

- ① For a uniform one-dimensional array located on the  $z$ -axis, the normalized AF is given by

$$AF_n = \frac{1}{N} \frac{\sin\left(\frac{N}{2}\psi\right)}{\sin\left(\frac{1}{2}\psi\right)} ; \quad \psi = kd \cos\theta + \alpha$$

and  $d$  is the distance between the elements so that  $\alpha$  represents the progressive phase.

- (a) Plot the magnitude of the  $AF_n$  for  $N = 4, 7, 10$  with respect to  $\psi$  from  $-4\pi$  to  $4\pi$  (3 linear plots). Indicate the null locations, number of side lobes and width of the main and side lobes by considering a single period. Do they match with the formulas?
- (b) What happens when  $N$  becomes larger? What do you expect as  $N$  goes to infinity?
- (c) Let  $\alpha = 0, \pi/2, \pi$  and let  $d = \lambda/2, \lambda$  and  $1.5\lambda$ . For each combination (there are 9 combinations for each  $N$ ) plot the normalized array factor ( $AF_n$ ) and clearly indicate the visible regions in terms of  $\psi$  (27 linear plots. You may use 'subplot' option to put the figures in some ordered form).
- (d) In each 27 combinations (indicated in part (c)), where is the location of the main lobe in terms of  $\theta$ ? What are the effects of  $\alpha$  and  $d$  in general?
- (e) Let  $\alpha = -kd \cos\theta_0$ . What is the location of the main lobe? Show that if

$$d < \frac{\lambda}{1 + |\cos\theta_0|}$$

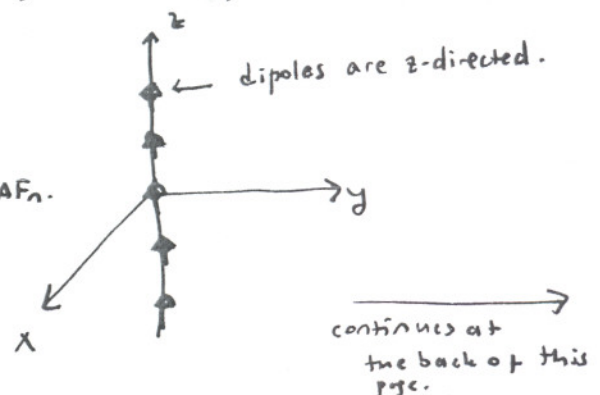
grating lobes will not appear in the visible region.

- (f) For  $[N=10; \alpha=\pi/2; d=\lambda/2]$  and  $[N=7; \alpha=\pi; d=\lambda]$  plot the magnitude of the normalized Array factor,  $AF_n$ , w.r. to  $\theta$  (2 linear plots). Compare these plots with the corresponding ones in (c).

- ② A five-element array of infinitesimal electric dipoles is placed along the  $z$ -axis as shown in the figure. The separation between the elements,  $d$ , and the amplitudes of the excitations are constant over the array.

- (a) Let  $d = \lambda/2$  and the progressive phase,  $\alpha = 0$ . Plot the followings:

- (i) The magnitude of the normalized array factor,  $AF_n$ .



(ii) The normalized radiation intensity of the single element.

(iii) The normalized radiation intensity of the array.

How does the radiation change from single element to array? (3 polar plots)

(b) Now let  $d = \lambda$  and see how grating lobes appear. Repeat the same plots indicated in part (a). Do the grating lobes change the total radiation effectively? Why? (2 or 3 polar plots; you may not re-plot the radiation of a single element).

(c) Let  $d = \lambda/2$  and the progressive phase,  $\alpha = \pi$  (end-fire array). Repeat the same plots indicated in part (a). How does the radiation change from a single element to this array? (2 or 3 polar plots; you may not re-plot the radiation of a single element).

(d) Let  $d = \lambda/2$  and the progressive phase,  $\alpha = \beta_0$ . Find  $\beta_0$  if the main lobe of the radiation is desired to occur at

(i)  $\theta = 60^\circ$ ,

(ii)  $\theta = 120^\circ$ .

Plot the normalized radiation intensities. (2 polar plots).

### SOME REMARKS

- \* This homework by itself is 3% of the overall grade. It is a relatively long one and may not be completed with a last-day-action. Thus, you are advised to adjust your time carefully.
- \* This homework includes 32 linear plots and 9 or 11 polar plots. Do not give extra or unnecessary plots.
- \* I personally want every student to work INDEPENDENTLY for this homework. If you have any questions about the homework contact me, not your friend.