Date: 15 March 2008, Saturday NAME:..
Time: 10:00-12:00

STUDENT NO:..

Math 206 Complex Calculus – Midterm Exam I

1	2	3	4	TOTAL
25	25	25	25	100

Please do not write anything inside the above boxes!

PLEASE READ:
Check that there are 4 questions on your exam booklet.
No correct answer without a satisfactory reasoning is accepted. Show your work in detail.
Write your name on the top of every page.

Q-1) Find the value of $\frac{d}{dz} sin(cosh^2 z)$ at $z = i\pi/4$.

Solution: $\frac{d}{dz} sin(cosh^2 z) = 2cos(cosh^2 z)\, cosh\, z\, sinh\, z$ and $cosh(i\pi/4) = cos(\pi/4) = 1/\sqrt{2},\ sinh(i\pi/4) = isin(\pi/4) = i/\sqrt{2}$ so that

$$\frac{d}{dz} sin(cosh^2 z)|_{z=i\pi/4} = i\, cos(1/2).$$

Q-2) Find the image in the $w = u + iv$ plane of the region $0 \leq x \leq ln(2), \ 0 \leq y \leq \pi$ in the $z = x + iy$ plane under the transformation $w = e^z$.

Solution: The image is the closed region in the upper half $uv-$plane bounded by semicircles of radii 1 and 2 and the line segments $-2 \leq x \leq -1$ and $1 \leq x \leq 2$ on the real line.

Q-3) Find the principal values of (a) $Log(-\sqrt{3}+i)^3$ and (b) $(-3)^i$. Give your answers in the form $a+ib$.

Solution: (a) We have $(-\sqrt{3}+i)^3 = 8i$. Hence

$$log(-\sqrt{3}+i)^3 = ln(8) + i(\pi/2 + 2n\pi)$$

for any integer n with principal value

$$Log(-\sqrt{3}+i)^3 = ln\, 8 + i\pi/2.$$

(b) The principal value is $(-3)^i = exp[i\, Log(-3)] = exp(i\, ln\, 3 - \pi) = e^{-\pi}[cos(ln\, 3) + i\, sin(ln\, 3)]$.

Q-4) Determine all solutions *in rectangular form* to the equation $sin\, z = -i$ (a) by equating the real and imaginary parts of two sides and (b) by using the inverse of the complex sine function.

(a) $sin\, z = -i$ if and only if $sin\, x\, cosh\, y = 0$, $cos\, x\, sinh\, y = -1$. The latter holds if and only if $x = n\pi$ and $sinh(y) = 1$ when n is odd and $sinh(y) = -1$ when n is even. Now, $sinh(y) = \frac{1}{2}(e^y - e^{-y}) = 1$ gives $y = ln\,(1+\sqrt{2})$. Since sinh function has symmetry w.r.t. the origin, we also have that $sinh(y) = -1$ implies $y = -ln\,(1+\sqrt{2})$. Combining

$$sin\, z = -i \quad \text{iff} \quad z = n\pi - i(-1)^n\, ln\,(1+\sqrt{2}),$$

for any integer n.
(b) See the Example in Section 35 of the textbook.