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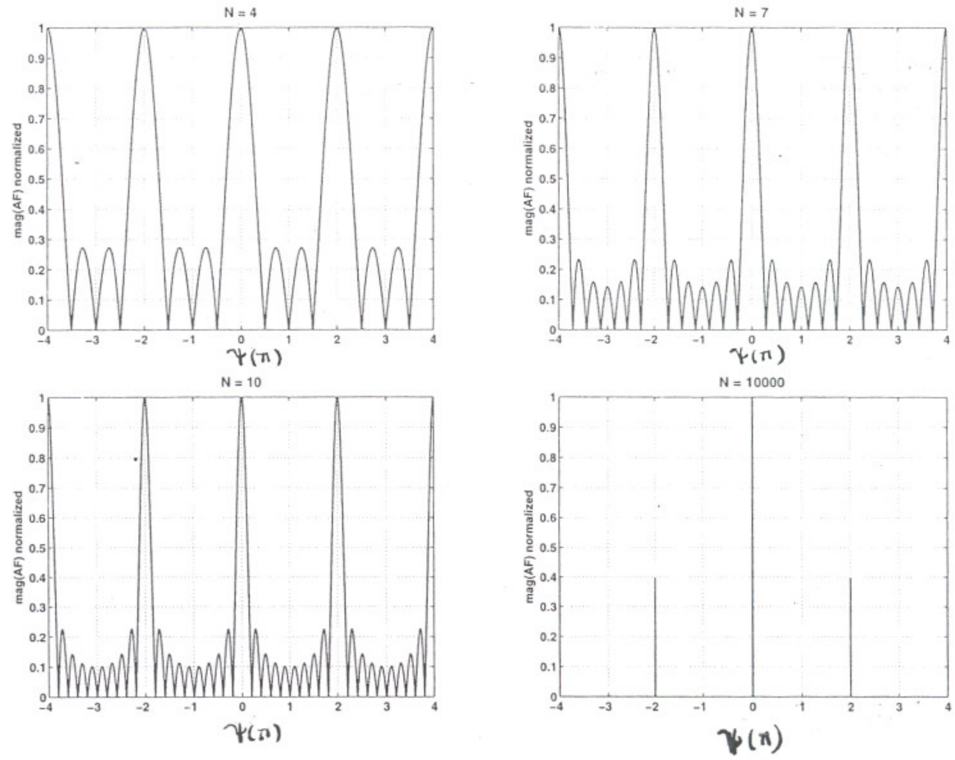
Solutions to HW # 12

1)

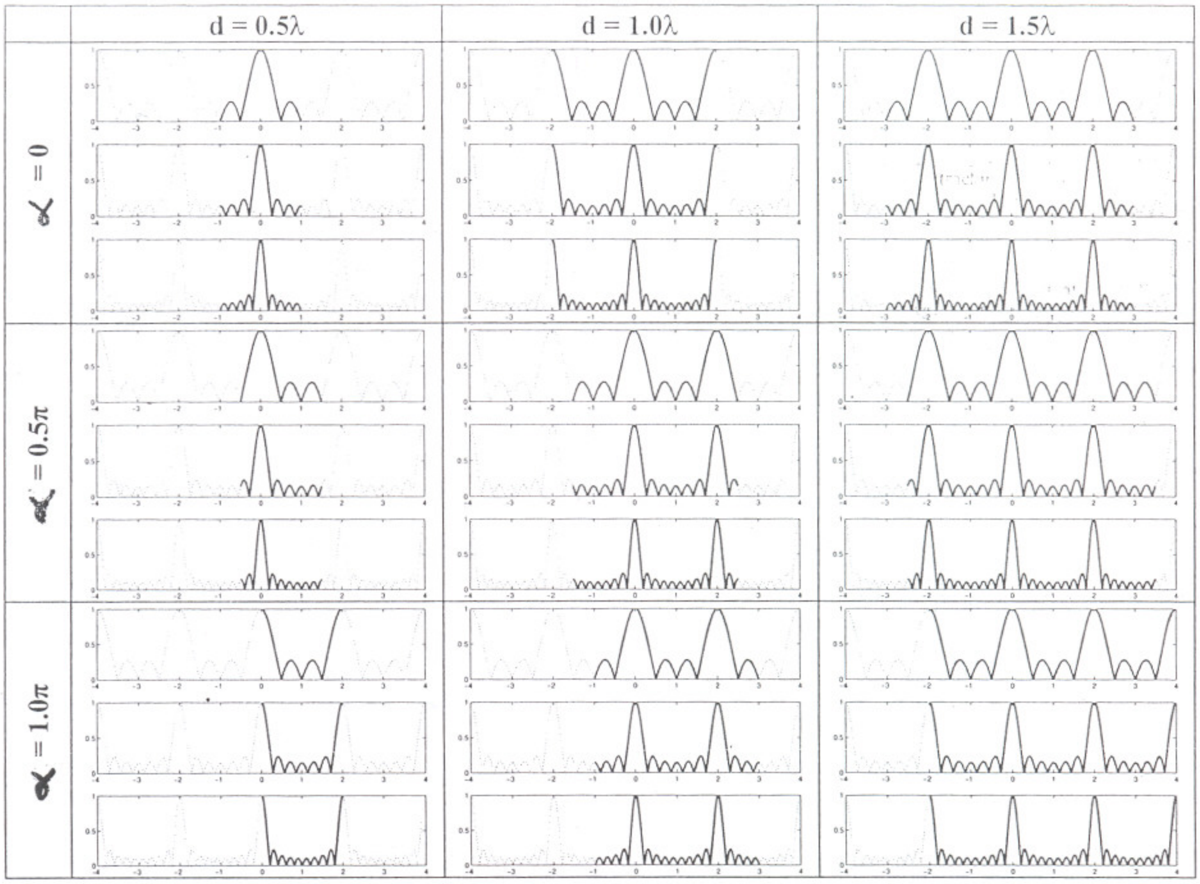
a, b)



