Due 20/02/2008

EEE 452/552

HW # 1

(1) Perform the following dot products and express the results in terms of $\sin\theta, \cos\theta, \sin\phi, \cos\phi, x', y', z'$ for parts (a), (b) and (c); $\sin\theta, \cos\theta, \sin\phi, \cos\phi, \rho'$ and $\sin\theta', \cos\theta', \sin\phi', \cos\phi'$ (if necessary) for part (d)

(20 pts)

(a) $(\hat{a}_y y' + \hat{a}_z z') \cdot \hat{a}_r$

(b) $(\hat{a}_x x' + \hat{a}_z z') \cdot \hat{a}_r$

(c) $(\hat{a}_x x' + \hat{a}_y y') \cdot \hat{a}_r$

(d) $\bar{\rho}' \cdot \hat{a}_r$

(2) Perform the following integrals (if necessary you can use integration tables)

(40 pts)

(a) $$\int_{-l/2}^{l/2} I_0 \sin\left[k\left(\frac{l}{2} - |z|\right)\right] e^{jkz\cos\theta}\, dz$$ (I_0, θ, k, l are constant w.r.t. z.)

simplify your result if $l = \lambda/2$ (remember $k = 2\pi/\lambda$)

(b) $$\int_{-b/2}^{b/2}\int_{-a/2}^{a/2} e^{jk(x'\sin\theta\cos\phi + y'\sin\theta\sin\phi)}\, dx'\, dy'$$ (θ & ϕ are independent from x' & y')

simplify your result if $\phi = \pi/2$, if $\phi = 0$

(+20 pts)

(c) BONUS → $$\int_0^{2\pi} e^{jk\rho'\sin\theta\cos(\phi - \phi')}\, d\phi'$$ (ρ', θ and ϕ are independent from ϕ')

(* Simplify your results as much as possible)

(3) Derive the following wave equations in time-domain:

(15 pts)

(a) Wave equation for $\bar{H}$-field in a source-free region (i.e., $\bar{J} = \rho = 0$)

(b) Wave equation for $\bar{E}$-field in a source-region (i.e., $\bar{J} \neq 0$, $\rho \neq 0$)

(c) Wave equation for $\bar{H}$-field in a source-region (i.e., $\bar{J} \neq 0$, $\rho \neq 0$)

(4) If $\vec{H}_e = j\omega\epsilon\, \nabla \times \vec{\Pi}_e$ where $\vec{\Pi}_e$ is the electric Hertzian potential, show that

(25 pts)

(a) $\nabla^2 \vec{\Pi}_e + k^2 \vec{\Pi}_e = j\frac{1}{\omega\epsilon}\vec{J}$; (b) $\vec{E}_e = k^2\vec{\Pi}_e + \nabla(\nabla\cdot\vec{\Pi}_e)$; (c) $\vec{\Pi}_e = -j\frac{1}{\omega\mu\epsilon}\vec{A}$

$\vec{E}_e$ & $\vec{H}_e$ are the electric and magnetic fields, respectively.