

HW MATH227/10

1. Which of the following sets of vectors in \mathbf{P}_2 are linearly dependent

- (a) $2 - x + 4x^2$, $3 + 6x + 2x^2$, $2 + 10x - 4x^2$.
- (b) $3 + x + x^2$, $2 - x + 5x^2$, $4 - 3x^2$.
- (c) $6 - x^2$, $1 + x + 4x^2$.
- (d) $1 + 3x + 3x^2$, $x + 4x^2$, $5 + 6x + 3x^2$, $7 + 2x - x^2$.

2. Assume that \mathbf{v}_1 , \mathbf{v}_2 , and \mathbf{v}_3 are vectors in \mathbf{R}^3 that have their initial points at the origin. In each part determine whether the three vectors lie in a plane.

- (a) $\mathbf{v}_1 = (2, -2, 0)$, $\mathbf{v}_2 = (6, 1, 4)$, $\mathbf{v}_3 = (2, 0, -4)$.
- (b) $\mathbf{v}_1 = (-6, 7, 2)$, $\mathbf{v}_2 = (3, 2, 4)$, $\mathbf{v}_3 = (4, -1, 2)$.

3. For which values of λ do the following vectors form a linearly dependent set in \mathbf{R}^3

$$\mathbf{v}_1 = (\lambda, -\frac{1}{2}, -\frac{1}{2}), \quad \mathbf{v}_2 = (-\frac{1}{2}, \lambda, -\frac{1}{2}), \quad \mathbf{v}_3 = (-\frac{1}{2}, -\frac{1}{2}, \lambda).$$

4. Which of the following sets of vectors are bases for \mathbf{R}^2 .

- (a) $(2, 1)$, $(3, 0)$.
- (b) $(4, 1)$, $(-7, -8)$.
- (c) $(0, 0)$, $(1, 3)$.
- (d) $(3, 9)$, $(-4, -12)$.

5. Find the coordinate vector of \mathbf{w} relative to the basis $S = \{\mathbf{u}_1, \mathbf{u}_2\}$ for \mathbf{R}^2 .

- (a) $\mathbf{u}_1 = (1, 0)$, $\mathbf{u}_2 = (0, 1)$; $\mathbf{w} = (3, -7)$.
- (b) $\mathbf{u}_1 = (2, -4)$, $\mathbf{u}_2 = (3, 8)$; $\mathbf{w} = (1, 1)$.
- (c) $\mathbf{u}_1 = (1, 1)$, $\mathbf{u}_2 = (0, 2)$; $\mathbf{w} = (a, b)$.

6. Determine the dimensions of the following subspaces of \mathbf{R}^4

- (a) all vectors of the form $(a, b, c, 0)$.
- (b) all vectors of the form (a, b, c, d) , where $d = a + b$ and $c = a - b$.
- (c) all vectors of the form (a, b, c, d) , where $a = b = c = d$.