	N = 4	**N = 7**	**N = 10**	**Formula**
Number of side lobes:	**2**	**5**	**8**	**N-2**
Null locations:	π/2, π, 3π/2	2π/7, 4π/7, 6π/7, 8π/7, 10π/7, 12π/7	π/5, 2π/5, 3π/5, 4π/5, π, 6π/5, 7π/5, 8π/5, 9π/5	2π(1+k)/N k=0, 1, …
Side lobe width:	π/2	2π/7	π/5	2π/N
Main lobe width:	π	4π/7	2π/5	4π/N

As N gets larger, the array occupies more area in the space. Due to Fourier Transform relation between the radiation and the current distribution, this gives rise to narrowing of the main lobe with the increase in the magnitude. In the limit case, AF reduces to periodic impulses with respect to ψ.

c)



d)
For any visible region, theta takes values from **π** to **0**. Therefore, location of the main lobes:

For α = 0, at $\theta = \pi/2$**;** independent of d.
For α = π/2, at $\theta = 2\pi/3$ for $d = 0.5\lambda$
at $\theta = 0.58\pi$ for $d = 1.0\lambda$
at $\theta = 0.55\pi$ for $d = 1.5\lambda$
For α = π, at $\theta = \pi$ (smaller end of the visible region) for $d = 0.5\lambda$
at $\theta = 2\pi/3$ for $d = 1.0\lambda$
at $\theta = 0.61\pi$ for $d = 1.5\lambda$

In general, α shifts the visible region, while d controls the width.

e)
If $\alpha = -kd\cos\theta_0$

$$\Psi = kd\cos\theta - kd\cos\theta_0 = kd\left(\cos\theta - \cos\theta_0\right)$$

and the main lobe occurs at $\theta = \theta_0$ (at $\Psi = 0$). Visible region is

$$-kd - kd\cos\theta_0 \le \Psi \le kd - kd\cos\theta_0.$$

The right-side grating lobe occurs at $\Psi = 2\pi$, but it should not be in the visible region:

$$kd - kd\cos\theta_0 \le 2\pi$$

$$d \le \frac{2\pi}{\frac{2\pi}{\lambda} - \frac{2\pi}{\lambda}\cos\theta_0} = \frac{\lambda}{1-\cos\theta_0}$$

The left-side grating lobe occurs at $\psi = -2\pi$, but it should not be in the visible region:

