Due 27/02/2008

EEE 452/552

HW #2

① Consider the Maxwell's equations in a lossy medium where

$$\nabla \times \bar{E} = -\frac{\partial \bar{B}}{\partial t} \; ; \quad \nabla \times \bar{H} = \bar{J}_i + \frac{\partial \bar{D}}{\partial t} + \sigma \bar{E} \; ; \quad \nabla \cdot \bar{D} = \rho \; ; \quad \nabla \cdot \bar{B} = 0$$

with $\bar{J}_i$ being the impressed current source and $\sigma \bar{E} = \bar{J}_c$: conduction current.

Derive a wave equation (both in time and phasor domains) for vector potential $\bar{A}$ and scalar potential Φ. Clearly show your gauge.

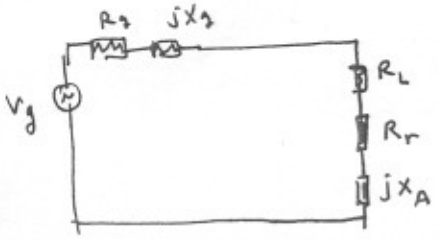
② In class we mentioned that any function in the form of $f(t - z/v)$ and $g(t + z/v)$ is a solution of one-dimensional wave equation given by $\frac{\partial^2 \psi}{\partial z^2} - \mu\epsilon \frac{\partial^2 \psi}{\partial t^2} = 0$

(a) Verify that $\psi(z;t) = \cos(t - z/v) + \cos(t + z/v)$ indeed satisfies $\frac{\partial^2 \psi}{\partial z^2} - \mu\epsilon \frac{\partial^2 \psi}{\partial t^2} = 0$

where $v = \frac{1}{\sqrt{\mu\epsilon}}$

(b) Verify that $\psi(z;t) = C_0^+ \cos(\omega t - kz) + C_0^- \cos(\omega t + kz)$ indeed satisfies the one-dimensional wave equation where C_0^+ & C_0^- are some real constants, ω is angular frequency ($\omega = 2\pi f$; f: frequency) and $k = \omega\sqrt{\mu\epsilon}$: wave number.

③ <u>Remember simple circuit theory:</u> Consider the following circuit driven by an AC voltage source V_g whose internal impedance is modelled by $Z_g = R_g + jX_g$. Determine the followings:



(a) Power dissipated on the resistance R_r (in terms of V_g, R_r, R_L, R_g, X_A and X_g)

(b) Power dissipated on the resistance R_L (in terms of V_g, R_r, R_L, R_g, X_A and X_g)

(c) " " " " " R_g (" " " " " " " " " ")

(d) Repeat parts (a), (b) and (c) when $X_A = -X_g$ and $R_g = (R_L + R_r)$

(e) Find the power supplied by the generator V_g when $X_A = -X_g$ and $R_g = (R_L + R_r)$