Due 03/04/2008

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Homework # 8

(1) Consider a circular wire loop antenna with a radius "a" located on the x-y plane as shown in Fig.1. The current on the wire is constant with a value of I_0.

(a) Calculate the far-zone $\bar{E}$ and $\bar{H}$ fields.

(b) If $a \ll \lambda$, $ka \ll 1$ perform small argument approximations and rewrite the $\bar{E}$ and $\bar{H}$ fields that you have found in part (a)

Hints: ① For the sake of simplicity, you can set $\phi = 0$. The result (i.e., the generality of the result) will not change. Can you explain why?

② You may use

$$j^n 2\pi J_n(x) = \int_0^{2\pi} e^{jx\cos\phi} e^{jn\phi} d\phi$$

$$j^n \pi J_n(x) = \int_0^{\pi} \cos(\phi n) e^{jx\cos\phi} d\phi$$

③ You may also use the result of HW#1

④ for part (b), use $J_1(x) \simeq \frac{1}{2}x - \frac{1}{16}x^3$ for $x \ll 1$ (neglect the second term)

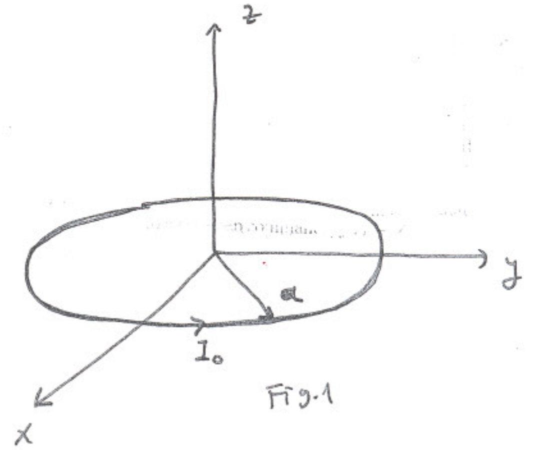


Fig.1

(2) An antenna having radiation intensity

$$U_1 = \begin{cases} B_1 \cos^3(\theta) & 0 \le \theta \le \pi/2, \ 0 \le \phi \le 2\pi \\ 0 & \text{otherwise} \end{cases}$$

is connected to a source having 2^V peak voltage and $50 + j75\Omega$ input impedance (Z_g). The antenna has 50Ω reactance (X_A), 50Ω radiation resistance (R_r) and 25Ω loss resistance (R_L)

(a) Find the power

(i) supplied by the generator, (ii) dissipated in the generator, (iii) delivered to the antenna

(iv) radiated by the antenna.

(b) Repeat part (a) if the input impedance of the source is changed and <u>matched</u> to the antenna. Compare the results with the ones in (a). (Matched means $Z_A = Z_g{}^a$)

(c) Find $B_?$ for both part (a) and part (b) and comment on the values.

(3) Figures 2. and 3. show two experiments, in which two antennas are used. In the first experiment, Antenna A (50Ω) is used as the transmitter and connected to a source with 50Ω terminal resistance. Antenna B (75Ω) is used as the receiver and connected to a 25Ω terminal resistance. In this experiment, the output power is found to be 1 Watt when the input power is 10 watts. In the second experiment, the roles are exchanged and Antenna A becomes the receiver while Antenna B becomes the transmitter.

