Due 12/03/2008

EEE 452/552

HW # 4

① Show that $\lim_{r \to 0} C \int_v e^{-jkr} r \sin\theta \, d\theta \, d\phi \, dr = 0$ where

C is a constant

② Derive the following relation: $\bar{E} = -j\omega\bar{A} - j \frac{1}{\omega\mu\epsilon} \nabla(\nabla \cdot \bar{A})$

③ For a given current distribution, the retarded vector potential $\bar{A}$ is given by

$$\bar{A}(\bar{r}, t) = \frac{\mu}{4\pi} \int_{v'} \frac{\bar{J}\left(t - \frac{\bar{r}-\bar{r}'}{v}\right)}{|\bar{r}-\bar{r}'|} dv' \quad \text{——} \quad (*)$$

Using an $e^{j\omega t}$ time convention, obtain the phasor form of (*). In other words, clearly show your work to obtain

$$\bar{A}(\bar{r}) = \frac{\mu}{4\pi} \int_{v'} \bar{J}(\bar{r}') \frac{e^{-jk|\bar{r}-\bar{r}'|}}{|\bar{r}-\bar{r}'|} dv'$$

④ Consider the E_r and E_θ components of the Hertzian dipole in the near-field region and show that in the limiting case where $\omega \to 0$ these expressions will recover the electric field intensity due to an elemental electric dipole of moment $\bar{p}_e$ in $\hat{a}_z$ direction.

Hint: $i(t) = \pm \frac{dq}{dt} = \pm j\omega q$ & $\bar{p}_e = \hat{a}_z q \ell$

+ charge at the upper portion of the origin.

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⑤ Answer all questions of the first quiz, provided on the next pages!