

## Homework #7

① Radiation intensities of two different lossless antennas are given by

$$U_1(\theta, \phi) = \begin{cases} B_1 \sin^2 \theta \sin^3 \phi & 0 \leq \theta \leq \pi, \quad 0 \leq \phi \leq \pi \\ 0 & \text{else} \end{cases}$$

$$U_2(\theta, \phi) = \begin{cases} B_2 \sin^3 \theta \sin^4 \phi & 0 \leq \theta \leq \pi, \quad 0 \leq \phi \leq \pi \\ 0 & \text{else} \end{cases}$$

- (a) Using MATLAB plot the radiation intensities at polar coordinates on x-y and y-z planes for both of the antennas. Normalize the plots as max. value to be 1.
- (b) Use MATLAB. Sample the radiation intensities with  $\pi/20$ -radian steps in both  $\theta$  and  $\phi$ . With total  $21 \times 21$  samples, determine the directivities (dimensionless) of both antennas numerically.
- (c) Find the directivities analytically and compare your results with the ones found in (b).
- (d) Find  $B_1$  and  $B_2$  if the total radiated power is required to be 2 Watts for both of the antennas.
- (e) Find the azimuthal and elevation plane half-power beam widths in degrees.

② Two lossless antennas have radiation intensities given by

$$U_1 = \begin{cases} 2 \sin \theta \sin \phi \left| \sin(\phi - \pi/5) \sin(\phi + \pi/5) \right| & 0 \leq \theta \leq \pi, \quad 0 \leq \phi \leq \pi \\ 0 & \text{otherwise} \end{cases}$$

$$U_2 = \begin{cases} 2 \sin \theta \sin \phi \left| \sin(\phi - \pi/7) \sin(\phi + \pi/7) \right| & 0 \leq \theta \leq \pi, \quad 0 \leq \phi \leq \pi \\ 0 & \text{otherwise} \end{cases}$$

- (a) Using MATLAB, plot the radiation intensity w.r. to  $\phi$  on the xy plane.
- (b) Using MATLAB find the azimuthal plane half-power beam width in degrees.
- (c) Find the sidelobe level (SLL) in dB (use MATLAB).
- (d) Find the maximum radiation density at a distance 1000m for each antenna (Assume far-field).

- ③ In Homework #6, question ④ you found the far-field expression of a very thin dipole whose current distribution is approximated as

$$I(x'=0, y'=0, z') = \begin{cases} \hat{a}_z I_0 \sin[k(\frac{l}{2} - z')] & ; 0 \leq z' \leq \frac{l}{2} \\ \hat{a}_z I_0 \sin[k(\frac{l}{2} + z')] & ; -\frac{l}{2} \leq z' \leq 0 \end{cases}$$

You can show that (not necessarily read pp. 170-173 of your book)

$$E_{\theta} \approx j\eta \frac{I_0 e^{-jkr}}{2\pi r} \left[ \frac{\cos(\frac{kl}{2} \cos\theta) - \cos(\frac{kl}{2})}{\sin\theta} \right]$$

$$H_{\phi} \approx \frac{E_{\theta}}{\eta}$$

Find the radiation intensity  $U(\theta, \phi)$  of this antenna, and for the elevation plane (for example  $z$ - $y$  or  $z$ - $x$  planes) plot the normalized power patterns for  $l = \lambda/2$ ,  $l = \lambda$ ,  $l = 1.25\lambda$  on the top of each other using MATLAB (I want a MATLAB figure). Check your results (i.e., your plots) with the ones given on pages 174 and 175 (see Fig. 4-6 and Fig. 4-7 (b)).

- ④ Consider an electrically small square-loop antenna as shown in the figure. The current has a constant amplitude  $I_0$  and a zero phase around the loop. Each side of the square loop is very short ( $l \ll \lambda$ ), uniform electric current segment that can be modeled as an ideal (infinitesimally Hertzian) dipole. For this geometry determine the far-zone  $\vec{E}$  and  $\vec{H}$  fields.

Hints:  $kl = \frac{2\pi}{\lambda} l \ll 1$

for  $|x| \ll 1$   $\sin(x) \approx x$

