Due 05/03/2008

EEE 452/552

HW # 3

① Clearly show that the integral, I, given by

$$I = \frac{1}{T}\int_0^T \bar{E}(\bar{r}) \times \bar{H}(\bar{r})\, e^{j2\omega t}\, dt = 0$$

, where $\bar{E}(\bar{r})$ and $\bar{H}(\bar{r})$ are time-independent.

② Show rigorously that for plane waves

$\bar{P}_{ave} = \frac{1}{2} Re\{\bar{E} \times \bar{H}^*\}$ becomes $\frac{|\bar{E}_0|^2}{2\eta} \hat{a}_k$ in lossless media.

$\bar{E}_0$: magnitude vector of the electric field.

③ The fields due to a short vertical current element $I dl$ located at the origin of a spherical coordinate system in free-space are given by

$$\bar{E}(r,\theta) = \hat{a}_\theta E_\theta(r,\theta) = \hat{a}_\theta \left(j \frac{60\pi I dl}{\lambda r} \sin\theta \right) e^{-jkr} \quad (V/m)$$

$$\bar{H}(r,\theta) = \hat{a}_\phi H_\phi(r,\theta) = \hat{a}_\phi \left(j \frac{I dl}{2\lambda r} \sin\theta \right) e^{-jkr} \quad (A/m)$$

(a) find the instantaneous poynting vector.

(b) the average power density vector.

(c) total average power radiated by the current element.

④ Assuming that the radiation electric field intensity of an antenna system is

$$\bar{E} = \hat{a}_\theta E_\theta + \hat{a}_\phi E_\phi$$

find the expression for the average outward power flow per unit area.

⑤ Find the poynting vector on the surface of a long, straight conducting wire (of radius b and conductivity σ) that carries a direct current I. Verify Poynting's theorem (for this problem verify that — surface integral of the — poynting vector is exactly equal to the ohmic power loss in the conducting wire).