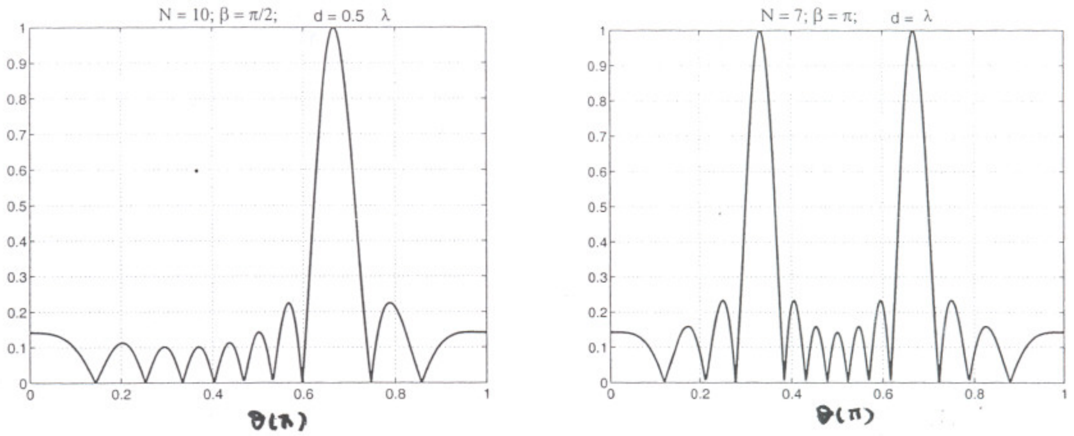
$$-kd - kd\cos\theta_0 \ge -2\pi \rightarrow kd + kd\cos\theta_0 \le 2\pi$$

$$d \le \frac{2\pi}{\frac{2\pi}{\lambda} + \frac{2\pi}{\lambda}\cos\theta_0} = \frac{\lambda}{1+\cos\theta_0}$$

Then,

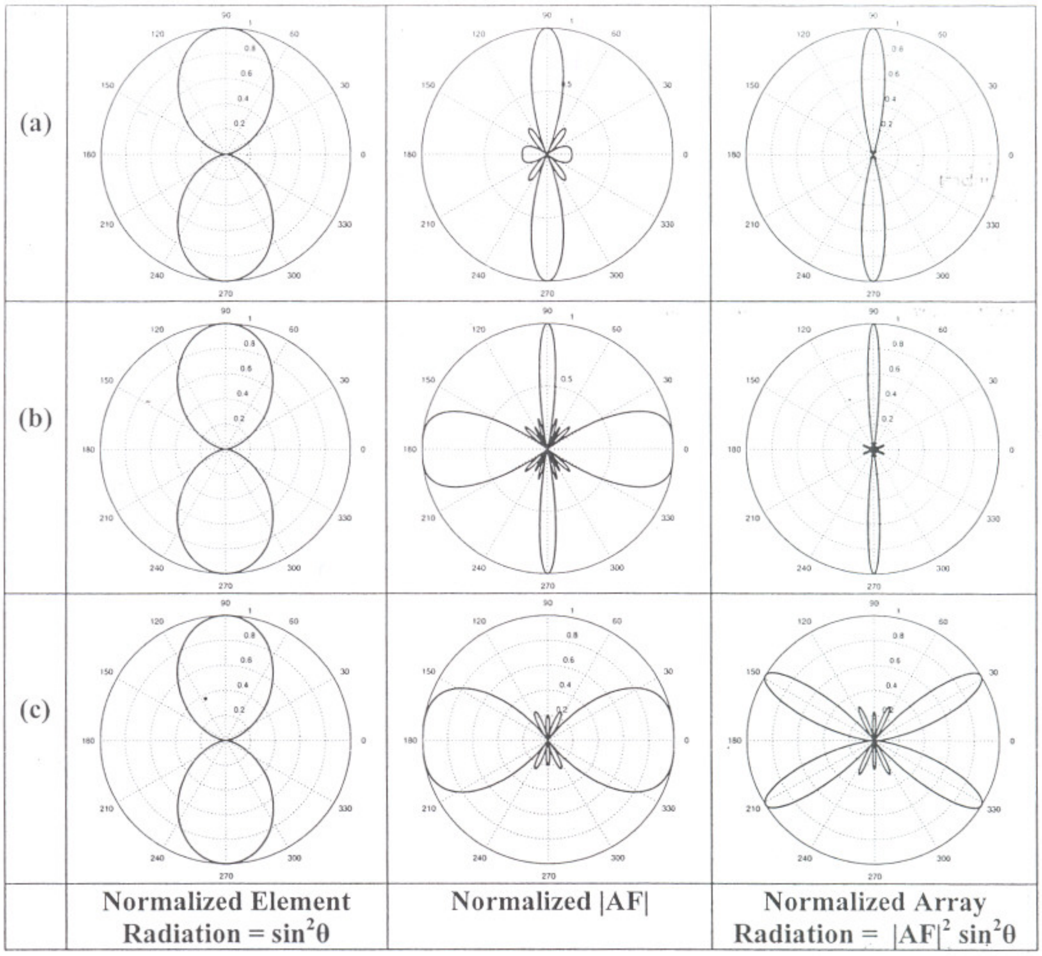
$$d \le \min\left\{\frac{\lambda}{1-\cos\theta_0}, \frac{\lambda}{1+\cos\theta_0}\right\} = \frac{\lambda}{1+|\cos\theta_0|}$$

