Problem 9.2 :-

(a) An exponentiation system is defined by input / output relation $y(t) = \exp[x(t+2)]$
$= e^{x(t+2)}$

(i) Linear
(ii) Time invariant
(iii) Stable
(iv) causal

(b) A phase modulator is a system whose input and output satisfy a relation of the form

$$y(t) = \cos[w_c t + x(t)]$$

(i) Linear
(ii) Time - invariant
(iii) stable
(iv) causal.

(c) amplitude modulator is a sytem whose input and output satisfy a relation of the form $y(t) = [A + x(t)] \cos(w_c t)$

(i) Linear
ii. Time - invariant
iii. stable
iv) Causal.

(d) A system that takes the even part of an input signal is defined by a relation of the form

$$y(t) = \mathcal{E}v\{x(t)\}$$
$$= \frac{x(t) + x(-t)}{2}$$

(i) Linear
ii) Time - invariant
iii) stable
iv) causal

Chapter 9

9.2, 9.3, 9.5, 9.6, 9.9, 9.22.

9.2: (a) $\delta(t-10) * [\delta(t+10) + 2e^{-t}u(t) + \cos(100\pi t)]$

$= \delta(t-10) * \delta(t+10) + \delta(t-10) * 2e^{-t}u(t) + \delta(t-10) * \cos(100\pi t)$

$= \delta(t) + 2e^{-(t-10)}u(t-10) + \cos[100\pi(t-10)]$

(b) $\cos(100\pi t)\,[\delta(t) + \delta(t-0.002)]$

$= \cos(100\pi \cdot 0)\,\delta(t) + \cos(100\pi(.002))\,\delta(t-.002)$

$= 1\,\delta(t) + \cos(0.2\pi)\,\delta(t-0.002)$

$= \delta(t) + 0.809\,\delta(t-0.002)$

(c) $\frac{d}{dt}[e^{-2(t-2)}u(t-2)]$ use formula for derivative of a product.

$= \frac{d}{dt}[e^{-2t}e^{4}u(t-2)] = e^{4}(-2)e^{-2t}u(t-2) + e^{-2t}e^{4}\delta(t-2)$

$= -2e^{4}e^{-2t}u(t-2) + e^{-2(2)}e^{4}\delta(t-2)$

$= -2e^{4}e^{-2t}u(t-2) + \delta(t-2)$

(d) $\int_{-\infty}^{t} \cos(100\pi\tau)\,[\delta(\tau) + \delta(\tau-0.002)]\,d\tau$

$= \int_{-\infty}^{t} \cos(100\pi \cdot 0)\,\delta(\tau)\,d\tau + \int_{-\infty}^{t} \cos(100\pi(0.002))\,\delta(\tau-0.002)\,d\tau$

$= u(t) + \cos(0.2\pi)\,u(t-0.002)$

$= u(t) + 0.809\,u(t-0.002)$

9.5: Solve for $h(t)$ in

$$[e^{-(t-4)} u(t-4)] * h(t) = 2e^{-t} u(t)$$

In order to find $h(t)$, use the shifting property of the impulse

$$x(t) * \delta(t-t_1) = x(t-t_1)$$

Thus we can write the first term above as

$$e^{-t} u(t) * \delta(t-4) = e^{-(t-4)} u(t-4)$$

Then we must solve:

$$e^{-t} u(t) * (\delta(t-4) * h(t)) = 2 e^{-t} u(t)$$

which requires that

$$\delta(t-4) * h(t) = 2 \delta(t)$$

Since $\delta(t-a) * \delta(t-b) = \delta(t-a-b)$, we can conclude that

$$h(t) = 2 \delta(t+4)$$

i.e $\delta(t+4) * 2\delta(t+4) = 2\delta(t)$

9.6

(a) $x(t)\,[\delta(t+1) + \delta(t-1)]$

$= x(t)\,\delta(t+1) + x(t)\,\delta(t-1)$ ← impulse at $t=1$

$= x(-1)\,\delta(t+1) + x(1)\,\delta(t-1)$

(b) $\int_{-\infty}^{\infty} x(\tau)\,\delta(\tau-1)\,d\tau$ ← impulse at $\tau = 1$

$= \int_{-\infty}^{\infty} x(1)\,\delta(\tau-1)\,d\tau$

$= x(1) \int_{-\infty}^{\infty} \delta(\tau-1)\,dt = x(1)$

(c) $\int_{-\infty}^{\infty} x(\tau)\,\delta(t-\tau)\,d\tau$ ← impulse at $\tau = t$

