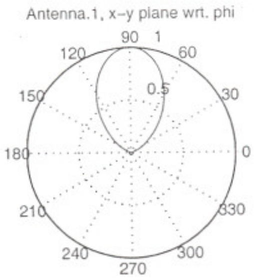
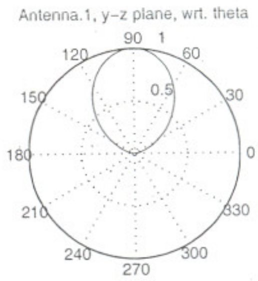
EEE 452/552 ANTENNA ENGINEERING
Spring 2008

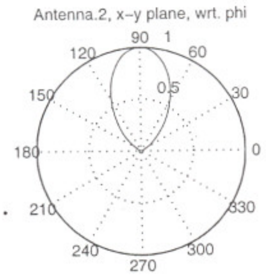
HW7 - SOLUTIONS

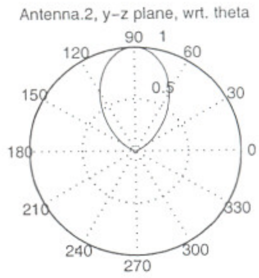
1)

1.a)









1.b) Normalizing the maximum radiation intensity to one:

$$D = \frac{4\pi}{P_{rad}} = \frac{4\pi}{\int_0^{2\pi}\int_0^{\pi} F(\theta,\phi)\sin(\theta)\,d\theta\, d\phi}$$

Dividing theta and phi with $\pi/20$ radian intervals:

$$D \approx \frac{4\pi}{\frac{2\pi^2}{882}\sum_{j=1}^{21}\sum_{i=1}^{21} F(\theta_i,\phi_j)\sin(\theta_i)}, \quad \theta_i = (i-1)\frac{\pi}{20}, \quad \phi_j = (j-1)\frac{\pi}{20}$$

The summation is taken over non-zero part of the intensity and we get:

D_1=7.793
D_2=9.982

However, this numerical integration uses pole values incorrectly that the values at two points (θ=0, π) are used 21 times. Correcting this mistake (note that pole values are already zero and they should be included only once):

$$D \approx \frac{4\pi}{\frac{2\pi^2}{800}\sum_{j=1}^{21}\sum_{i=2}^{20} F(\theta_i,\phi_j)\sin(\theta_i)}, \quad \theta_i = (i-1)\frac{\pi}{20}, \quad \phi_j = (j-1)\frac{\pi}{20}$$

And we get:

$D_1 \approx 7.069$
$D_2 \approx 9.054$

1.c)

$$P_{rad}^{1} = \int_0^{\pi}\int_0^{\pi} \sin^3(\theta)\sin^3(\phi)\,d\theta d\phi = \int_0^{\pi}\sin^3(\phi)d\phi\int_0^{\pi}\sin^3(\theta)d\theta = \frac{16}{9}$$

$$\rightarrow D_1 = \frac{9\pi}{4} \approx 7.069$$

$$P_{rad}^{2} = \int_0^{\pi}\int_0^{\pi} \sin^4(\theta)\sin^4(\phi)\,d\theta d\phi = \int_0^{\pi}\sin^4(\phi)d\phi\int_0^{\pi}\sin^4(\theta)d\theta = \frac{9\pi^2}{64}$$

$$\rightarrow D_2 = \frac{256}{9\pi} \approx 9.054$$

Numerical results perfectly agree with the analytical results.

