MATH 225 Linear Algebra and Differential Equations

Fall 2007 MATLAB Homework 4

Due date: Friday, December 14, 17:30

The singular value decomposition (SVD) of a matrix **A** is given as $\mathbf{A}=\mathbf{USV}^{\mathbf{T}}$. One important feature of SVD is that it provides the best low-rank approximations to a matrix. To see this fact, write **S** as a sum of *r* matrices $\mathbf{S_j}$, where $\mathbf{S_j}=diag(0,\ldots,0,\sigma_j,0,\ldots,0)$ and σ_j is the jth singular value of **A**. Then, it follows that $\mathbf{A}=\sum_{j=1}^{r}\sigma_j u_j v_j^T$, where u_j and v_j are the *j*th columns of **U** and **V**, respectively. Here, the *k*th partial sum $\mathbf{A_k}=\sum_{j=1}^{k}\sigma_j u_j v_j^T$ captures as much information of **A** as possible. We will use this property to examine a head phantom image shown in Figure 1.

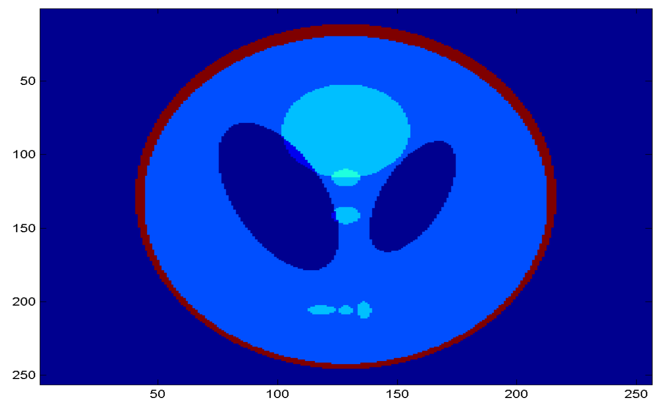

Figure 1: A head phantom image.

(a) Read this image into a matrix using the command

```
P=phantom('Modified Shepp-Logan');
```

Find the rank of the matrix *r*. Why dou you think it is lower than 256? Then, plot the low-rank approximations of the image for *k*= 5,10,15,25,50,75,100, and *r*. Use the `imagesc` and `subplot` functions to insert images on a 4 by 2 plot. Use `title` and `int2str` functions for proper identification of the images. Comment on the resulting images.

(b) In real-life such images are almost always accompanied by a troublesome noise. Implement such a noise using the command `P=P+sigma*randn(size(P))`. What is the rank of this noisy matrix? (You should easily guess this with the help of your previous homework!). Again insert this noisy image as well as its 7 low-rank approximations to a 4 by 2 plot. Which kind of effect did the low-rank approximations perform on the noise? What is the closest image do you think now to the original one? Why is the situation is different from part (a)?