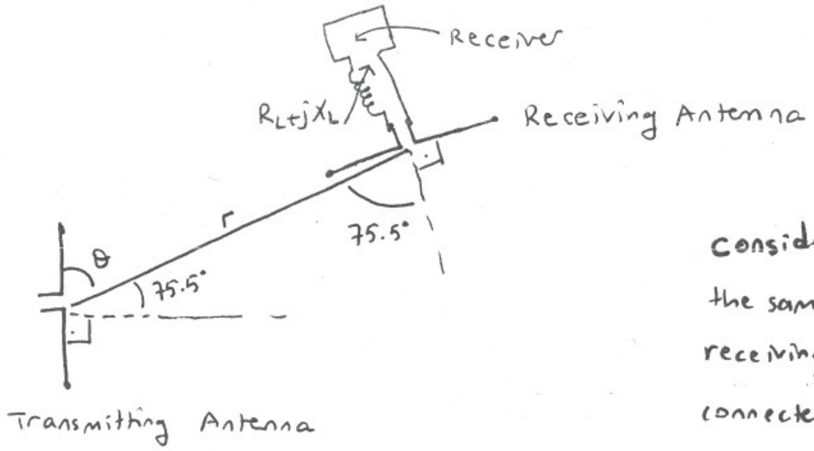
Due 30/04/2008

EEE 452/552

Homework # 10

(1)



Consider 2 half-wave dipoles situated on the same plane as shown in the figure. The receiving dipole of effective length 0.3m is connected to a receiver having 300Ω resistive input through a tuning coil which is used to tune out the reactive component of the antenna impedance. The radiation resistance of the antenna is 70Ω and its radiation efficiency is 97.22%. The impedance of the tuning coil, when properly adjusted, is 50+j800Ω. The RMS signal field strength is 40mW/meter (at the receiving antenna terminals). What is the RMS signal voltage at the receiver input?

(2) Antennas 'A' and 'B' have radiation intensities given as:

$$U(\theta,\phi) = \begin{cases} U_0 \cos^4(\theta) & 0^\circ \le \theta \le 20^\circ;\ 0^\circ \le \phi \le 180^\circ \\ 0 & \text{otherwise} \end{cases}$$

(a) Assume that antennas A and B are separated by a distance of 1km (assume far-field) and they're oriented to get max. power at the receiving side. Radiation efficiencies are given as 1 for both and they are perfectly matched to their circuits. Find the received power at 300 MHz, when the input power is 500 Watts if

(i) Both are circularly polarized but one is RHCP and the other is LHCP.
(ii) Both are RHCP.
(iii) One is circularly polarized but the other is linearly polarized.

(b) Assume that antenna A is operated as both receiver and transmitter. Radiation efficiency is given as 1 and it's matched to its circuit. It is aligned properly so that it looks at an object at 1km distance (assume far-zone) and receives 1mW power, when the input power is 100 Watts at 300 MHz. Calculate the radar cross section (RCS) of the object,

(3) Question 2.97 on page 131 of your textbook.

④ When two equations that describe the behavior of two different variables are of the same mathematical form, their solutions will also be identical. The variables in the two equations that occupy identical positions are known as <u>dual quantities</u>, and a solution of one can be formed by a systematic interchange of symbols to the others. This concept is known as the <u>duality theorem</u>.

Compare the following equations:

$$\bar{H}_A = \frac{1}{\mu}\nabla\times\bar{A} \quad\longleftrightarrow\quad \bar{E}_F = -\frac{1}{\epsilon}\nabla\times\bar{F}$$

$$\nabla\times\bar{E}_A = -j\omega\mu\bar{H}_A \quad\longleftrightarrow\quad \nabla\times\bar{H}_F = j\omega\epsilon\bar{E}_F$$

$$\nabla\times\bar{H}_A = \bar{J} + j\omega\epsilon\bar{E}_A \quad\longleftrightarrow\quad \nabla\times\bar{E}_F = -\bar{M} - j\omega\mu\bar{H}_F$$

$$\nabla^2\bar{A} + k^2\bar{A} = -\mu\bar{J} \quad\longleftrightarrow\quad \nabla^2\bar{F} + k^2\bar{F} = -\epsilon\bar{M}$$

$$\bar{E}_A = -j\omega\bar{A} - j\frac{1}{\omega\mu\epsilon}\nabla(\nabla\cdot\bar{A}) \quad\longleftrightarrow\quad \bar{H}_F = -j\omega\bar{F} - \frac{j}{\omega\mu\epsilon}\nabla(\nabla\cdot\bar{F})$$

