hat{\omega}=\pi/2$ and $\hat{\omega}=-\pi/2$.