MATH 225 Linear Algebra and Differential Equations

Fall 2007 MATLAB Homework 4 Solutions

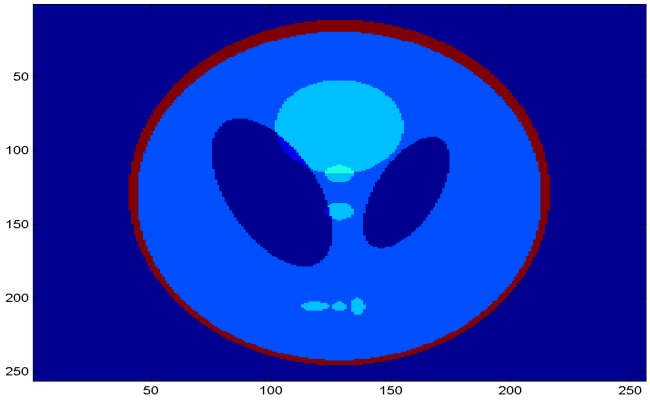


Figure 1: A head phantom image.

(a) Read this image into a matrix using the command

```
P=phantom('Modified Shepp-Logan');
```

Find the rank of the matrix *r*. Why dou you think it is lower than 256?

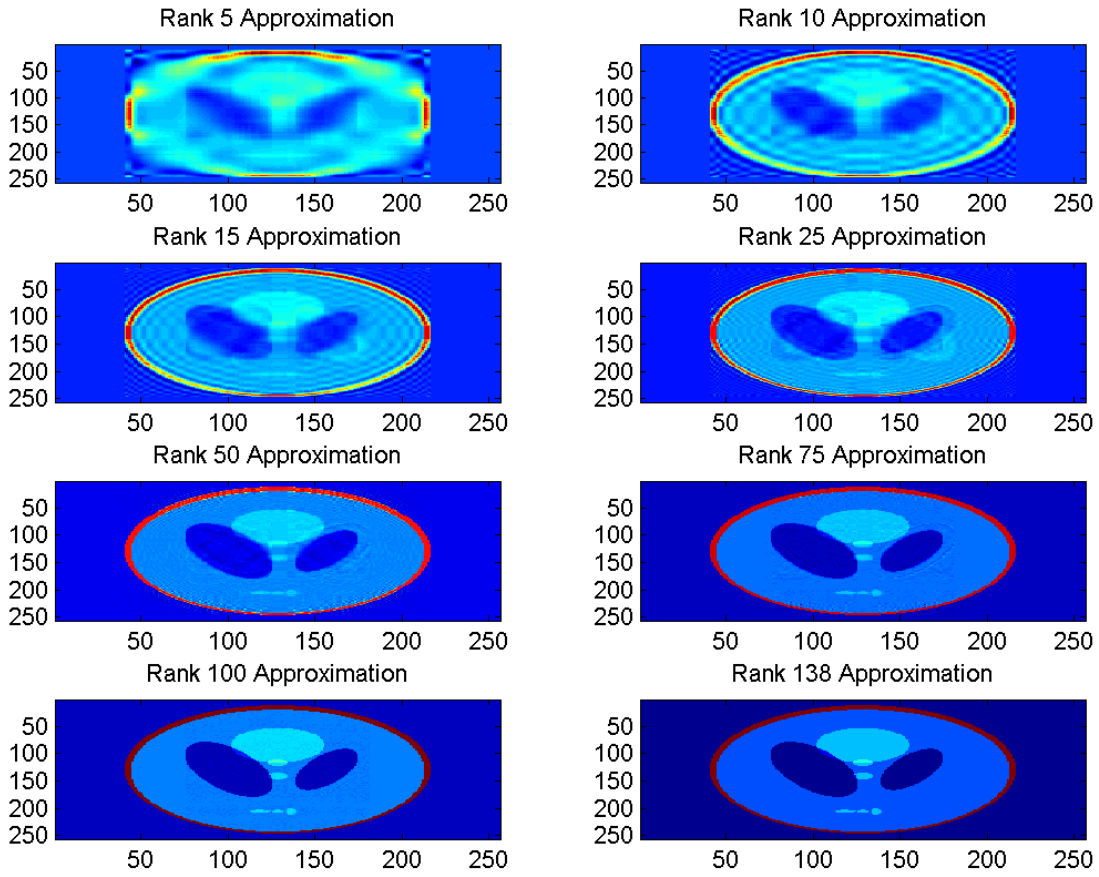
There are columns in the image which are the same, signaling dependent matrix columns.

Then, plot the low-rank approximations of the image for *k*= 5,10,15,25,50,75,100, and *r*.

