MATH 225 Linear Algebra and Differential Equations

Fall 2007 MATLAB Homework 1

Due date: Friday, October 19, 17:30

1. For the numerical solution of the differential equation

$$\frac{dy}{dx} = f(x, y), \tag{1.1}$$

the improved Euler method provides better accuracy compared to the Euler method at the cost of one more function call to $f(x, y)$. Another way to improve the Euler Method is *the midpoint method*. One step of this algorithm consists of the following steps

$$k_1 = f(x_n, y_n) \text{ the value of the slope evaluated at } (x_n, y_n)$$

$$y_{n+1/2} = y_n + \frac{h}{2} f(x_n, y_n) \text{ estimate the solution at the midpoint}$$

$$k_2 = f(x_n + \frac{h}{2}, y_n + \frac{h}{2} k_1) \text{ estimate the slope at the midpoint}$$

$$y_{n+1} = y_n + hk_2$$

Implement the midpoint method in an m-file called midpoint.m by making small changes to the listing Figure 2.5.11 in your textbook. Also write the listing in Figure 2.4.11 in another m-file called euler.m. To verify your codes, find the exact solution of the initial value problem

$$\frac{dy}{dx} = -y, \quad y(0) = 1, \quad 0 \le x \le 1. \tag{1.2}$$

Then, write another driver routine called `compareEM`, which calls `euler` and `midpoint` subroutines for step sizes $h = [0.2\ 0.1\ 0.05\ 0.025\ 0.0125\ 0.00625]$ and prints the errors and elapsed time. Comment on the results. Here are a few hints:

- Find errors using the formula `max(abs(y_e-y_exact))`, where `y_e` is the solution vector obtained from (for example) the Euler method and `y_exact` is the exact solution vector discretized with corresponding *h*.
- You can use the `tic` and `toc` function calls of matlab to find the time elapsed. However, the time passed between the subroutine calls will be too small to get accurate results. To prevent this situation, make the calls to subroutines 1,000 times and then divide resulting elapsed time by 1,000. Output the time in microseconds.
- For proper formatting output data use the following `fprintf` routines:

```
fprintf('\n   h       timeE        errE       timeM        errM\n');
fprintf('%8.5f %7d %11.2e %7d %11.2e\n',h,time_e,err_e,time_m,err_m).
```

"`_e`" is the abbreviation for the Euler method and "`_m`" is for the midpoint method.

2. Solve the following initial-value problems by hand. Then, plot exact, euler, midpoint, and `ode45` (Matlab routine) solutions using *h*=0.1 step size. For `ode45` routine, refer to Matlab help or to the manual at the web page. *Comment on the solutions.* Repeat with *h*=0.01.

(a) $\frac{dy}{dx} + y = x^2, \quad y(0) = 1, \quad 0 \le x \le 4$

(b) $\frac{dy}{dx} + y = Q(x), \quad y(0) = 0, \quad 0 \le x \le 4$, where $Q(x) = \begin{cases} 1, & 0 \le x \le 1 \\ 0, & x < 0 \text{ or } x > 1 \end{cases}$

For this question use the `hold on` feature to insert exact, euler, midpoint, and `ode45` solutions on the same plot. Hence, in addition to your solutions and comments, provide only four plots.

3. For the differential equation $\frac{dy}{dx} = -200xy^2$, solve the initial-value problem for two initial conditions, $y(0) = 1$ and $y(-3) = 1/901$. Then, plot the solution using `ode45` until $x = 1$. Finally, try to obtain the solutions with direction fields using the `dfield` program from http://math.rice.edu/~dfield/index.html. Do you have any difficulties in obtaining one of the solution using direction fields? Why? Provide your calculation and the solutions obtained with `ode45` and `dfield`.

4. Solve the differential equation $\frac{dy}{dx} = \sqrt{y}, \quad y(0) = 0, \quad 0 \le x \le 2$ using `ode45` and `dfield`. What happens if $y(0) = \varepsilon$ (a very small number such as 10^{-8}) is used as the initial condition? Why? Provide the solution and two plots; one from `ode45` and one from `dfield`.