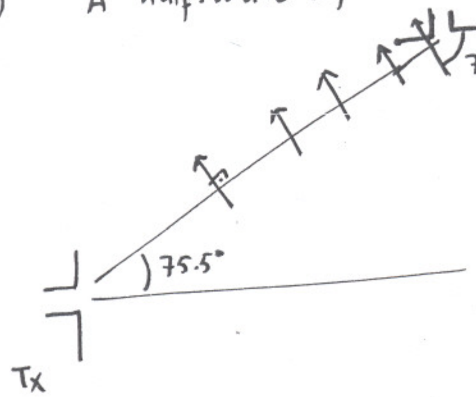
Solutions to HW # 10

(1) A half-wave dipole will generate E_θ



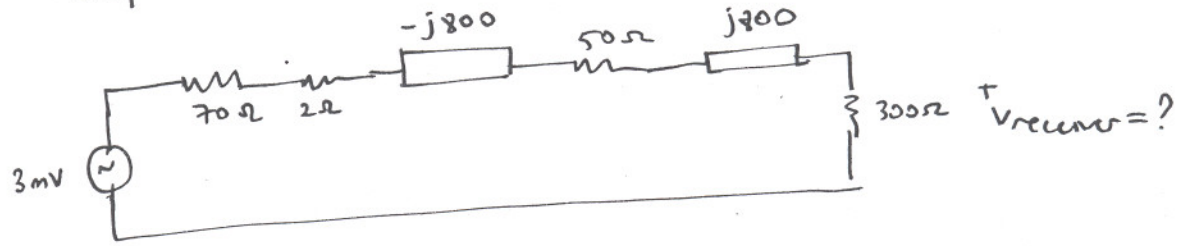
$\Rightarrow$ Based on this picture at the receiving antenna

$$V_{oc} = |\bar{E} \cdot \bar{l}_e| = E\, l_e \cos\alpha$$

$$\Rightarrow V_{oc} = 40 \times 0.3 \cdot \cos(75.5^\circ) = 3\,mV$$

$$e_{cd} = 97.22\% \Rightarrow \frac{R_r}{R_r + R_L} = 0.9722 \Rightarrow R_L \approx 2\,\Omega$$

The equivalent circuit then becomes



$$V_r = \frac{3\,mV}{72 - j800 + j800 + 50 + 300} \times 300 = \underline{\underline{2.13\,mV}}$$

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(2) (a) $$P_{rad} = \int_0^{\pi} d\phi \int_0^{\pi/9} U_o \cos^4(\theta) \sin\theta\, d\theta = U_o \pi \left[1 - \frac{\cos^5(\pi/9)}{5}\right] \cong U_o \pi\, 0.05346$$

$$\Rightarrow D = \frac{4\pi U_o}{U_o \pi\, 0.05346} = 74.82$$

$$P_{rec} = P_{in} \left(\frac{\lambda}{4\pi r}\right)^2 D_t D_r |\hat{a}_t \cdot \hat{a}_r|^2 = 500 \left(\frac{1}{16\pi^2 10^6}\right) 74.82^2 |\hat{a}_t \cdot \hat{a}_r|^2$$

$$\Rightarrow P_{rec} \cong 0.0177 |\hat{a}_t \cdot \hat{a}_r|^2 = 17.7 |\hat{a}_t \cdot \hat{a}_r|^2\, mW$$

(i) 0 mW ; (ii) 17.7 mW ; (iii) 8.66 mW

(b) $$P_{rec} = P_{in} \frac{\lambda^2}{(4\pi)^3} \frac{1}{r^4} D_t^2 \sigma \Rightarrow 10^{-3} = 10^2 \frac{1}{64\pi^3} \frac{1}{10^{12}} 74.82^2 \sigma$$

$$\Rightarrow \sigma \cong 0.3545 \times 10^7\, m^2 = 3.545\, km^2$$

(note PLF is not entering the picture)

3 $\sigma = 0.86\lambda^2$

$$\frac{P_{rec}}{P_{in}} = \sigma \frac{G_{ot} G_{or}}{4\pi} \left(\frac{\lambda}{4\pi r_1 r_2} \right)^2 |\hat{a}_w \cdot \hat{a}_r|^2$$

$f = 3\,GHz \Rightarrow \lambda = 0.1\,m.$

$G_{ot} = G_{or} = 15\,dB = 10^{1.5} = 31.6228$

$r_1 = r_2 = 100m. \Rightarrow r_1 = r_2 = 1000\lambda$

$|\hat{a}_w \cdot \hat{a}_r|^2 = -1\,dB \Rightarrow |\hat{a}_w \cdot \hat{a}_r| = 0.7943$

$$\Rightarrow \frac{P_{rec}}{P_{in}} = 0.86\lambda^2 \cdot \frac{(31.6228)^2}{4\pi} \left(\frac{\lambda^2}{16\pi^2 10^6 \lambda^2} \right) 0.7943$$

with $P_{in} = 100W$

$\Rightarrow P_{rec} = \underline{34.02 \times 10^{-12}}$ watts $=$ $\underline{34.02 \text{ picowatts}}$

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$\bar{E}_A \longleftrightarrow \bar{H}_F$

$\bar{H}_A \longleftrightarrow -\bar{E}_F$

$\bar{J} \longleftrightarrow \bar{M}$

$\bar{A} \longleftrightarrow \bar{F}$

$\epsilon \longleftrightarrow \mu$

$\mu \longleftrightarrow \epsilon$

$k \longleftrightarrow k$

$\eta \longleftrightarrow 1/\eta$

5 Reading assignment

6 (a) $$Z_{A,monopole} = \frac{V_{A,monopole}}{I_{A,monopole}} = \frac{\frac{1}{2} V_{A,dipole}}{I_{A,dipole}} = \frac{1}{2} Z_{A,dipole}$$

$$\therefore \boxed{Z_{A,monopole} = \frac{1}{2} Z_{A,dipole}.}$$

(b) Since the fields of a monopole only extend over a hemisphere, the power radiated is only half that of a dipole with the same current

$$\Rightarrow R_{r,monopole} = \frac{P_{monopole}}{\frac{1}{2}|I_{A,monopole}|^2} = \frac{\frac{1}{2} P_{dipole}}{\frac{1}{2}|I_{A,dipole}|^2} = \frac{1}{2} R_{r,dipole}$$

$$\therefore \quad \boxed{R_{r,\,\text{monopole}} = \frac{1}{2} R_{r,\,\text{dipole}}}$$

(c) A monopole fed against a perfect ground plane radiates one-half the total power of a similar dipole in free-space because the power is distributed in the same fashion but only over half as much space [as part (b)]. As a result, the beam solid angle of a monopole above a perfect ground plane is one-half that of a similar dipole in free-space

$$\Rightarrow D_{\text{monopole}} = \frac{4\pi}{\Omega_{A,\,\text{monopole}}} = \frac{4\pi}{\frac{1}{2}\Omega_{A,\,\text{dipole}}} = 2 D_{\text{dipole}}$$

OR

$$D_{\text{dipole}} = \frac{U_m}{U_{ave}} = \frac{U_m}{P/4\pi} \quad \& \quad D_{\text{monopole}} = \frac{U_m}{\frac{1}{2} P/4\pi} = 2 D_{\text{dipole}}$$

$$\Rightarrow \boxed{D_{\text{monopole}} = 2 D_{\text{dipole}}}$$