Code: `P=phantom('Modified Shepp-Logan');`

```
r=rank(P);
[U,S,V]=svd(P);
kArray=[5 10 15 25 50 75 100 r];
n=length(kArray);
for i=1:n
    k=kArray(i);
    PLowRank=U(:,1:k)*S(1:k,1:k)*V(:,1:k)';
    subplot(4,2,i);
    imagesc(PLowRank);
    title(['Rank ' num2str(k) ' Approximation']);
end
print -dpng 'q1' % save figure as a png image
```

Yields



We see that we get better and better approximations as k approximates r. However, k=50 is a fairly good apprroximation.

(b) In real-life such images are almost always accompanied by a troublesome noise. Implement such a noise using the command `P=P+sigma*randn(size(P))`. What is the rank of this noisy matrix? (You should easily guess this with the help of your previous homework!).

It is 256 as you guessed.

Again insert this noisy image as well as its 7 low-rank approximations to a 4 by 2 plot.

Code:

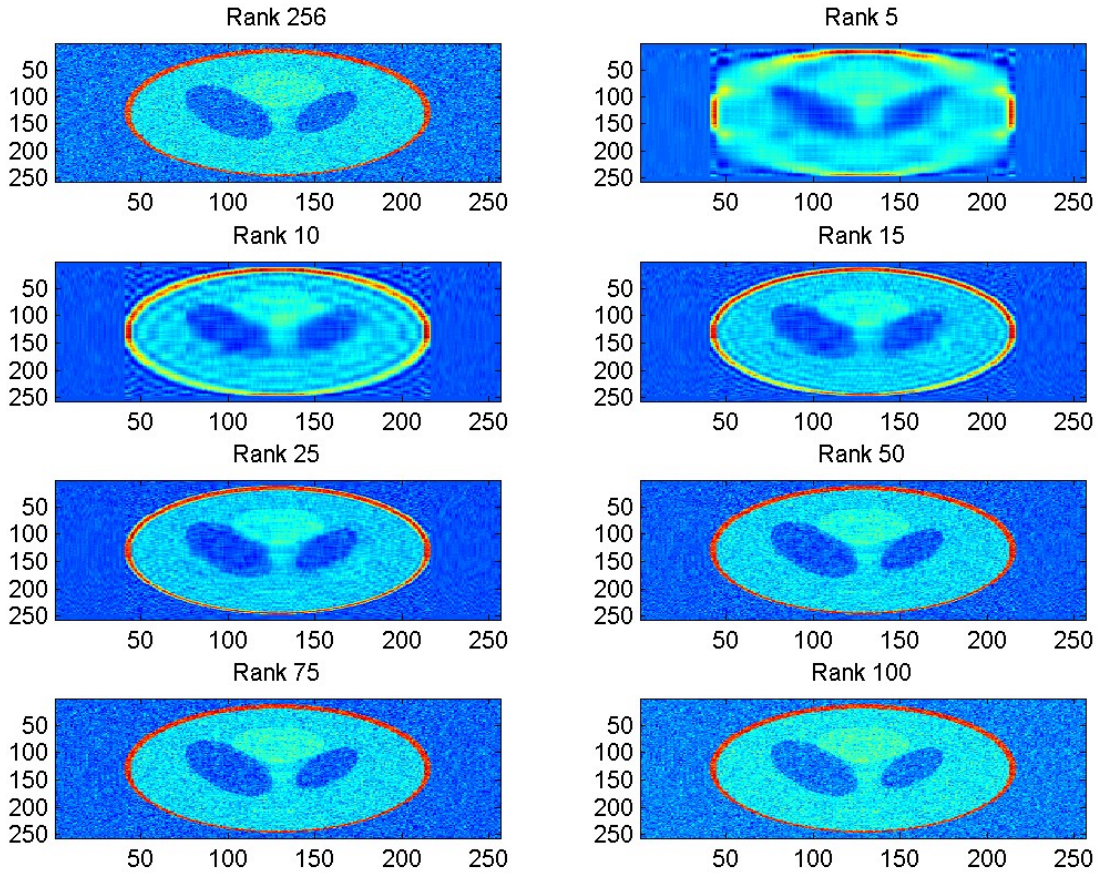
```
P=phantom('Modified Shepp-Logan');

sigma=0.1;
P=P+sigma*randn(size(P));
rankP=rank(P);

subplot(4,2,1);
imagesc(P);
title(['Rank ' num2str(rankP)]);
[U,S,V]=svd(P);
rs=[5 10 15 25 50 75 100];
n=length(rs);
for i=1:n
```

```
    r=rs(i);
    PLowRank=U(:,1:r)*S(1:r,1:r)*V(:,1:r)';
    subplot(4,2,i+1);
    imagesc(PLowRank);
    title(['Rank ' num2str(r)]);
end
print -dpng 'q2'
```

Yields



Which kind of effect did the low-rank approximations perform on the noise? What is the closest image do you think now to the original one? Why is the situation is different from part (a)?

The low-rank approximations actually filtered the noise. Now rank-10 or rank-15 approximations are better approximations to the original image since the effect of noise is less for these low-rank approximations.