f)



Note that these figures (x axis is theta/π now) are some reversed (also shifted) versions of the corresponding visible AFs in the previous figure.

2)



a) Broadside array configuration increases the directivity of single element.

b) Although the grating lobes are very significant in the array factor, they add very small side lobes in the total radiation.

c) End-fire array configuration splits the main lobe into two lobes, which is not desired in the antenna designs.

d)

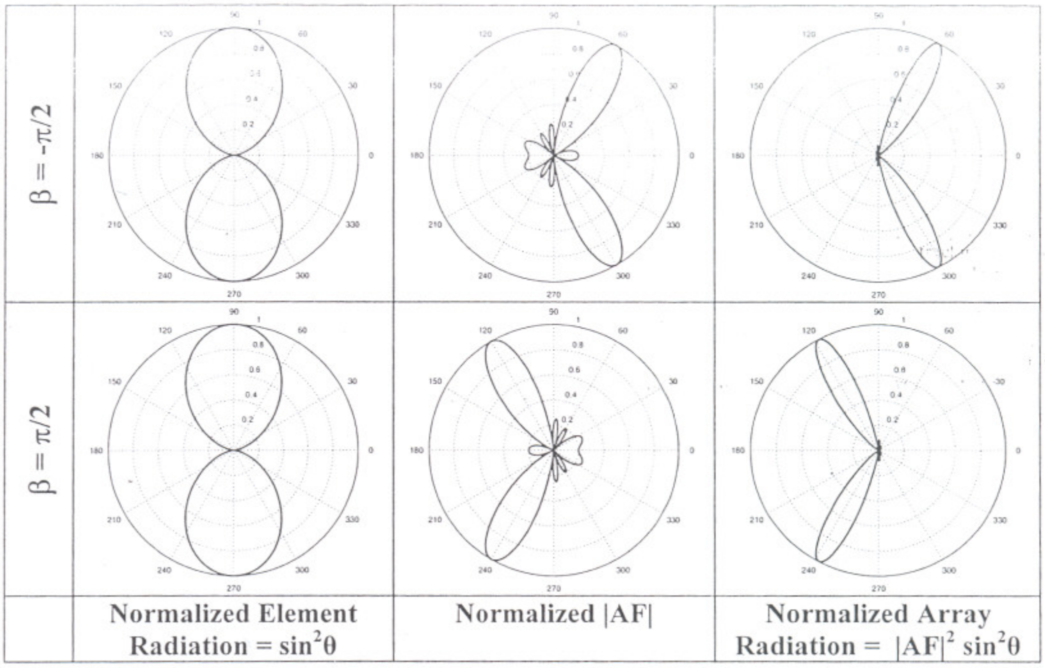
$$\Psi = kd\cos\theta + \beta_0 \rightarrow \text{ main lobe: } \Psi = 0$$

$$\rightarrow \beta_0 = -kd\cos\theta_{\text{main lobe}}$$

$$\text{if } \theta_{\text{main lobe}} = 60° \rightarrow \beta_0 = -\frac{kd}{2} = -\frac{\pi}{2}$$

$$\text{if } \theta_{\text{main lobe}} = 120° \rightarrow \beta_0 = \frac{kd}{2} = \frac{\pi}{2}$$

It should be noted that this analysis ignores the small shift due to element radiation. It can be observed that the main lobes are a little shifted from the desired angles (60° and 120°). A more accurate anaysis can be done by optimizing the total radiation instead of the array factor, but the array factor analysis is still very good since it is the dominating function in terms of the oscillation.


β = -π/2
β = π/2
Normalized Element Radiation = sin²θ
Normalized |AF|
Normalized Array Radiation = |AF|² sin²θ