$\therefore$ replace τ with t in $x(\tau)$

$= \int_{-\infty}^{\infty} x(t)\,\delta(t-\tau)\,d\tau$

$= x(t) \int_{-\infty}^{\infty} \delta(t-\tau)\,dt = x(t)$

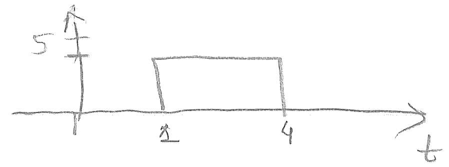
(d) $\delta^{(1)}(t) * x(t-1) = x^{(1)}(t-1)$ using eq (9.47)

Recall that $x^{(1)}(t)$ is the first derivative

Thus,

$$x^{(1)}(t-1) = \frac{d}{dt}\,x(t-1)$$

9.9: $h(t) = 5u(t-1) - 5u(t-4)$



(a) $y(t) = u(t) * h(t)$

$= u(t) * [5u(t-1) - 5u(t-4)]$

$= 5u(t) * u(t-1) - 5u(t) * u(t-4)$

use the fact that $u(t) * u(t) = t\,u(t)$ which can be combined with the shift property to write

$$u(t) * u(t-a) = (t-a)\,u(t-a)$$

Thus

$$y(t) = 5(t-1)\,u(t-1) - 5(t-4)\,u(t-4)$$

(b) The three regions are:

$t < 1$, $1 \le t \le 4$ and $t > 4$

when $t < 1$, both unit-step signals are zero, so $y(t) = 0$.

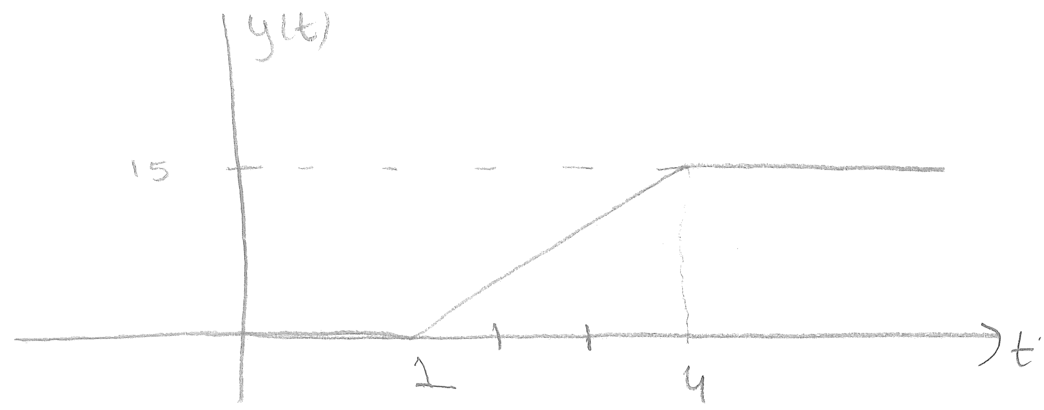
For $1 \le t \le 4$, $u(t-1) = 1$ and $u(t-4) = 0$, so $y(t) = 5t - 5$

For $t > 4$, $u(t-1) = 1$ and $u(t-4) = 1$ so, $y(t) = 5t - 5 - 5t + 20 = 15$

In summary,

$$y(t) = \begin{cases} 0 & t < 1 \\ 5t - 5 & 1 \le t \le 4 \\ 15 & 4 < t \end{cases}$$

(c)

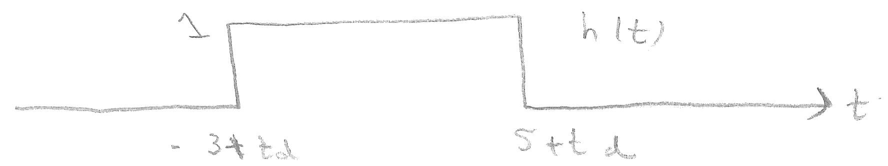


Problem 9.22 :-

(a) $v(t) = u(t+3) - u(t-5)$

$h(t) = y(t) \Big|_{x(t) = \delta(t)} = v(t) * \delta(t - t_d) = \delta(t - t_d) * v(t)$

$\therefore h(t) = u(t+3-t_d) - u(t-5-t_d)$



b) $t_d \geq 3$ because $h(t) = 0$ for $t < 0$

c) #1 and #2 are not stable because

$\int_{-\infty}^{\infty} |h_1(t)|\, dt \to \infty$ and $\int_{-\infty}^{\infty} |h_2(t)|\, dt \to \infty$

#3 is stable

The overall system is stable because

$\int_{-\infty}^{\infty} |h(t)|\, dt < \infty$.