Experiment 1

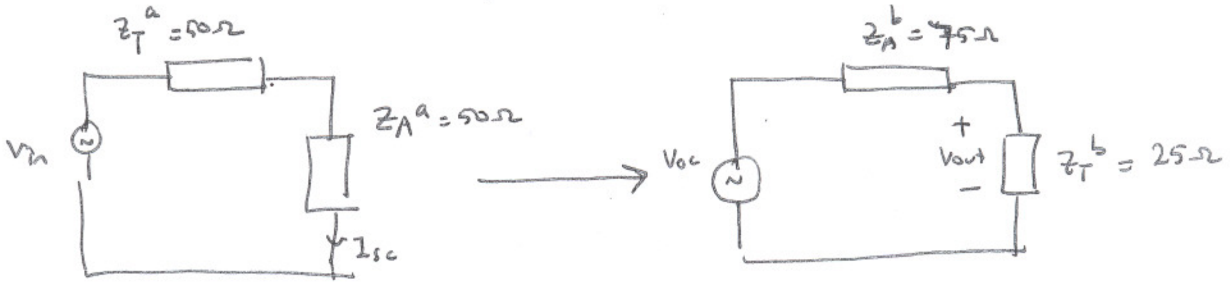


Fig.2

Experiment 2

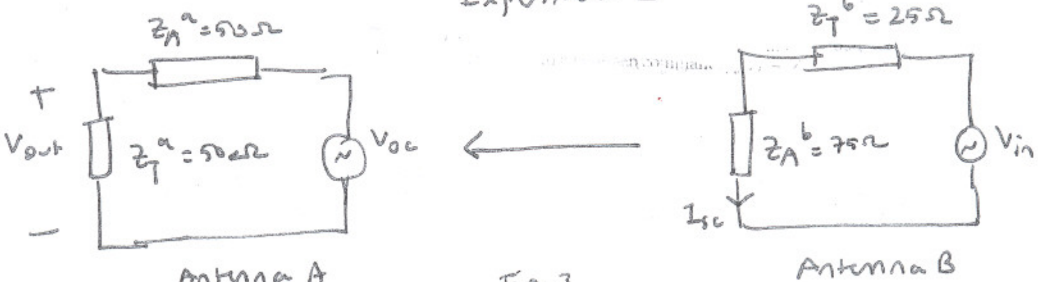


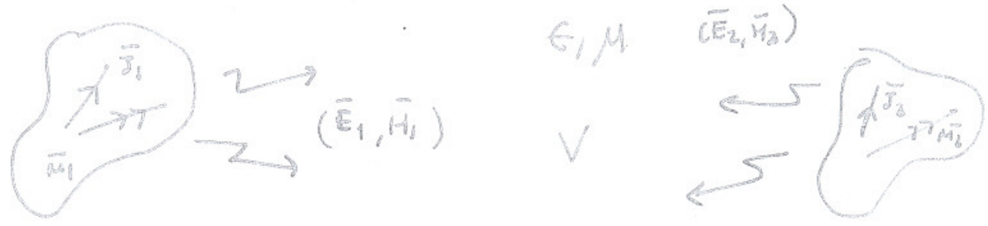
Fig.3

(a) Find the output power in the second experiment when the input power is again 10 watts.

(b) What should be the terminal resistance (Z_T) of the transmitter while others remain the same in the second experiment so that the output power is 1 watt, when the input power is again 10 watts.

(c) What should be the terminal resistance (Z_T) of the receiver while others remain the same in the second experiment so that the output power is 1 watt, when the input power is again 10 watts.

④



$$\nabla \times \bar{E}_1 = -\bar{M}_1 - j\omega\mu \bar{H}_1 \qquad \nabla \times \bar{E}_2 = -\bar{M}_2 - j\omega\mu \bar{H}_2$$

$$\nabla \times \bar{H}_1 = \bar{J}_1 + j\omega\epsilon \bar{E}_1 \qquad \nabla \times \bar{H}_2 = \bar{J}_2 + j\omega\epsilon \bar{E}_2$$

With the given curl equations for $(\bar{E}_1, \bar{H}_1)$ and $(\bar{E}_2, \bar{H}_2)$ pairs for $(\bar{J}_1, \bar{M}_1)$ and $(\bar{J}_2, \bar{M}_2)$ pairs, respectively, derive the following equations

$$\oint_S (\bar{E}_1 \times \bar{H}_2 - \bar{E}_2 \times \bar{H}_1) \cdot d\bar{s} = \int_V (\bar{J}_1 \cdot \bar{E}_2 + \bar{M}_2 \cdot \bar{H}_1 - \bar{J}_2 \cdot \bar{E}_1 - \bar{M}_1 \cdot \bar{H}_2)\, dv$$

where S is the surface that bounds volume V.

Use $\nabla \cdot (\bar{A} \times \bar{B}) = \bar{B} \cdot \nabla \times \bar{A} - \bar{A} \cdot \nabla \times \bar{B}$