1.d)

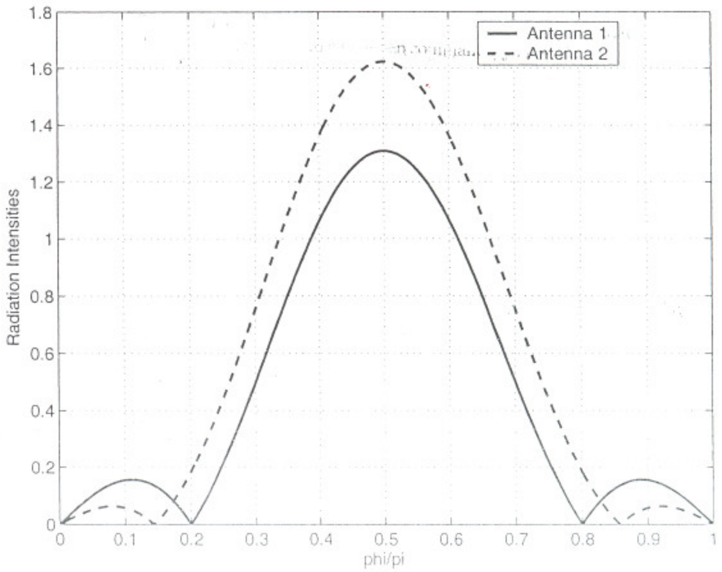
$B_1 \frac{16}{9} = 2 \rightarrow B_1 = \frac{18}{16} = 1.1250$ Watts/Unit Solid Angle

$B_2 \frac{9\pi^2}{64} = 2 \rightarrow B_2 = \frac{128}{9\pi^2} \approx 1.441$ Watts/Unit Solid Angle

Note that for the same total radiated power, the second antenna has higher maximum power intensity.

1.e) HP. Beam widths:
Antenna 1: 74.94° azimuthal, 90° elevation
Antenna 2: 65.52° azimuthal, 74.94° elevation

2)
2.a)



2.b) $U_1 \approx 0.655$ W / U.S.Angle = $U_{1max}/2$ at
$\Phi \approx 58.64°$ and $121.38°$, HPBW $\approx 62.7°$

$U_2 \approx 0.812$ W / U.S.Angle = $U_{2max}/2$ at
$\Phi \approx 55.61°$ and $124.37°$, HPBW $\approx 68.8°$

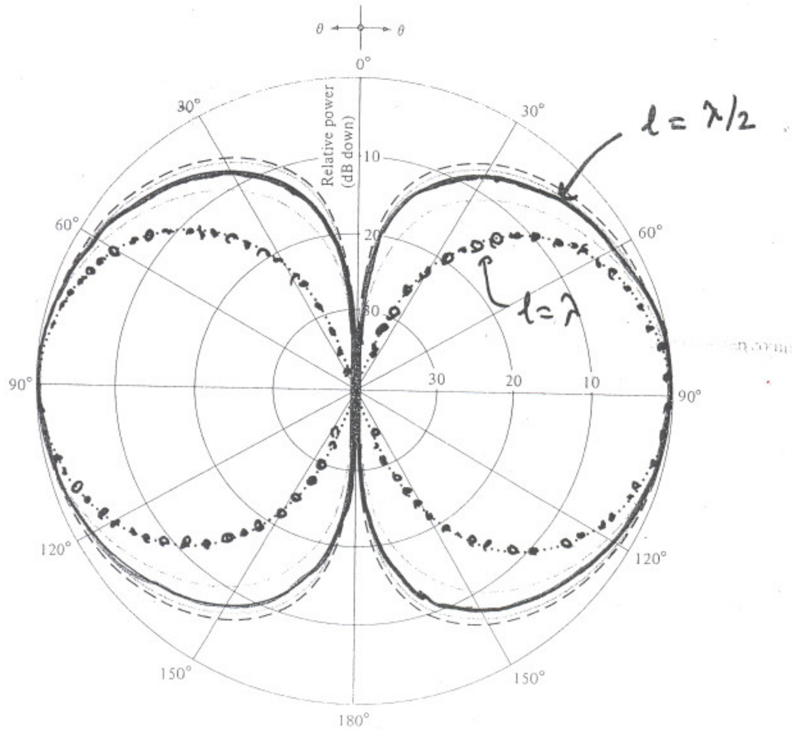
2.c) Antenna 1: $U_{1max}/U_{1side} \approx 1.31/0.156 \approx 8.397$ $\quad E_{1max}/E_{1side} \approx 2.898 \approx 4.621$ dB
Antenna 2: $U_{2max}/U_{2side} \approx 1.62/0.063 \approx 25.71$ $\quad E_{2max}/E_{2side} \approx 5.071 \approx 7.051$ dB

