MATH 225 Linear Algebra and Differential Equations

Homework 3 Solutions

1. The code that performs all questions in (a)-(e)

```
%prepare an mxn0 matrix with rank=r
clear
m=10; n=20; r=3;
B=10*rand(m,n);
[U,S,V]=svd(B);
for i=r+1:m
    S(i,i)=0;
end
A = U*S*V'
rankA=rank(A)

rrefA=rref(A);
rowBasis=rrefA(1:rankA,:)'

rrefAt=rref(A'); %to find the column subspace use the transpose of A
colBasis=rrefAt(1:rankA,:)'

nullSpace=null(A)
rowCheck=rowBasis'*nullSpace %if two sets are orthogonal, the inner product
%of their columns should be 0.
spaceOrthogonalToColBasis=null(A') %null A transpose gives us a set
%orthogonal to col(A)
colCheck=colBasis'*spaceOrthogonalToColBasis

noisyA=A+1e-6*rand(m,n)
rankNoisyA=rank(noisyA)
```

Check with m=4 n=6 r=2

```
A =

    7.6358    7.3396    8.8156    8.8038   11.3258    1.6669
    5.7397    5.5942    6.4704    5.9787    7.8525    2.4050
    5.4209    5.6813    5.3053    2.3507    4.0073    8.2133
    4.4254    4.1608    5.2972    5.8718    7.3598   -0.4211


rankA =

     2


rowBasis =

    1.0000         0
         0    1.0000
    3.1009   -2.0249
    9.1157   -8.2840
    9.7193   -8.5684
  -14.1368   14.9345


colBasis =

    1.0000         0
         0    1.0000
```

```
   -3.8772    6.1024
    1.4847   -1.2042


nullSpace =

    0.8127   -0.0235    0.2640    0.3068
   -0.0103    0.4043    0.2761   -0.7594
    0.1052   -0.3754   -0.7665   -0.2429
   -0.1975    0.6783   -0.2617    0.4025
   -0.4202   -0.4843    0.4449    0.1161
   -0.3357    0.0204   -0.0123    0.3078


rowCheck =

  1.0e-014 *

    0.5329    0.1832    0.2054    0.0888
   -0.5329   -0.2220   -0.1887   -0.0888


spaceOrthogonalToColBasis =

    0.1223    0.6814
   -0.8004   -0.3464
    0.2371   -0.0697
    0.5368   -0.6410


colCheck =

  1.0e-015 *

    0.9992    0.8882
   -0.8882   -0.8882


noisyA =

    7.6358    7.3396    8.8156    8.8038   11.3258    1.6669
    5.7397    5.5942    6.4704    5.9787    7.8525    2.4050
    5.4209    5.6813    5.3053    2.3507    4.0073    8.2133
    4.4254    4.1608    5.2972    5.8718    7.3598   -0.4211


rankNoisyA =

     4
```

We see that a little noise highly effects the matrix and makes it full rank!

2. Show *analytically* that

(i) $q_1 \perp q_2$

A direct multiplication yields ${q_1}^T q_2 = 0$.

(ii) span$\{q_1, q_2\}$ =span$\{a_1, a_2\}$.

Again, inserting the expressions of q_1 and q_2,

$\text{span}\{q_1, q_2\} = \alpha q_1 + \beta q_2 = \tilde{\alpha} a_1 + \tilde{\beta} a_2 = \text{span}\{\text{a}_1,\text{a}_2\}$.

(b) Code:

```
%
function Q=orthogonalizeA(A)
n=length(A);
Q=[];
for i=1:n
    v=A(:,i);
    for j=1:i-1
        v=v-(Q(:,j)'*v)*Q(:,j); %orthogonalize wrt previous vectors
    end
    q=v/norm(v); %normalize
    Q=[Q q]; % add the new q
end
```

Check:

```
>> Q=orthogonalizeA(A)

Q =

    0.2582    0.6583    0.0000    0.7071
   -0.2582   -0.6583   -0.0000    0.7071
   -0.5164    0.2025   -0.8321    0.0000
   -0.7746    0.3038    0.5547    0.0000
```

The built-in `qr` function should yield the same up to signs:

```
>> [Q,R] = qr(A)

Q =

   -0.2582    0.6583    0.0000    0.7071
    0.2582   -0.6583    0.0000    0.7071
    0.5164    0.2025   -0.8321    0.0000
    0.7746    0.3038    0.5547   -0.0000
```