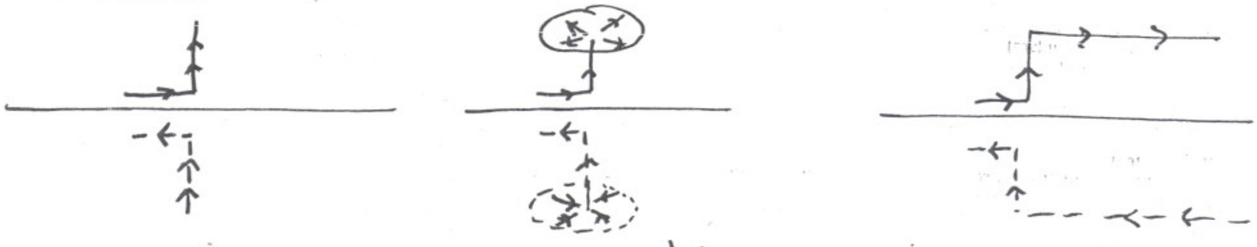
It is evident that they are to each other dual equations and their variables are dual quantities. Thus, knowing the solution to one set (i.e., $\bar{J}\neq 0, \bar{M}=0$), the solution to the other set ($\bar{J}=0, \bar{M}\neq 0$) can be formed by a proper interchange of quantities. Below, the quantities for $\bar{J}\neq 0, \bar{M}=0$ are given. Write down their duals for $\bar{J}=0, \bar{M}\neq 0$

Electric Sources ($\bar{J}\neq 0$ $\bar{M}=0$)	Magnetic Sources ($\bar{J}=0$, $\bar{M}\neq 0$)
$\bar{E}_A$	?
$\bar{H}_A$	?
$\bar{J}$	?
$\bar{A}$	?
ϵ	?
μ	?
k	?
η	?

Note that duality only serves as a guide to form mathematical solutions. As of today there are no magnetic charges or currents in nature.

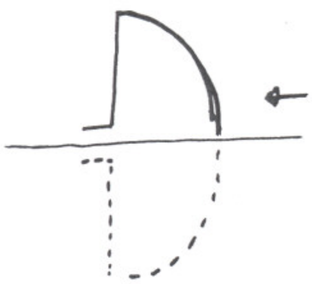
⑤ Read "Image Theory" (4.7.1) on page 184–187. Especially focus on Fig. 4.13 (try to understand this figure).

(6) The principles of image theory are illustrated in this question with several forms of the <u>monopole antenna</u>. A <u>monopole</u> is a dipole that has been divided in half at its center feed point and fed against a ground plane. Three monopoles and their images in a perfect ground plane are shown below:



Monopole Antenna — Capacitor plate Monopole — Transmission line monopole

The current and charges on a monopole are the same as on the upper half of its dipole counterpart, but the terminal voltage is only half that of the dipole because the gap width of the input terminals is half that of the dipole, and the same $\bar{E}$-field over half the distance gives half the voltage.



← equivalent dipole model shown with currents.

(a) Write down $Z_{A,\text{monopole}}$ (impedance of a monopole in terms of impedance of a dipole, $Z_{A,\text{dipole}}$.

(b) Considering the fact that in a monopole the fields extend only over a hemisphere write down the radiation resistance, $R_{r,\text{monopole}}$, of a monopole in terms of the radiation resistance of a dipole, $R_{r,\text{dipole}}$.

(c) The radiation pattern of a monopole above a perfect ground plane is the same as that of a dipole similarly positioned in free-space since the fields above the image plane are the same. Write down the directivity of a monopole, D_{monopole}, in terms of directivity of a dipole, D_{dipole}.