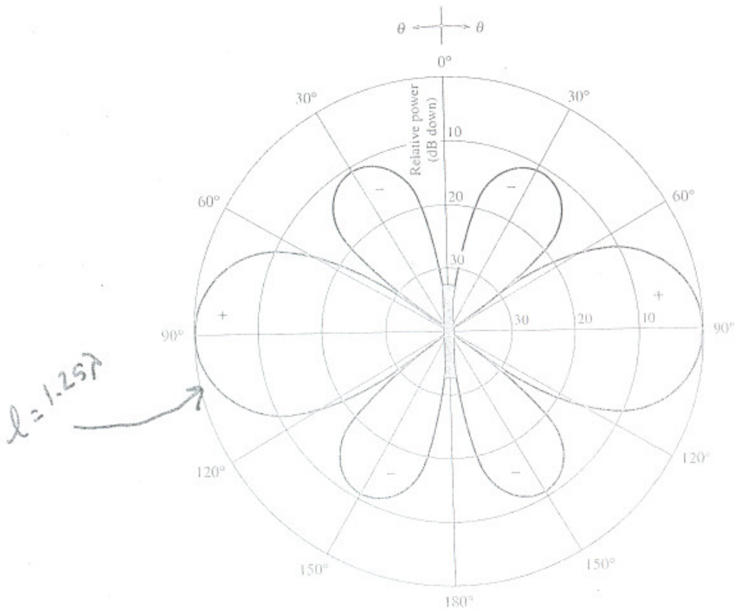
2.d)
Antenna 1: $W_{1max} = U_{1max}/r^2 \approx 1.31 \times 10^{-6}$ Watts/m^2
Antenna 2: $W_{2max} = U_{2max}/r^2 \approx 1.62 \times 10^{-6}$ Watts/m^2

3 Using the given E_θ and H_ϕ expressions

$$\overline{W}_{ave} = \frac{1}{2}\mathrm{Re}\{\bar{E}\times\bar{H}^*\} = \frac{1}{2}\mathrm{Re}\{\hat{a}_\theta E_\theta \times \hat{a}_\phi H_\phi^*\} = \hat{a}_r \frac{1}{2\eta}|E_\theta|^2 = \hat{a}_r \eta \frac{|I_0|^2}{8\pi^2 r^2}\left[\frac{\cos\left(\frac{kl}{2}\cos\theta\right) - \cos\left(\frac{kl}{2}\right)}{\sin\theta}\right]^2$$

$$\text{and } U(\theta,\phi) = r^2 W_{ave} = \eta \frac{|I_0|^2}{8\pi^2}\left[\frac{\cos\left(\frac{kl}{2}\cos\theta\right) - \cos\left(\frac{kl}{2}\right)}{\sin\theta}\right]^2$$





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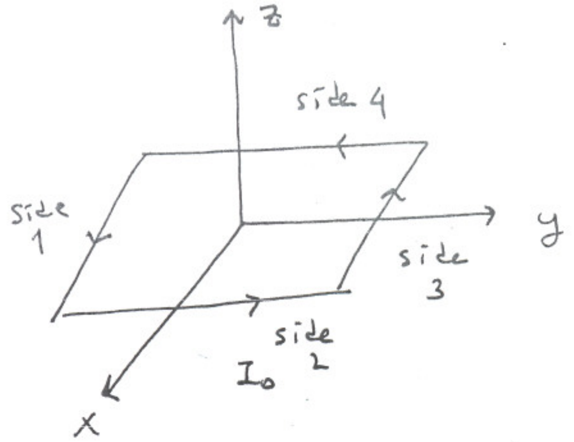

For an infinitesimal dipole $\bar{A}(\bar{r}) \simeq \hat{a}_u \dfrac{\mu I l}{4\pi r} e^{-jkr}$ $\quad \hat{a}_u = \hat{a}_x$ or $\hat{a}_y$ or $\hat{a}_z$

Thus, if each side of this loop is treated like an infinitesimal dipole

From side 1 and side 3

$$A_x \simeq \frac{\mu I_o l}{4\pi}\left[\frac{e^{-jkR_1}}{R_1} - \frac{e^{-jkR_3}}{R_3}\right]$$

From side 2 and sid 4

$$A_y \simeq \frac{\mu I_o l}{4\pi}\left[\frac{e^{-jkR_2}}{R_2} - \frac{e^{-jkR_4}}{R_4}\right]$$

For the far-zone

in the denominator $R_1 = R_2 = R_3 = R_4 = r$

in the exponent $\quad R_1 \simeq r - \hat{a}_r \cdot \bar{r}' = r + \hat{a}_r \cdot \hat{a}_y \frac{l}{2} = r + \frac{l}{2}\sin\theta\sin\phi$

$$R_2 \simeq r - \hat{a}_r \cdot \bar{r}' = r - \hat{a}_r \cdot \hat{a}_x \frac{l}{2} = r - \frac{l}{2}\sin\theta\cos\phi$$

$$R_3 \simeq r - \hat{a}_r \cdot \bar{r}' = r - \hat{a}_r \cdot \hat{a}_y \frac{l}{2} = r - \frac{l}{2}\sin\theta\sin\phi$$

$$R_4 \simeq r - \hat{a}_r \cdot \bar{r}' = r + \hat{a}_r \hat{a}_x \frac{l}{2} = r + \frac{l}{2}\sin\theta\cos\phi$$

$$\Rightarrow A_x \simeq \frac{\mu I_o l}{4\pi r} e^{-jkr}\left(e^{-jk\frac{l}{2}\sin\theta\sin\phi} - e^{jk\frac{l}{2}\sin\theta\sin\phi}\right)$$

$$\Rightarrow A_y \simeq \frac{\mu I_o l}{4\pi r} e^{-jkr}\left(e^{jk\frac{l}{2}\sin\theta\cos\phi} - e^{-jk\frac{l}{2}\sin\theta\cos\phi}\right)$$

$$\Rightarrow \begin{cases} A_x \simeq -2j \dfrac{\mu I_0 l}{4\pi r} e^{-jkr} \sin\left(\dfrac{kl}{2} \sin\theta \sin\phi\right) \simeq -j \dfrac{\mu I_0}{4\pi} \dfrac{e^{-jkr}}{r} kl^2 \sin\theta \sin\phi \\ A_y \simeq 2j \dfrac{\mu I_0 l}{4\pi r} e^{-jkr} \sin\left(\dfrac{kl}{2} \sin\theta \cos\phi\right) \simeq j \dfrac{\mu I_0}{4\pi} \dfrac{e^{-jkr}}{r} kl^2 \sin\theta \cos\phi \end{cases}$$

$$\Rightarrow \bar{A} = A_x \hat{a}_x + A_y \hat{a}_y \simeq jkl^2 \frac{\mu I_0}{4\pi} \frac{e^{-jkr}}{r} \sin\theta \underbrace{\left[-\hat{a}_x \sin\phi + \hat{a}_y \cos\phi\right]}_{\hat{a}_\phi}$$

$$\Rightarrow \bar{A} \simeq jkl^2 \frac{\mu I}{4\pi} \frac{e^{-jkr}}{r} \sin\theta \, \hat{a}_\phi$$

$$\bar{E} = -j\omega\bar{A} \Rightarrow \boxed{E_\phi = \eta \frac{k^2 l^2 I_0}{4\pi} \frac{e^{-jkr}}{r} \sin\theta}$$

$$\boxed{H_\theta = -\frac{E_\phi}{\eta} = -\frac{k^2 l^2 I_0}{4\pi} \frac{e^{-jkr}}{r} \sin\theta}$$